Programming + Microprocessors
≡
Superpower!

What are your super powerful programs/processors doing?

Logic and Proofs!

Induction
≡
Recursion.

What can computers do?

Work with discrete objects.

Discrete Math =⇒ immense application.

Computers learn and interact with the world?
E.g. machine learning, data analysis, robotics, ...

Probability!
Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
   Logic and Proofs!
   Induction $\equiv$ Recursion.

What can computers do?
   Work with discrete objects.
   **Discrete Math** $\implies$ immense application.

Computers learn and interact with the world?
   E.g. machine learning, data analysis, robotics, ...
   **Probability!**
My hopes and dreams.

We teach you to think more clearly and more powerfully.
My hopes and dreams.

We teach you to think more clearly and more powerfully.
..And to deal clearly with uncertainty itself.
Probability Unit

• How can we predict unknown future events (e.g., gambling profit, next week rainfall, traffic congestion, ...)?
  – Constructive Models: Model the overall system (including the sources of uncertainty).
    ▪ For modeling uncertainty, we’ll develop probabilistic models and techniques for analyzing them.
  – Deductive Models: Extract the “trend” from the previous outcomes (e.g., linear regression).
Course Webpage: http://www.eecs70.org/
Course Webpage: http://www.eecs70.org/

Explains policies, has office hours, homework, midterm dates, etc.
Course Webpage: http://www.eecs70.org/

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.
Course Webpage: [http://www.eecs70.org/](http://www.eecs70.org/)

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.

midterm.
Course Webpage: http://www.eecs70.org/
Explains policies, has office hours, homework, midterm dates, etc.
One midterm, final.
midterm.

Questions ⇒ piazza: Logistics, etc.
Content Support: other students!
Plus Piazza hours.
Weekly Post. It's weekly. Read it!!!!
Announcements, logistics, critical advice.
Course Webpage: http://www.eecs70.org/

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.

midterm.

Questions
Course Webpage: http://www.eecs70.org/

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.

Questions ⇒ piazza:
Course Webpage: http://www.eecs70.org/

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.

Questions ➞ piazza:

- **Logistics, etc.**
- **Content Support: other students!**
  - Plus Piazza hours.
Course Webpage: [http://www.eecs70.org/](http://www.eecs70.org/)

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.

Questions ➞ piazza:

- **Logistics, etc.**
- **Content Support: other students!**
  - Plus Piazza hours.

Weekly Post.
Course Webpage: [http://www.eecs70.org/](http://www.eecs70.org/)

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.

Questions ➔ piazza:

- **Logistics, etc.**
- **Content Support: other students!**
  - Plus Piazza hours.

**Weekly** Post.

It’s **weekly**.
Course Webpage: http://www.eecs70.org/

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.

Questions ➞ piazza:

- Logistics, etc.
- Content Support: other students!
  - Plus Piazza hours.

Weekly Post.

It’s weekly.
Read it!!!!
Course Webpage: http://www.eecs70.org/

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.

Questions ➞ piazza:
   Logistics, etc.
   Content Support: other students!
   Plus Piazza hours.

Weekly Post.

It’s weekly.
Read it!!!!
Announcements, logistics, critical advice.
Wason’s experiment: 1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
Wason’s experiment: 1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person’s destination on one side, and mode of travel.

Answer: (A), (B), (C), (D). Later.
Wason’s experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person’s destination on one side, and mode of travel.
- Consider the theory:
Wason’s experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person’s destination on one side, and mode of travel.
- Consider the theory: “If a person travels to Chicago, they flies.”
Wason’s experiment: 1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person’s destination on one side, and mode of travel.
- Consider the theory: “If a person travels to Chicago, they flies.”
Wason’s experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person’s destination on one side, and mode of travel.
- Consider the theory: “If a person travels to Chicago, they flies.”
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

<table>
<thead>
<tr>
<th>Alice</th>
<th>Bob</th>
<th>Charlie</th>
<th>Donna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baltimore</td>
<td>drove</td>
<td>Chicago</td>
<td>flew</td>
</tr>
</tbody>
</table>
Wason’s experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person’s destination on one side, and mode of travel.
- Consider the theory: “If a person travels to Chicago, they flies.”
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards must you flip to test the theory?
Wason’s experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person’s destination on one side, and mode of travel.
- Consider the theory: “If a person travels to Chicago, they flies.”
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards must you flip to test the theory?

Answer: (A), (B), (C), (D).
Wason’s experiment:1

Suppose we have four cards on a table:

▶ 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
▶ Card contains person’s destination on one side, and mode of travel.
▶ Consider the theory: “If a person travels to Chicago, they flies.”
▶ Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

![Card placeholders]

▶ Which cards must you flip to test the theory?

Answer: (A), (B), (C), (D). Later.
Today: Note 1.
Today: Note 1. Note 0 is background.
Today: Note 1.  Note 0 is background. Do read it.
Today: Note 1.  Note 0 is background. Do read it.
The language of proofs!
Today: Note 1. Note 0 is background. Do read it.

The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan’s Laws
Propositions: Statements that are true or false.

√2 is irrational
2 + 2 = 4
2 + 2 = 3
826th digit of pi is 4
Johnny Depp is a good actor
Any even > 2 is sum of 2 primes
4 + 5
x + x
Alice travelled to Chicago
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
2+2 = 4
2+2 = 3
826th digit of pi is 4
Johnny Depp is a good actor
Any even > 2 is sum of 2 primes
4 + 5
x + x
Alice travelled to Chicago
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
\[ 2 + 2 = 4 \]
\[ 2 + 2 = 3 \]
\[ 826\text{th digit of pi is 4} \]
\[ \text{Johnny Depp is a good actor} \]
\[ \text{Any even } > 2 \text{ is sum of 2 primes} \]
\[ 4 + 5 \]
\[ x + x \]
\[ \text{Alice travelled to Chicago} \]
Propositions: Statements that are true or false.

