

Discussion 1A

CS 70, Summer 2024

1 Translation

(a) Convert the following statements from English to first-order logic. For each statement, state whether it is true or false with brief justification.

(i) There are real numbers which aren't rational.

(ii) A natural number is divisible by 6 only if it is divisible by both 3 and 2.

(b) Convert the following statements from first-order logic to English. For each statement, state whether it is true or false with brief justification.

(i) $(\forall q \in \mathbb{Q})(q \in \mathbb{Z})$.

(ii) $(\forall k \in \mathbb{Z})(\exists n, m \in \mathbb{N})(k = n - m)$.

2 Truth Tables

(a) Prove whether each of the following propositions is tautologically true.

(i) $((P \vee Q) \wedge \neg P) \implies Q$.

(ii) $\neg(P \vee Q) \implies (\neg P \vee \neg Q)$.

(b) Prove whether each of the following pairs of propositions is tautologically equivalent.

(i) $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge (P \vee R)$.

(ii) $\neg(P \implies Q)$ and $P \wedge \neg Q$.

3 Logical Implication

Determine which of the following logical implications are true. For those that are true, prove that they are; for those that are false, provide a model in which they are false.

(a) $\exists x P(x) \vee \exists x Q(x) \implies \exists x (P(x) \vee Q(x))$.

(b) $\exists x P(x) \wedge \exists x Q(x) \implies \exists x (P(x) \wedge Q(x))$.

(c) $\forall x \exists y R(x, y) \implies \exists y \forall x R(x, y)$.