70: Discrete Math and Probability Theory

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Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
Logic and Proofs!
Induction $\equiv$ Recursion.

What can computers do?
Work with discrete objects.
Discrete Math $\implies$ immense application.

Computers learn and interact with the world?
E.g. machine learning, data analysis, robotics, ...
Probability!

My hopes and dreams.

We teach you to think more clearly and more powerfully.

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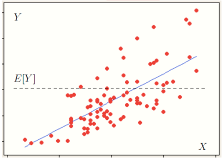
We teach you to think more clearly and more powerfully.

..And to deal clearly with uncertainty itself.

Probability Unit

- How can we predict unknown future events (e.g., gambling profit, next week rainfall, traffic congestion, …)?
  - Constructive Models: Model the overall system (including the sources of uncertainty).
    - For modeling uncertainty, we'll develop probabilistic models and techniques for analyzing them.
  - Deductive Models: Extract the "trend" from the previous outcomes (e.g., linear regression).



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Explains policies, has office hours, homework, midterm dates, etc.

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Questions

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Questions $\implies$ piazza:

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Read it!!!!
Announcements, logistics, critical advice.

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.

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Suppose we have four cards on a table:

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- Suppose you see that Alice went to Baltimore, Bob drove,
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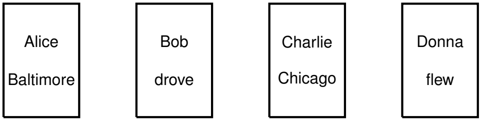

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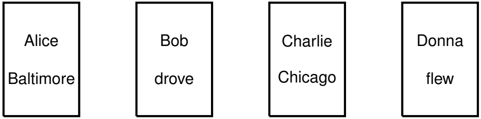

- Which cards must you flip to test the theory?

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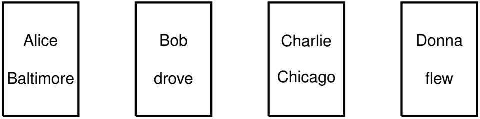

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Answer: (A), (B), (C), (D).

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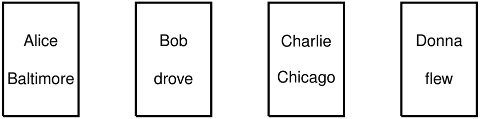

- Which cards must you flip to test the theory?

Answer: (A), (B), (C), (D). Later.

CS70: Lecture 1. Outline.

Today: Note 1.

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The language of proofs!

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The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational
2+2 = 4
2+2 = 3
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Any even > 2 is sum of 2 primes
$4 + 5$
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Propositional Forms.

Put propositions together to make another...

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Put them together..

Propositions:

P_1 - Person 1 rides the bus.

Put them together..

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P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

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Put them together..

Propositions:
P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
....

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

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$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

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Can person 3 ride the bus?

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Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

Put them together..

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Can person 3 and person 4 ride the bus together?

Put them together..

Propositions:

P_1 - Person 1 rides the bus.
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But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

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We need a way to keep track!

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True if
both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	
F	T	
F	F	

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Check: $\wedge$ and $\vee$ are commutative.

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DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q)$

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P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
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F	F	T	T

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T	F	F	F
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T	T	F	F
T	F	F	F
F	T	F	F
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DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$ $\qquad$ $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Is $(T \wedge Q) \equiv Q$?

P	Q	$P \vee Q$
T	T	T
T	F	T
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F	F	F

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes?

P	Q	$P \vee Q$
T	T	T
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Quick Questions

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T	T	T
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Quick Questions

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T	T	T
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P	Q	$P \vee Q$
T	T	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes!

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
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P	Q	$P \vee Q$
T	T	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
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What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

Quick Questions

P	Q	$P \wedge Q$
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P	Q	$P \wedge Q$
T	T	T
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F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

What is $(F \vee Q)$? Q

Distributive?

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)?$$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$,

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

Distributive?

$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$?

Simplify: $(T \land Q) \equiv Q$, $(F \land Q) \equiv F$.

Cases:

P is True .

LHS: $T \land (Q \lor R) \equiv (Q \lor R)$.

RHS: $(T \land Q) \lor (T \land R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
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P is False .

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$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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P is True .
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P is False .
LHS: $F \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

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P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$,

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

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P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Distributive?

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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Distributive?

$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$?

Simplify: $(T \land Q) \equiv Q$, $(F \land Q) \equiv F$.

Cases:

P is True .

LHS: $T \land (Q \lor R) \equiv (Q \lor R)$.

RHS: $(T \land Q) \lor (T \land R) \equiv (Q \lor R)$.

P is False .

LHS: $F \land (Q \lor R) \equiv F$.

RHS: $(F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F$.

$P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)$?

Simplify: $T \lor Q \equiv T$, $F \lor Q \equiv Q$. ...

Foil 1:

$(A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D)$?