\sqrt{2} \text{ is irrational} \quad \text{Proposition} \quad \text{True}

2 + 2 = 4 \quad \text{Proposition}

2 + 2 = 3 \quad \text{Proposition}

826\text{th digit of pi is 4} \quad \text{Proposition}

Johnny Depp is a good actor \quad \text{Not Proposition}

Any even \ > 2 \ is \ sum \ of \ 2 \ primes \quad \text{Proposition}

4 + 5 \quad \text{Not Proposition}

x + x \quad \text{Not Proposition}

Alice travelled to Chicago
Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational

$2 + 2 = 4$

$2 + 2 = 3$

826th digit of pi is 4

Johnny Depp is a good actor

Any even $> 2$ is sum of 2 primes

$4 + 5$

$x + x$

Alice travelled to Chicago
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
\[ 2 + 2 = 4 \] Proposition True
\[ 2 + 2 = 3 \] Proposition True
\[ 826\text{th digit of pi is 4} \] Proposition False
\[ \text{Johnny Depp is a good actor} \] Proposition False
\[ \text{Any even > 2 is sum of 2 primes} \] Proposition False
\[ 4 + 5 \] Proposition
\[ x + x \] Proposition
\[ \text{Alice travelled to Chicago} \] Proposition
Propositions: Statements that are true or false.

- $\sqrt{2}$ is irrational: Proposition, True
- $2+2 = 4$: Proposition, True
- $2+2 = 3$: Proposition, False
- 826th digit of pi is 4: Proposition, False
- Johnny Depp is a good actor: Not Proposition
- Any even $> 2$ is sum of 2 primes: Proposition, False
- $4 + 5$: Not Proposition
- $x + x$: Not Proposition
- Alice travelled to Chicago: Proposition, False
Propositions: Statements that are true or false.

- $\sqrt{2}$ is irrational
- $2 + 2 = 4$ (Proposition, True)
- $2 + 2 = 3$ (Proposition, False)
- 826th digit of pi is 4
- Johnny Depp is a good actor
- Any even $> 2$ is sum of 2 primes
- $4 + 5$
- $x + x$
- Alice travelled to Chicago
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
Proposition True

\[ 2 + 2 = 4 \]
Proposition True

\[ 2 + 2 = 3 \]
Proposition False

\[ 826\text{th digit of pi is 4} \]
Proposition False

Johnny Depp is a good actor
Proposition False

Any even \( > 2 \) is sum of 2 primes
Proposition False

\[ 4 + 5 \]
Not a Proposition

\[ x + x \]
Not a Proposition

Alice travelled to Chicago
Proposition
Propositions: Statements that are true or false.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Proposition</th>
<th>Truth Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{2}$ is irrational</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>$2 + 2 = 4$</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>$2 + 2 = 3$</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>826th digit of $\pi$ is 4</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>Johnny Depp is a good actor</td>
<td>Not Proposition</td>
<td></td>
</tr>
<tr>
<td>Any even $&gt; 2$ is sum of 2 primes</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>$4 + 5$</td>
<td>Not a Proposition</td>
<td></td>
</tr>
<tr>
<td>$x + x$</td>
<td>Not a Proposition</td>
<td></td>
</tr>
<tr>
<td>Alice travelled to Chicago</td>
<td>Proposition</td>
<td>False</td>
</tr>
</tbody>
</table>
Propositions: Statements that are true or false.

\sqrt{2} \text{ is irrational} \quad \text{Proposition} \quad \text{True}

2 + 2 = 4 \quad \text{Proposition} \quad \text{True}

2 + 2 = 3 \quad \text{Proposition} \quad \text{False}

826th digit of pi is 4 \quad \text{Proposition} \quad \text{False}

Johnny Depp is a good actor \quad \text{Not Proposition} \quad \text{False}

Any even \( > 2 \) is sum of 2 primes \quad \text{Proposition}

4 + 5 \quad \text{Proposition}

x + x \quad \text{Not Proposition}

Alice travelled to Chicago

I love you. \quad \text{Hmmm.} \quad \text{It's complicated.}

Again: “value” of a proposition is... True or False
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \] Proposition True
\[ 2+2 = 4 \] Proposition True
\[ 2+2 = 3 \] Proposition False
\[ 826\text{th digit of pi is } 4 \] Proposition False
Johnny Depp is a good actor Not Proposition False
Any even \( > 2 \) is sum of 2 primes Proposition False
\[ 4 + 5 \] Proposition False
\[ x + x \] Proposition False
Alice travelled to Chicago Proposition False
<table>
<thead>
<tr>
<th>Proposition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{2}$ is irrational</td>
<td>True</td>
</tr>
<tr>
<td>$2 + 2 = 4$</td>
<td>True</td>
</tr>
<tr>
<td>$2 + 2 = 3$</td>
<td>False</td>
</tr>
<tr>
<td>826th digit of pi is 4</td>
<td>False</td>
</tr>
<tr>
<td>Johnny Depp is a good actor</td>
<td>False</td>
</tr>
<tr>
<td>Any even $&gt; 2$ is sum of 2 primes</td>
<td>False</td>
</tr>
<tr>
<td>$4 + 5$</td>
<td></td>
</tr>
<tr>
<td>$x + x$</td>
<td></td>
</tr>
<tr>
<td>Alice travelled to Chicago</td>
<td></td>
</tr>
</tbody>
</table>
Propositions: Statements that are true or false.

- $\sqrt{2}$ is irrational: Proposition, True
- $2 + 2 = 4$: Proposition, True
- $2 + 2 = 3$: Proposition, False
- 826th digit of pi is 4: Proposition, False
- Johnny Depp is a good actor: Not Proposition, False
- Any even $> 2$ is sum of 2 primes: Proposition, False
- $4 + 5$: Not Proposition.
- $x + x$: Not a Proposition.
- Alice travelled to Chicago: Not a Proposition.
# Propositions: Statements that are true or false.

