1 Elevator Variance

A building has $n$ upper floors numbered 1, 2, ..., $n$, plus a ground floor $G$. At the ground floor, $m$ people get on the elevator together, and each person gets off at one of the $n$ upper floors uniformly at random and independently of everyone else. What is the variance of the number of floors the elevator does not stop at?

2 Covariance

(a) We have a bag of 5 red and 5 blue balls. We take two balls uniformly at random from the bag without replacement. Let $X_1$ and $X_2$ be indicator random variables for the events of the first and second ball being red, respectively. What is $\text{cov}(X_1, X_2)$? Recall that $\text{cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$. 
(b) Now, we have two bags A and B, with 5 red and 5 blue balls each. Draw a ball uniformly at random from A, record its color, and then place it in B. Then draw a ball uniformly at random from B and record its color. Let $X_1$ and $X_2$ be indicator random variables for the events of the first and second draws being red, respectively. What is $\text{cov}(X_1, X_2)$?

3 Number Game

Sinho and Vrettos are playing a game where they each choose an integer uniformly at random from $[0, 100]$, then whoever has the larger number wins (in the event of a tie, they replay). However, Vrettos doesn’t like losing, so he’s rigged his random number generator such that it instead picks randomly from the integers between Sinho’s number and 100. Let $S$ be Sinho’s number and $V$ be Vrettos’ number.

(a) What is $E[S]$?

(b) What is $E[V \mid S = s]$, where $s$ is any constant such that $0 \leq s \leq 100$?
(c) What is $E[V]$?