1 Dice Variance

(a) Let $X$ be a random variable representing the outcome of the roll of one fair 6-sided die. What is $\text{Var}(X)$?

(b) Let $Z$ be a random variable representing the average of $n$ rolls of a fair 6-sided die. What is $\text{Var}(Z)$?

2 Elevator Variance

A building has $n$ upper floors numbered 1, 2, . . . , $n$, plus a ground floor $G$. At the ground floor, $m$ people get on the elevator together, and each person gets off at one of the $n$ upper floors uniformly at random and independently of everyone else. What is the variance of the number of floors the elevator does not stop at?
3 Covariance

(a) We have a bag of 5 red and 5 blue balls. We take two balls uniformly at random from the bag without replacement. Let $X_1$ and $X_2$ be indicator random variables for the events of the first and second ball being red, respectively. What is $\text{cov}(X_1, X_2)$? Recall that $\text{cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X] \mathbb{E}[Y]$.

(b) Now, we have two bags A and B, with 5 red and 5 blue balls each. Draw a ball uniformly at random from A, record its color, and then place it in B. Then draw a ball uniformly at random from B and record its color. Let $X_1$ and $X_2$ be indicator random variables for the events of the first and second draws being red, respectively. What is $\text{cov}(X_1, X_2)$?