<table>
<thead>
<tr>
<th>Proposition</th>
<th>True/False</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{2}$ is irrational</td>
<td>True</td>
</tr>
<tr>
<td>$2+2 = 4$</td>
<td>True</td>
</tr>
<tr>
<td>$2+2 = 3$</td>
<td>False</td>
</tr>
<tr>
<td>826th digit of pi is 4</td>
<td>False</td>
</tr>
<tr>
<td>Johnny Depp is a good actor</td>
<td>False</td>
</tr>
<tr>
<td>Any even $&gt; 2$ is sum of 2 primes</td>
<td>False</td>
</tr>
<tr>
<td>$4 + 5$</td>
<td>False</td>
</tr>
<tr>
<td>$x + x$</td>
<td>False</td>
</tr>
<tr>
<td>Alice travelled to Chicago</td>
<td>True</td>
</tr>
</tbody>
</table>
## Propositions: Statements that are true or false.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{2}$ is irrational</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>$2+2 = 4$</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>$2+2 = 3$</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>826th digit of pi is 4</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>Johnny Depp is a good actor</td>
<td>Not Proposition</td>
<td>False</td>
</tr>
<tr>
<td>Any even $&gt; 2$ is sum of 2 primes</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>$4 + 5$</td>
<td>Not Proposition</td>
<td></td>
</tr>
<tr>
<td>$x + x$</td>
<td>Not a Proposition.</td>
<td></td>
</tr>
<tr>
<td>Alice travelled to Chicago</td>
<td>Proposition</td>
<td>False</td>
</tr>
</tbody>
</table>
Propositions: Statements that are true or false.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{2}$ is irrational</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>$2+2 = 4$</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>$2+2 = 3$</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>826th digit of pi is 4</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>Johnny Depp is a good actor</td>
<td>Not Proposition</td>
<td>False</td>
</tr>
<tr>
<td>Any even $&gt; 2$ is sum of 2 primes</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>$4 + 5$</td>
<td>Not Proposition</td>
<td>False</td>
</tr>
<tr>
<td>$x + x$</td>
<td>Not a Proposition.</td>
<td>False</td>
</tr>
<tr>
<td>Alice travelled to Chicago</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>I love you.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Propositions: Statements that are true or false.

- $\sqrt{2}$ is irrational: True
- $2+2 = 4$: True
- $2+2 = 3$: False
- 826th digit of pi is 4: False
- Johnny Depp is a good actor: False
- Any even $> 2$ is sum of 2 primes: False
- $4 + 5$: Not Proposition
- $x + x$: Not a Proposition
- Alice travelled to Chicago: False
- I love you: Hmmm.
Propositions: Statements that are true or false.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Proposition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{2}$ is irrational</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>$2+2 = 4$</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>$2+2 = 3$</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>826th digit of pi is 4</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>Johnny Depp is a good actor</td>
<td>Not Proposition</td>
<td>False</td>
</tr>
<tr>
<td>Any even $&gt; 2$ is sum of 2 primes</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>$4 + 5$</td>
<td>Not Proposition</td>
<td>False</td>
</tr>
<tr>
<td>$x + x$</td>
<td>Not a Proposition</td>
<td>False</td>
</tr>
<tr>
<td>Alice travelled to Chicago</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>I love you.</td>
<td>Hmmmm.</td>
<td></td>
</tr>
</tbody>
</table>

Again: “value” of a proposition is ...
### Propositions: Statements that are true or false.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{2} ) is irrational</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>( 2 + 2 = 4 )</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>( 2 + 2 = 3 )</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>826th digit of pi is 4</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>Johnny Depp is a good actor</td>
<td>Not Proposition</td>
<td>False</td>
</tr>
<tr>
<td>Any even &gt; 2 is sum of 2 primes</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>( 4 + 5 )</td>
<td>Not Proposition</td>
<td></td>
</tr>
<tr>
<td>( x + x )</td>
<td>Not a Proposition</td>
<td></td>
</tr>
<tr>
<td>Alice travelled to Chicago</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>I love you.</td>
<td>Hmmm.</td>
<td></td>
</tr>
</tbody>
</table>

Again: “value” of a proposition is ... True or False
Propositions: Statements that are true or false.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{2} ) is irrational</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>( 2 + 2 = 4 )</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>( 2 + 2 = 3 )</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>826th digit of pi is 4</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>Johnny Depp is a good actor</td>
<td>Not Proposition</td>
<td></td>
</tr>
<tr>
<td>Any even &gt; 2 is sum of 2 primes</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>( 4 + 5 )</td>
<td>Not Proposition</td>
<td></td>
</tr>
<tr>
<td>( x + x )</td>
<td>Not a Proposition</td>
<td></td>
</tr>
<tr>
<td>Alice travelled to Chicago</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>I love you.</td>
<td>Hmmmm.</td>
<td></td>
</tr>
</tbody>
</table>

Again: “value” of a proposition is ... True or False
Propositional Forms.

Put propositions together to make another...
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): $P \land Q$
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True if both $P$ and $Q$ are True.
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True if both $P$ and $Q$ are True. Else False.
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): $P \land Q$

“$P \land Q$” is True if both $P$ and $Q$ are True. Else False.

Disjunction (“or”): $P \lor Q$

Examples:

$\neg (2 + 2 = 4)$ – a proposition that is False.

“$2 + 2 = 3$” $\land$ “$2 + 2 = 4$” – a proposition that is False.

“$2 + 2 = 3$” $\lor$ “$2 + 2 = 4$” – a proposition that is True.
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): $P \land Q$

"$P \land Q$" is True if both $P$ and $Q$ are True. Else False.

Disjunction (“or”): $P \lor Q$

"$P \lor Q$" is True if at least one $P$ or $Q$ is True. Else False.
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): \( P \land Q \)

“\( P \land Q \)” is True if both \( P \) and \( Q \) are True. Else False.

Disjunction (“or”): \( P \lor Q \)

“\( P \lor Q \)” is True if at least one \( P \) or \( Q \) is True. Else False.

Negation (“not”): \( \neg P \)
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True if both $P$ and $Q$ are True. Else False.

Disjunction ("or"): $P \lor Q$

"$P \lor Q$" is True if at least one $P$ or $Q$ is True. Else False.

Negation ("not"): $\neg P$

"$\neg P$" is True if $P$ is False.
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): $P \land Q$

“$P \land Q$” is True if both $P$ and $Q$ are True. Else False.

Disjunction (“or”): $P \lor Q$

“$P \lor Q$” is True if at least one $P$ or $Q$ is True. Else False.

Negation (“not”): $\neg P$

“$\neg P$” is True if $P$ is False. Else False.
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True if both $P$ and $Q$ are True. Else False.

Disjunction ("or"): $P \lor Q$

"$P \lor Q$" is True if at least one $P$ or $Q$ is True. Else False.

Negation ("not"): $\neg P$

"$\neg P$" is True if $P$ is False. Else False.

Examples:
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): \( P \land Q \)

“\( P \land Q \)” is True if both \( P \) and \( Q \) are True. Else False.

