

1 Head Count

Consider a coin with $\mathbb{P}[\text{Heads}] = 2/5$. Suppose you flip the coin 20 times, and define X to be the number of heads.

(a) What is $\mathbb{P}[X = k]$, for some $0 \leq k \leq 20$?

(b) Name the distribution of X and what its parameters are.

(c) What is $\mathbb{P}[X \geq 1]$? Hint: You should be able to do this without a summation.

(d) What is $\mathbb{P}[12 \leq X \leq 14]$?

2 Family Planning

Mr. and Mrs. Brown decide to continue having children until they either have their first girl or until they have three children. Assume that each child is equally likely to be a boy or a girl, independent of all other children, and that there are no multiple births. Let G denote the numbers of girls that the Browns have. Let C be the total number of children they have.

(a) Determine the sample space, along with the probability of each sample point.

(b) Compute the joint distribution of G and C . Fill in the table below.

	$C = 1$	$C = 2$	$C = 3$
$G = 0$			
$G = 1$			

(c) Use the joint distribution to compute the marginal distributions of G and C and confirm that the values are as you'd expect. Fill in the tables below.

$\mathbb{P}(G = 0)$		$\mathbb{P}(C = 1)$	$\mathbb{P}(C = 2)$	$\mathbb{P}(C = 3)$
$\mathbb{P}(G = 1)$				

(d) Are G and C independent?

(e) What is the expected number of girls the Browns will have? What is the expected number of children that the Browns will have?

3 Binary Conditional Probabilities

Let us consider a sample space $\Omega = \{\omega_1, \dots, \omega_N\}$ of size $N > 2$, and two probability functions \mathbb{P}_1 and \mathbb{P}_2 on it. That is, we have two probability spaces: (Ω, \mathbb{P}_1) and (Ω, \mathbb{P}_2) .

If for every subset $A \subset \Omega$ of size $|A| = 2$ and every outcome $\omega \in \Omega$ it is true that $\mathbb{P}_1(\omega | A) = \mathbb{P}_2(\omega | A)$, then is it necessarily true that $\mathbb{P}_1(\omega) = \mathbb{P}_2(\omega)$ for all $\omega \in \Omega$? That is, if \mathbb{P}_1 and \mathbb{P}_2 are equal conditional on events of size 2, are they equal unconditionally? (*Hint*: Remember that probabilities must add up to 1.)