1 Variance

(a) Let X be a random variable representing the outcome of the roll of one fair 6-sided die. What is $\operatorname{Var}(X)$?

(b) Let Z be a random variable representing the average of n rolls of a fair die 6-sided die. What is $\operatorname{Var}(Z)$?

2 Coupon Collector Variance

It's that time of the year again - Safeway is offering its Monopoly Card promotion. Each time you visit Safeway, you are given one of n different Monopoly Cards with equal probability. You need to collect them all to redeem the grand prize.

Let X be the number of visits you have to make before you can redeem the grand prize. Show that $\operatorname{Var}(X) = n^2\left(\sum_{i=1}^{n} i^{-2}\right) - \mathbb{E}(X)$. *[Hint: Try to break this problem down using indicators as with the coupon collector's problem. Are the indicators independent?]*

3 Shuttles and Taxis at Airport

In front of terminal 3 at San Francisco Airport is a pickup area where shuttles and taxis arrive according to a Poisson process. The shuttles arrive at a rate $\lambda_1 = 1/20$ (i.e. 1 shuttle per 20 minutes) and the taxis arrive at a rate $\lambda_2 = 1/10$ (i.e. 1 taxi per 10 minutes) starting at 00:00. The shuttles and the taxis arrive independently.

(a) What is the distribution of the following:

  (i) The number of taxis that arrive between times 00:00 and 00:20?

  (ii) The number of shuttles that arrive between times 00:00 and 00:20?

  (iii) The total number of pickup vehicles that arrive between times 00:00 and 00:20?

(b) What is the probability that exactly 1 shuttle and 3 taxis arrive between times 00:00 and 00:20?

(c) Given that exactly 1 pickup vehicle arrived between times 00:00 and 00:20, what is the conditional probability that this vehicle was a taxi?

(d) Suppose you reach the pickup area at 00:20. You learn that you missed 3 taxis and 1 shuttle in those 20 minutes. What is the probability that you need to wait for more than 10 mins until either a shuttle or a taxi arrives?