Disjunction (“or”): \( P \lor Q \)

“\( P \lor Q \)” is True if at least one \( P \) or \( Q \) is True. Else False.

Negation (“not”): \( \neg P \)

“\( \neg P \)” is True if \( P \) is False. Else False.

Examples:

\( \neg \left( (2 + 2 = 4) \right) \) – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): $P \land Q$

“$P \land Q$” is True if both $P$ and $Q$ are True. Else False.

Disjunction (“or”): $P \lor Q$

“$P \lor Q$” is True if at least one $P$ or $Q$ is True. Else False.

Negation (“not”): $\neg P$

“$\neg P$” is True if $P$ is False. Else False.

Examples:

$\neg \left( 2 + 2 = 4 \right)$ – a proposition that is ... False
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): $P \land Q$

“$P \land Q$” is True if both $P$ and $Q$ are True. Else False.

Disjunction (“or”): $P \lor Q$

“$P \lor Q$” is True if at least one $P$ or $Q$ is True. Else False.

Negation (“not”): $\neg P$

“$\neg P$” is True if $P$ is False. Else False.

Examples:

$\neg \left(2 + 2 = 4\right)$ – a proposition that is ... False

“$2 + 2 = 3” \land “2 + 2 = 4$” – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True if both $P$ and $Q$ are True. Else False.

Disjunction ("or"): $P \lor Q$

"$P \lor Q$" is True if at least one $P$ or $Q$ is True. Else False.

Negation ("not"): $\neg P$

"$\neg P$" is True if $P$ is False. Else False.

Examples:

$\neg \text{"(2 + 2 = 4)"}$ — a proposition that is ... False

"2 + 2 = 3" $\land$ "2 + 2 = 4" — a proposition that is ... False
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True if both $P$ and $Q$ are True. Else False.

Disjunction ("or"): $P \lor Q$

"$P \lor Q$" is True if at least one $P$ or $Q$ is True. Else False.

Negation ("not"): $\neg P$

"$\neg P$" is True if $P$ is False. Else False.

Examples:

$\neg "(2 + 2 = 4)"$ – a proposition that is ... False

"$2 + 2 = 3$" $\land$ "$2 + 2 = 4$" – a proposition that is ... False

"$2 + 2 = 3$" $\lor$ "$2 + 2 = 4$" – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): \( P \land Q \)

“\( P \land Q \)” is True if both \( P \) and \( Q \) are True. Else False.

Disjunction (“or”): \( P \lor Q \)

“\( P \lor Q \)” is True if at least one \( P \) or \( Q \) is True. Else False.

Negation (“not”): \( \neg P \)

“\( \neg P \)” is True if \( P \) is False. Else False.

Examples:

\( \neg \) “\( 2 + 2 = 4 \)” – a proposition that is ... False

“\( 2 + 2 = 3 \)” \( \land \) “\( 2 + 2 = 4 \)” – a proposition that is ... False

“\( 2 + 2 = 3 \)” \( \lor \) “\( 2 + 2 = 4 \)” – a proposition that is ... True
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True if both $P$ and $Q$ are True. Else False.

Disjunction ("or"): $P \lor Q$

"$P \lor Q$" is True if at least one $P$ or $Q$ is True. Else False.

Negation ("not"): $\neg P$

"$\neg P$" is True if $P$ is False. Else False.

Examples:

$\neg "(2 + 2 = 4)"$ – a proposition that is ... False

"2 + 2 = 3" \land "2 + 2 = 4" – a proposition that is ... False

"2 + 2 = 3" \lor "2 + 2 = 4" – a proposition that is ... True
Put them together..

Propositions:

\[ P_1 \] - Person 1 rides the bus.
Put them together..

Propositions:

\( P_1 \) - Person 1 rides the bus.
\( P_2 \) - Person 2 rides the bus.

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

Propositional Form:

\[ \neg ((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5)) \]

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?
This seems complicated.

We can program!!!! We need a way to keep track!
Put them together..

Propositions:

$P_1$ - Person 1 rides the bus.

$P_2$ - Person 2 rides the bus.

....

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

Propositional Form:

$$\neg ((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5))$$

Can person 3 ride the bus?

Can person 3 and person 4 ride the bus together?

This seems complicated.

We can program!!!! We need a way to keep track!
Propositions:
\( P_1 \) - Person 1 rides the bus.
\( P_2 \) - Person 2 rides the bus.

But we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn’t.
Put them together..

Propositions:
$P_1$ - Person 1 rides the bus.
$P_2$ - Person 2 rides the bus.

But we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn’t.

Propositional Form:
$\neg((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5))$
Put them together..

Propositions:
P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
....

But we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn’t.

Propositional Form:
\neg(((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5)))

Can person 3 ride the bus?
Put them together..

Propositions:
\[ P_1 \] - Person 1 rides the bus.
\[ P_2 \] - Person 2 rides the bus.

But we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn’t.

Propositional Form:
\[
\neg((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5))
\]

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?
Put them together..

Propositions:
$P_1$ - Person 1 rides the bus.
$P_2$ - Person 2 rides the bus.

But we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn’t.

Propositional Form:
$\neg(((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5)))$

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?
Put them together..

Propositions:
\( P_1 \) - Person 1 rides the bus.
\( P_2 \) - Person 2 rides the bus.

....

But we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn’t.

Propositional Form:
\[ \neg(((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5))) \]

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

This seems ...
Put them together..

Propositions:
$P_1$ - Person 1 rides the bus.
$P_2$ - Person 2 rides the bus.
....

But we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn’t.

Propositional Form:
\[ \neg \left( ((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5)) \right) \]

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

This seems ...complicated.
Propositions:

\( P_1 \) - Person 1 rides the bus.
\( P_2 \) - Person 2 rides the bus.

But we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn’t.

Propositional Form:

\[
\neg((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5))
\]

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

This seems ...complicated.

We can program!!!!
Propositions:

$P_1$ - Person 1 rides the bus.
$P_2$ - Person 2 rides the bus.

But we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn’t.

Propositional Form:

$$\neg(((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5)))$$

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

This seems ...complicated.

We can program!!!!

We need a way to keep track!
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Truth Tables for Propositional Forms.

"$P \land Q$" is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“$P \lor Q$” is True if ≥ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg (P \land Q)$ logically equivalent to $\neg P \lor \neg Q$.

