

## Discussion 7B

*CS 70, Summer 2024*

### 1 Random Variables with Density

Suppose that the random variable  $X$  has the density

$$f(x) = \begin{cases} c(1 - x^2) & -1 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find  $c$ .

(b) Find the cumulative distribution function of  $X$ .

(c) Find  $P(|X| < 0.1)$ .

(d) Find  $E[X]$ .

(e) Find  $\text{Var}[X]$ .

## 2 Exponential Minima

Let  $X_1, \dots, X_n$  be independent random variables such that  $X_i \stackrel{\text{ind}}{\sim} \text{Exponential}(\lambda_i)$  for each  $i \in \{1, \dots, n\}$  and  $\lambda_1, \dots, \lambda_n > 0$ . Find the distribution of  $V = \min\{X_1, \dots, X_n\}$ . Identify it as one of the famous ones and provide its name and parameters.

## 3 Competing Uniforms

(a) Suppose  $X \sim \text{Uniform}(0, 2)$  and  $Y \sim \text{Uniform}(1, 2)$  are independent. Find  $P(X > Y)$ .

(b) For  $n \in \mathbb{Z}^+$ , suppose  $X \sim \text{Uniform}\{0, \dots, n\}$  and  $Y \sim \text{Uniform}(0, n)$  are independent. Find  $P(X > Y)$ .

## 4 Functions of Uniforms

In this question, we'll find the distributions of some functions of uniform random variables. Let  $U \sim \text{Uniform}(0, 1)$  and  $V \sim \text{Uniform}(-1, 1)$ .

For each of the following, remember to start with the possible values.

(a) Find the distribution of  $U^2$ .

(b) Find the distribution of  $|V|$ .

(c) Find the distribution of  $1/U$ .