1 Strings

What is the number of strings you can construct given:

(a) n ones, and m zeroes?

(b) n_1 A's, n_2 B's and n_3 C's?

(c) $n_1, n_2, \ldots, n_k$ respectively of k different letters?

2 The Count

(a) How many of the first 100 positive integers are divisible by 2, 3, or 5?

(b) The Count is trying to choose his new 7-digit phone number. Since he is picky about his numbers, he wants it to have the property that the digits are non-increasing when read from left to right. For example, 9973220 is a valid phone number, but 9876545 is not. How many choices for a new phone number does he have?

(c) Now instead of non-increasing, they must be strictly decreasing. So 9983220 is no longer valid, while 9753210 is valid. How many choices for a new phone number does he have now?

3 Digits

(a) How many 7-digit numbers have no two adjacent digits equal?

(b) How many 5-digit palindromes are there? (A palindrome is a number that reads the same way forwards and backwards. For example, 27872 and 48484 are palindromes, but 28389 and 12541 are not.)

4 Divisor Graph Colorings

Define G where we have $V = \{2,3,4,5,6,7,8,9\}$, and we add an edge between vertex i and vertex j if i divides j, or j divides i.

(a) Draw G.

(b) Explain why we cannot vertex-color G with only 2 colors.

(c) How many ways can we vertex-color G with 3 colors?