Same Truth Table!

DeMorgan’s Law’s for Negation: distribute and flip!

$\neg (P \land Q) \equiv \neg P \lor \neg Q$

$\neg (P \lor Q) \equiv \neg P \land \neg Q$
Truth Tables for Propositional Forms.

"$P \land Q$" is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

"$P \lor Q$" is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example:

$\neg (P \land Q)$ is logically equivalent to $\neg P \lor \neg Q$.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg (P \lor Q)$</th>
<th>$\neg P \land \neg Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

DeMorgan's Law's for Negation: distribute and flip!

$\neg (P \land Q)$ $\equiv \neg P \lor \neg Q$

$\neg (P \lor Q)$ $\equiv \neg P \land \neg Q$
**Truth Tables for Propositional Forms.**

---

**“**$P \land Q$**” is True if both $P$ and $Q$ are True.**

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

---

**“**$P \lor Q$**” is True if $\geq$ one of $P$ or $Q$ is True.**

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!
Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. 
Truth Tables for Propositional Forms.

"$P \land Q$" is True if both $P$ and $Q$ are True.

\[ \begin{array}{ccc}
   P & Q & P \land Q \\
   T & T & T \\
   T & F & F \\
   F & T & F \\
   F & F & F \\
\end{array} \]

"$P \lor Q$" is True if $\geq$ one of $P$ or $Q$ is True.

\[ \begin{array}{ccc}
   P & Q & P \lor Q \\
   T & T & T \\
   T & F & T \\
   F & T & T \\
   F & F & F \\
\end{array} \]

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg (P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms! Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg(P \lor Q)$</th>
<th>$\neg P \land \neg Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

```
<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P \land Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
```

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

```
<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P \lor Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
```

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!

```
<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>\neg(P \lor Q)</th>
<th>\neg P \land \neg Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
```
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg(P \lor Q)$</th>
<th>$\neg P \land \neg Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!
Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg(P \lor Q)$</th>
<th>$\neg P \land \neg Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Truth Tables for Propositional Forms.

“\( P \land Q \) is True if both \( P \) and \( Q \) are True .

<table>
<thead>
<tr>
<th>( P )</th>
<th>( Q )</th>
<th>( P \land Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“\( P \lor Q \) is True if \geq one of \( P \) or \( Q \) is True .

<table>
<thead>
<tr>
<th>( P )</th>
<th>( Q )</th>
<th>( P \lor Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: \( \land \) and \( \lor \) are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: \( \neg(P \land Q) \) logically equivalent to \( \neg P \lor \neg Q \). Same Truth Table!
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!
Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!
Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg(P \lor Q)$</th>
<th>$\neg P \land \neg Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>
Truth Tables for Propositional Forms.

“\( P \land Q \)” is True if both \( P \) and \( Q \) are True.

<table>
<thead>
<tr>
<th>( P )</th>
<th>( Q )</th>
<th>( P \land Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“\( P \lor Q \)” is True if \( \geq \) one of \( P \) or \( Q \) is True.

<table>
<thead>
<tr>
<th>( P )</th>
<th>( Q )</th>
<th>( P \lor Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: \( \land \) and \( \lor \) are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: \( \neg (P \land Q) \) logically equivalent to \( \neg P \lor \neg Q \). Same Truth Table!

<table>
<thead>
<tr>
<th>( P )</th>
<th>( Q )</th>
<th>( \neg (P \lor Q) )</th>
<th>( \neg P \land \neg Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>
Truth Tables for Propositional Forms.

“\( P \land Q \)” is True if both \( P \) and \( Q \) are True.

<table>
<thead>
<tr>
<th>( P )</th>
<th>( Q )</th>
<th>( P \land Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“\( P \lor Q \)” is True if \( \geq \) one of \( P \) or \( Q \) is True.

<table>
<thead>
<tr>
<th>( P )</th>
<th>( Q )</th>
<th>( P \lor Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: \( \land \) and \( \lor \) are commutative.

One use for truth tables: Logical Equivalence of propositional forms! Example: \( \neg(P \land Q) \) logically equivalent to \( \neg P \lor \neg Q \). Same Truth Table!

DeMorgan’s Law’s for Negation: distribute and flip!

\[ \neg(P \land Q) \]
Truth Tables for Propositional Forms.

"$P \land Q$" is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

"$P \lor Q$" is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!
Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg(P \lor Q)$</th>
<th>$\neg P \land \neg Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

DeMorgan’s Law’s for Negation: distribute and flip!

$\neg(P \land Q) \equiv \neg P \lor \neg Q$
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms! Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg(P \lor Q)$</th>
<th>$\neg P \land \neg Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

DeMorgan’s Law’s for Negation: distribute and flip!

$\neg(P \land Q) \equiv \neg P \lor \neg Q$  $\neg(P \lor Q)$
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg(P \lor Q)$</th>
<th>$\neg P \land \neg Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

DeMorgan’s Law’s for Negation: distribute and flip!

$\neg(P \land Q) \equiv \neg P \lor \neg Q$  \quad \quad \quad \quad $\neg(P \lor Q) \equiv \neg P \land \neg Q$
Quick Questions

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Quick Questions

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Is $(T \land Q) \equiv Q$?
Quick Questions

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Is $(T \land Q) \equiv Q$? Yes?
Quick Questions

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Is $(T \land Q) \equiv Q$? Yes? No?

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Quick Questions

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Is $(T \land Q) \equiv Q$? Yes? No?
Yes!
Quick Questions

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Is $(T \land Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$. 
Quick Questions

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Is $(T \land Q) \equiv Q$? Yes? No?
Yes! Look at rows in truth table for $P = T$.
What is $(F \land Q)$?
Quick Questions

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Is $(T \land Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \land Q)$? F or False.
Quick Questions

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$T$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$T$</td>
<td>$F$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
</tbody>
</table>

Is $(T \land Q) \equiv Q$? Yes? No?
Yes! Look at rows in truth table for $P = T$.

What is $(F \land Q)$? F or False.

What is $(T \lor Q)$?
Quick Questions

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Is $(T \land Q) \equiv Q$? Yes? No?
Yes! Look at rows in truth table for $P = T$.