Foil 2:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:

$(A \wedge B) \vee (C \wedge D) \equiv (A \vee C) \wedge (A \vee D) \wedge (B \vee C) \wedge (B \vee D)$?

Implication.

$P \implies Q$ interpreted as

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If P, then Q.

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Examples:

Statement: If you stand in the rain, then you'll get wet.

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Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"

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Examples:

Statement: If you stand in the rain, then you'll get wet.
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Examples:

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Statement: "Stand in the rain"

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Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
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Statement: "Stand in the rain"
Can conclude: "you'll get wet."

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Statement:
If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

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P = "a right triangle has sidelengths $a \leq b \leq c$",
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Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

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only is False if P is True and Q is False .

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Some Fun: use propositional formulas to describe implication?

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Some Fun: use propositional formulas to describe implication?
$((P \implies Q) \wedge P) \implies Q$.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.

Implication and English.

$P \implies Q$

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- If P, then Q.
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Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.

  Just reversing the order.
- P only if Q.

  Remember if P is true then Q must be true.

  this suggests that P can only be true if Q is true.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
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  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.

Implication and English.

$P \implies Q$

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- If P, then Q.
- Q if P.

  Just reversing the order.
- P only if Q.

  Remember if P is true then Q must be true.

  this suggests that P can only be true if Q is true.

  since if Q is false P must have been false.
- P is sufficient for Q.

Implication and English.

$P \implies Q$

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  Just reversing the order.
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Implication and English.

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  to conclude that Q is true.

  Example: Showing $n > 4$ is sufficient for showing $n > 3$.

Implication and English.

$P \implies Q$

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- Q if P.
  Just reversing the order.
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  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
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- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.
  Example: Showing $n > 4$ is sufficient for showing $n > 3$.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.
  Example: Showing $n > 4$ is sufficient for showing $n > 3$.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.
  Example: It is necessary that $n > 3$ for $n > 4$.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	
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Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P \vee Q$
T	T	
T	F	
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
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T	T	T
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T	T	T
T	F	F
F	T	T
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P	Q	$\neg P \lor Q$
T	T	T
T	F	F
F	T	T
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$\neg P \lor Q \quad \equiv P \implies Q$.

Truth Table: implication.

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T	T	T
T	F	F
F	T	T
F	F	T

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T	T	T
T	F	F
F	T	T
F	F	T

$\neg P \vee Q \quad \equiv P \implies Q.$

These two propositional forms are logically equivalent!

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute.

Contrapositive, Converse

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- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
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Next: Statements about boolean valued functions!!

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Universe examples include..

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- $\mathbb{Z}^+$ (positive integers)
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Proposition has **universe**: "the natural numbers".

Universe examples include..

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
- $\mathbb{Z}^+$ (positive integers)
- $\mathbb{R}$ (real numbers)
- Any set: $S = \{Alice, Bob, Charlie, Donna\}$.
- See note 0 for more!

Other proposition notation(for discussion):
"$d|n$" means d divides n
or $\exists k \in \mathbb{Z}, n = kd$.
$2|4$? True.
$4|2$? False.

Back to: Wason's experiment:1

Theory:

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Theory: "If a person travels to Chicago, he/she flies."

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Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x)$

Back to: Wason's experiment:1

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Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

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$Chicago(A)$ = False .

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$Chicago(A)$ = False . Do we care about $Flew(A)$?
No.

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No. $Chicago(A) \implies Flew(A)$ is true.
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Yes.

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So $Chicago(Bob)$ must be False .

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$Chicago(C)$ = True .

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So $Chicago(Bob)$ must be False .

$Chicago(C)$ = True . Do we care about $Flew(C)$?
Yes.

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So $Chicago(Bob)$ must be False .

$Chicago(C)$ = True . Do we care about $Flew(C)$?
Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

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Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

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Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

$Flew(D)$ = True . Do we care about $Chicago(D)$?
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No. $Chicago(D) \implies Flew(D)$ is true if $Flew(D)$ is true.

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Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

$Flew(D)$ = True . Do we care about $Chicago(D)$?
No. $Chicago(D) \implies Flew(D)$ is true if $Flew(D)$ is true.

Only have to turn over cards for Bob and Charlie.

More for all quantifiers examples.

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- "Square of any natural number greater than 5 is greater than 25."

More for all quantifiers examples.

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- "Square of any natural number greater than 5 is greater than 25."

  $$(\forall x \in N)(x > 5 \implies x^2 > 25).$$

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  Idea alert: Restrict domain using implication.

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  Idea alert: Restrict domain using implication.

Later we may omit universe if clear from context.

Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

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- In English: “there is a natural number that is the square of every natural number”.

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Quantifiers....negation...DeMorgan again.

Consider

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What we do in this course! We consider claims.

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Claim: $(\forall x)\ P(x)$

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Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."

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What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.

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Consider

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English: there is an x in S where $P(x)$ does not hold.

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Next Time: proofs!