1 Such High Expectations

Suppose $X$ and $Y$ are independently drawn from a Geometric distribution with parameter $p$.

(a) Compute $E[\max(X,Y)]$.

(b) Compute $E[\min(X,Y)]$.

2 Number Game

Sinho and Vrettos are playing a game where they each choose an integer uniformly at random from $[0, 100]$, then whoever has the larger number wins (in the event of a tie, they replay). However, Vrettos doesn’t like losing, so he’s rigged his random number generator such that it instead picks randomly from the integers between Sinho’s number and 100. Let $S$ be Sinho’s number and $V$ be Vrettos’ number.

(a) What is $E[S]$?

(b) What is $E[V \mid S = s]$, where $s$ is any constant such that $0 \leq s \leq 100$?
3 Number of Ones

In this problem, we will revisit dice-rolling, except with conditional expectation. (Hint: for both of these subparts, the law of total expectation may be helpful.)

(a) If we roll a die until we see a 6, how many ones should we expect to see?

(b) If we roll a die until we see a number greater than 3, how many ones should we expect to see?