What is $(F \land Q)$? F or False.
What is $(T \lor Q)$? T
Quick Questions

|   |   |  
|---|---|---|
| $P$ | $Q$ | $P \land Q$ |
| T  | T  | T  |
| T  | F  | F  |
| F  | T  | F  |
| F  | F  | F  |

|   |   |  
|---|---|---|
| $P$ | $Q$ | $P \lor Q$ |
| T  | T  | T  |
| T  | F  | T  |
| F  | T  | T  |
| F  | F  | F  |

Is $(T \land Q) \equiv Q$? Yes? No?
Yes! Look at rows in truth table for $P = T$.
What is $(F \land Q)$? F or False.
What is $(T \lor Q)$? T
What is $(F \lor Q)$?
Quick Questions

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Q</td>
<td>P ∧ Q</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Q</td>
<td>P ∨ Q</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Is \((T ∧ Q) ≡ Q\)? Yes? No?
Yes! Look at rows in truth table for \(P = T\).
What is \((F ∧ Q)\)? F or False.
What is \((T ∨ Q)\)? T
What is \((F ∨ Q)\)? Q
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) \]?
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) \]?

Simplify: \( (T \land Q) \equiv Q \),
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \( T \land Q \equiv Q, \ (F \land Q) \equiv F. \)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

Cases:
- \(P\) is True.
- LHS: \(T \land (Q \lor R)\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\]

Cases:
- \(P\) is True.
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F \).

Cases:

\( P \) is True.

LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).

RHS: \( (T \land Q) \lor (T \land R) \)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

\(P\) is **True**.

- **LHS:** \(T \land (Q \lor R) \equiv (Q \lor R).\)
- **RHS:** \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\]

Cases:

- \(P\) is True.
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\]
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\]
- \(P\) is False.
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F \).

Cases:

\( P \) is **True** .

\begin{itemize}
  \item LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).
  \item RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R) \).
\end{itemize}

\( P \) is **False** .

\begin{itemize}
  \item LHS: \( F \land (Q \lor R) \)
\end{itemize}
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\]

Cases:

- \(P\) is True .
  
  LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\]
  
  RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\]

- \(P\) is False .
  
  LHS: \(F \land (Q \lor R) \equiv F.\]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

**Cases:**
- **\(P\) is True.**
  - **LHS:** \(T \land (Q \lor R) \equiv (Q \lor R).\)
  - **RHS:** \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)
- **\(P\) is False.**
  - **LHS:** \(F \land (Q \lor R) \equiv F.\)
  - **RHS:** \((F \land Q) \lor (F \land R)\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

- \(P\) is True.
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)

- \(P\) is False.
  - LHS: \(F \land (Q \lor R) \equiv F.\)
  - RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F)\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

Cases:

\(P\) is True.

\[ \text{LHS: } T \land (Q \lor R) \equiv (Q \lor R). \]
\[ \text{RHS: } (T \land Q) \lor (T \land R) \equiv (Q \lor R). \]

\(P\) is False.

\[ \text{LHS: } F \land (Q \lor R) \equiv F. \]
\[ \text{RHS: } (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F. \]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

Cases:

\(P\) is True.
- LHS: \(T \land (Q \lor R) \equiv (Q \lor R)\).
- RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R)\).

\(P\) is False.
- LHS: \(F \land (Q \lor R) \equiv F\).
- RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F\).
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:
- \(P\) is True .
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)
- \(P\) is False .
  - LHS: \(F \land (Q \lor R) \equiv F.\)
  - RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F.\)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

\(P\) is True.
LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)

\(P\) is False.
LHS: \(F \land (Q \lor R) \equiv F.\)
RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F.\)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R) ? \]

Simplify: \(T \lor Q \equiv T,\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \( (T \land Q) \equiv Q, \ (F \land Q) \equiv F. \)

Cases:

\( P \) is True .
- LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).
- RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R) \).

\( P \) is False .
- LHS: \( F \land (Q \lor R) \equiv F \).
- RHS: \( (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F \).

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \( T \lor Q \equiv T, \ (F \lor Q) \equiv Q. \ ...)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

Cases:

\(P\) is True .

LHS: \(T \land (Q \lor R) \equiv (Q \lor R)\).

RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R)\).

\(P\) is False .

LHS: \(F \land (Q \lor R) \equiv F\).

RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F\).

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \(T \lor Q \equiv T, F \lor Q \equiv Q\). ...

Foil 1:
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\]

Cases:
- \(P\) is True.
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)
- \(P\) is False.
  - LHS: \(F \land (Q \lor R) \equiv F.\)
  - RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F.\)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R) ? \]

Simplify: \(T \lor Q \equiv T, F \lor Q \equiv Q.\ ...

Foil 1:
\[(A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D) ?\]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

\(P\) is True.

LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)

RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)

\(P\) is False.

LHS: \(F \land (Q \lor R) \equiv F.\)

RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F.\)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \(T \lor Q \equiv T, F \lor Q \equiv Q. \ldots\)

Foil 1:

\((A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D)?\)

Foil 2:
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F. \)

Cases:

\( P \) is True.
\[ \text{LHS: } T \land (Q \lor R) \equiv (Q \lor R). \]
\[ \text{RHS: } (T \land Q) \lor (T \land R) \equiv (Q \lor R). \]

\( P \) is False.
\[ \text{LHS: } F \land (Q \lor R) \equiv F. \]
\[ \text{RHS: } (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F. \]

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \( T \lor Q \equiv T, F \lor Q \equiv Q. \) ...

Foil 1:
\[ (A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D)? \]

Foil 2:
\[ (A \land B) \lor (C \land D) \equiv (A \lor C) \land (A \lor D) \land (B \lor C) \land (B \lor D)? \]
Implication.

\[ P \implies Q \text{ interpreted as} \]

Examples:

Statement: If you stand in the rain, then you'll get wet.

\[ P = \text{"you stand in the rain"}, \quad Q = \text{"you will get wet"}. \]

Statement: "Stand in the rain"

Can conclude: "you'll get wet."

Statement:

If a right triangle has sidelengths \( a \leq b \leq c \), then \( a^2 + b^2 = c^2 \).

\[ P = \text{"a right triangle has sidelengths } a \leq b \leq c\text{"}, \quad Q = \text{"} a^2 + b^2 = c^2 \text{"}. \]
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).
Implication.

\[ P \implies Q \text{ interpreted as } \]

If \( P \), then \( Q \).
Implication.

\[ P \implies Q \] interpreted as
If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).

