Discussion 1A

CS 70, Summer 2024

1 Translation

(a) **(i)** $(\exists x \in \mathbb{R})(x \notin \mathbb{Q})$. True.

(ii) $(\forall n \in \mathbb{N})\big(6 \mid n \implies (3 \mid n \land 2 \mid n)\big)$. True.

(b) **(i)** All rational numbers are integers. False.

(ii) Every integer is the difference of two natural numbers. True.

2 Truth Tables

(a) **(i)** True, using the truth tables.

(ii) True, using the truth tables.

(b) **(i)** Equivalent, using the truth tables.

(ii) Equivalent, using the truth tables.

3 Logical Implication

(a) True. Suppose that $\exists x P(x) \lor \exists x Q(x)$. Then either $\exists x P(x)$ or $\exists x Q(x)$.

If the first is true, existential instantiation gets some a such that $P(a)$. But if $P(a)$, it's definitely true that $P(a) \lor Q(a)$. So by existential generalization, $\exists x(P(x) \lor Q(x))$.

If the second is true, existential instantiation gets some b such that $Q(b)$. But if $Q(b)$, it's definitely true that $P(b) \lor Q(b)$. So by existential generalization, $\exists x(P(x) \lor Q(x))$.

In either case, $\exists x(P(x) \lor Q(x))$. So we have that it is true regardless of which one actually was true.

Since by assuming $\exists x P(x) \lor \exists x Q(x)$ we were able to get that $\exists x(P(x) \lor Q(x))$, we have that

$$\exists x P(x) \lor \exists x Q(x) \implies \exists x(P(x) \lor Q(x)).$$

(b) False. Consider a model with two elements, a and b, where $P(a)$ and $Q(b)$.

(c) False. Consider a model with two elements, a and b, where $R(a, b)$ and $R(b, a)$.