

Discussion 5C

CS 70, Summer 2024

1 Pullout Balls

Suppose you have a bag containing four balls numbered 1, 2, 3, 4.

- (a) You perform the following experiment: pull out a single ball and record its number. What is the expected value of the number that you record?

- (b) You repeat the experiment from part (a), except this time you pull out two balls together and record the product of their numbers. What is the expected value of the total that you record?

2 Number Game

Sinho and Vrettos are playing a game where they each choose an integer uniformly at random from $[0, 100]$, then whoever has the larger number wins (in the event of a tie, they replay). However, Vrettos doesn't like losing, so he's rigged his random number generator such that it instead picks randomly from the integers between Sinho's number and 100. Let S be Sinho's number and V be Vrettos' number.

- (a) What is $E[S]$?

- (b) What is $E[V \mid S = s]$, where s is any constant such that $0 \leq s \leq 100$?

(c) What is $E[V]$?

3 Linearity

Solve each of the following problems using linearity of expectation. Explain your methods clearly.

(a) In an arcade, you play game A 10 times and game B 20 times. Each time you play game A , you win with probability $1/3$ (independently of the other times), and if you win you get 3 tickets (redeemable for prizes), and if you lose you get 0 tickets. Game B is similar, but you win with probability $1/5$, and if you win you get 4 tickets. What is the expected total number of tickets you receive?

(b) A monkey types at a 26-letter keyboard with one key corresponding to each of the lower-case English letters. Each keystroke is chosen independently and uniformly at random from the 26 possibilities. If the monkey types 1 million letters, what is the expected number of times the sequence “book” appears? (*Hint*: Consider where the sequence “book” can appear in the string.)

4 Number of Ones

If we roll a die until we see a 6, how many ones should we expect to see? (Hint: You may use the self-referencing trick.)