Examples:
Statement: If you stand in the rain, then you’ll get wet.
\( P = ”you stand in the rain” \), \( Q = ”you will get wet” \).

Statement: “Stand in the rain”
Can conclude: “you’ll get wet.”

Statement: If a right triangle has sidelengths \( a \leq b \leq c \), then \( a^2 + b^2 = c^2 \).
\( P = ”a right triangle has sidelengths \( a \leq b \leq c \)” \), \( Q = ”a^2 + b^2 = c^2” \).
Implication.

\[ P \implies Q \] interpreted as
If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Examples:
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Examples:

Statement: If you stand in the rain, then you’ll get wet.

\[ \text{Statement: If you stand in the rain, then you’ll get wet.} \]
Implication.

\[ P \implies Q \] interpreted as
If \( P \), then \( Q \).

True Statements: \( P, \ P \implies Q \).
Conclude: \( Q \) is true.

Examples:

Statement: If you stand in the rain, then you’ll get wet.
\( P = \) “you stand in the rain”
Implication.

\[ P \implies Q \text{ interpreted as} \]
\[ \text{If } P, \text{ then } Q. \]

True Statements: \( P, P \implies Q. \)
Conclude: \( Q \) is true.

Examples:

Statement: If you stand in the rain, then you’ll get wet.
\[ P = \text{“you stand in the rain”} \]
\[ Q = \text{“you will get wet”} \]
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Examples:

Statement: If you stand in the rain, then you’ll get wet.
\[ P = \text{“you stand in the rain”} \]
\[ Q = \text{“you will get wet”} \]

Statement: “Stand in the rain”
Implication.

\[ P \implies Q \] interpreted as
If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Examples:
Statement: If you stand in the rain, then you’ll get wet.
\( P = “you stand in the rain” \)
\( Q = “you will get wet” \)
Statement: “Stand in the rain”
Can conclude: “you’ll get wet.”
Implication.

\[ P \implies Q \] interpreted as
If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Examples:

Statement: If you stand in the rain, then you’ll get wet.
\( P = \) “you stand in the rain”
\( Q = \) “you will get wet”

Statement: “Stand in the rain”
Can conclude: “you’ll get wet.”

Statement:
If a right triangle has sidelengths \( a \leq b \leq c \), then \( a^2 + b^2 = c^2 \).
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Examples:

Statement: If you stand in the rain, then you’ll get wet.
\( P = \) “you stand in the rain”
\( Q = \) “you will get wet”

Statement: “Stand in the rain”
Can conclude: “you’ll get wet.”

Statement:
If a right triangle has sidelengths \( a \leq b \leq c \), then \( a^2 + b^2 = c^2 \).
\( P = \) “a right triangle has sidelengths \( a \leq b \leq c \),
Implication.

\[ P \implies Q \] interpreted as
If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Examples:

Statement: If you stand in the rain, then you’ll get wet.
\( P = \) “you stand in the rain”
\( Q = \) “you will get wet”

Statement: “Stand in the rain”
Can conclude: “you’ll get wet.”

Statement:
If a right triangle has sidelengths \( a \leq b \leq c \), then \( a^2 + b^2 = c^2 \).

\( P = \) “a right triangle has sidelengths \( a \leq b \leq c \)”,
\( Q = \) “\( a^2 + b^2 = c^2 \)”.
Non-Consequences/consequences of Implication

The statement “$P \implies Q$” only is False if $P$ is True and $Q$ is False.
False implies nothing. $P$ False means $Q$ can be True or False.
Anything implies true. $P$ can be True or False if $Q$ is True.

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?
Not necessarily.

$P \implies Q$ and $Q$ are True does not mean $P$ is True.
Be careful!
Instead we have: $P \implies Q$ and $P$ are True does mean $Q$ is True.
The chemical plant pollutes river. Can we conclude fish die?

Some Fun: use propositional formulas to describe implication?

$(P \implies Q) \land P \iff Q$. 
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

---

"False implies nothing" $P$ False means $Q$ can be True or False.

"Anything implies true." $P$ can be True or False if $Q$ is True.

If chemical plant pollutes river, fish die.

If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and $Q$ are True does not mean $P$ is True.

Be careful!

Instead we have: $P \implies Q$ and $P$ are True does mean $Q$ is True.

The chemical plant pollutes river.

Can we conclude fish die?

Some Fun: use propositional formulas to describe implication?

$((P \implies Q) \land P) = \implies Q$. 
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
P False means
Non-Consequences/consequences of Implication

The statement “\( P \implies Q \)”

only is False if \( P \) is True and \( Q \) is False.

False implies nothing
P False means \( Q \) can be True
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing

P False means $Q$ can be True or False
The statement “\( P \implies Q \)” only is False if \( P \) is True and \( Q \) is False.

False implies nothing.

\( P \text{ False} \) means \( Q \) can be True or False.

Anything implies true.
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
$P$ False means $Q$ can be True or False
Anything implies true.
$P$ can be True or False if
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is **False** if $P$ is **True** and $Q$ is **False**.

False implies nothing

$P$ **False** means $Q$ can be **True** or **False**

Anything implies true.

$P$ can be **True** or **False** if $Q$ is **True**
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
$P$ False means $Q$ can be True or False
Anything implies true.
$P$ can be True or False if $Q$ is True

If chemical plant pollutes river, fish die.
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
$P$ False means $Q$ can be True or False
Anything implies true.
$P$ can be True or False if $Q$ is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False if Q is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?
Not necessarily.
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is **False** if $P$ is **True** and $Q$ is **False**.

False implies nothing

$P$ False means $Q$ can be **True** or **False**

Anything implies true.

$P$ can be **True** or **False** if $Q$ is **True**

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and $Q$ are **True** does not mean $P$ is **True**
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

- only is False if $P$ is True and $Q$ is False.

  False implies nothing
  $P$ False means $Q$ can be True or False
  Anything implies true.
  $P$ can be True or False if $Q$ is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?
Not necessarily.

$P \implies Q$ and $Q$ are True does not mean $P$ is True

Be careful!
Non-Consequences/consequences of Implication

The statement “\( P \implies Q \)”

only is False if \( P \) is True and \( Q \) is False.

False implies nothing
\( P \) False means \( Q \) can be True or False
Anything implies true.
\( P \) can be True or False if \( Q \) is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?
Not necessarily.

\( P \implies Q \) and \( Q \) are True does not mean \( P \) is True

Be careful!

Instead we have:
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
P False means $Q$ can be True or False
Anything implies true.
$P$ can be True or False if $Q$ is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and $Q$ are True does not mean $P$ is True

Be careful!

Instead we have:
$P \implies Q$ and $P$ are True does mean $Q$ is True.
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
$P$ False means $Q$ can be True or False
Anything implies true.
$P$ can be True or False if $Q$ is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and $Q$ are True does not mean $P$ is True

Be careful!

Instead we have:
$P \implies Q$ and $P$ are True does mean $Q$ is True.

The chemical plant pollutes river.
The statement \( P \implies Q \)

only is \textbf{False} if \( P \) is \textbf{True} and \( Q \) is \textbf{False}.

False implies nothing.

P \textbf{False} means \( Q \) can be \textbf{True} or \textbf{False}

Anything implies true.

\( P \) can be \textbf{True} or \textbf{False} if \( Q \) is \textbf{True}

If chemical plant pollutes river, fish die.

If fish die, did chemical plant pollute river?

Not necessarily.

\( P \implies Q \) and \( Q \) are \textbf{True} does not mean \( P \) is \textbf{True}

Be careful!

Instead we have:

\( P \implies Q \) and \( P \) are \textbf{True} \textit{does} mean \( Q \) is \textbf{True}.

The chemical plant pollutes river. Can we conclude fish die?
Non-Consequences/consequences of Implication

The statement \("P \implies Q\"")

only is False if \(P\) is True and \(Q\) is False.

False implies nothing
\(P\) False means \(Q\) can be True or False
Anything implies true.
\(P\) can be True or False if \(Q\) is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

\(P \implies Q\) and \(Q\) are True does not mean \(P\) is True

Be careful!

Instead we have:
\(P \implies Q\) and \(P\) are True does mean \(Q\) is True.

The chemical plant pollutes river. Can we conclude fish die?
Non-Consequences/consequences of Implication

The statement “\( P \implies Q \)”

- only is **False** if \( P \) is **True** and \( Q \) is **False**.
- False implies nothing
- \( P \) **False** means \( Q \) can be **True** or **False**
- Anything implies true.
- \( P \) can be **True** or **False** if \( Q \) is **True**

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?
Not necessarily.

\( P \implies Q \) and \( Q \) are **True** does **not** mean \( P \) is **True**

Be careful!

Instead we have:
\( P \implies Q \) and \( P \) are **True** **does** mean \( Q \) is **True**.

The chemical plant pollutes river. Can we conclude fish die?

Some Fun: use propositional formulas to describe implication?
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
P False means $Q$ can be True or False
Anything implies true.
P can be True or False if $Q$ is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and $Q$ are True does not mean $P$ is True

Be careful!

Instead we have:
$P \implies Q$ and $P$ are True does mean $Q$ is True.

The chemical plant pollutes river. Can we conclude fish die?

Some Fun: use propositional formulas to describe implication?

$\left((P \implies Q) \land P\right) \implies Q$. 
Implication and English.

\[ P \implies Q \]
Poll.

▶ If \( P \), then \( Q \).

- \( P \) only if \( Q \).
- \( P \) is sufficient for \( Q \).
- \( Q \) is necessary for \( P \).
Implication and English.

\[ P \implies Q \]

Poll.

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  
  Just reversing the order.
Implication and English.

\[ P \implies Q \]
Poll.

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  Just reversing the order.
Implication and English.

\[ P \implies Q \]

Poll.

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  Just reversing the order.
- \( P \) only if \( Q \).
- \( Q \) is necessary for \( P \).
  For \( P \) to be true it is necessary that \( Q \) is true.
  Or if \( Q \) is false then we know that \( P \) is false.
- \( P \) is sufficient for \( Q \).
  This means that proving \( P \) allows you to conclude that \( Q \) is true.
  Example: Showing \( n > 4 \) is sufficient for showing \( n > 3 \).
Implication and English.

\( P \implies Q \)

Poll.

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  Just reversing the order.
- \( P \) only if \( Q \).
  Remember if \( P \) is true then \( Q \) must be true.

---

Example: Showing \( n > 4 \) is sufficient for showing \( n > 3 \).

- \( Q \) is necessary for \( P \).
  For \( P \) to be true it is necessary that \( Q \) is true.
  Or if \( Q \) is false then we know that \( P \) is false.
  Example: It is necessary that \( n > 3 \) for \( n > 4 \).
Implication and English.

\[ P \implies Q \]

Poll.

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  
  Just reversing the order.

- \( P \) only if \( Q \).
  
  Remember if \( P \) is true then \( Q \) must be true.
  
  this suggests that \( P \) can only be true if \( Q \) is true.

- \( Q \) is necessary for \( P \).
  
  For \( P \) to be true it is necessary that \( Q \) is true.
  
  Or if \( Q \) is false then we know that \( P \) is false.

- \( P \) is sufficient for \( Q \).
  
  This means that proving \( P \) allows you to conclude that \( Q \) is true.

Example: Showing \( n > 4 \) is sufficient for showing \( n > 3 \).

- \( Q \) is necessary for \( P \).

Example: It is necessary that \( n > 3 \) for \( n > 4 \).
Implication and English.

\[ P \implies Q \]

Poll.

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  
  Just reversing the order.
- \( P \) only if \( Q \).
  
  Remember if \( P \) is true then \( Q \) must be true.  
  this suggests that \( P \) can only be true if \( Q \) is true. 
  since if \( Q \) is false \( P \) must have been false.
Implication and English.

\[ P \implies Q \]

Poll.

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  Just reversing the order.
- \( P \) only if \( Q \).
  Remember if \( P \) is true then \( Q \) must be true.
  this suggests that \( P \) can only be true if \( Q \) is true.
  since if \( Q \) is false \( P \) must have been false.
- \( P \) is sufficient for \( Q \).
Implication and English.

\[ P \implies Q \]

Poll.

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  Just reversing the order.
- \( P \) only if \( Q \).
  Remember if \( P \) is true then \( Q \) must be true.
  this suggests that \( P \) can only be true if \( Q \) is true.
  since if \( Q \) is false \( P \) must have been false.
- \( P \) is sufficient for \( Q \).
  This means that proving \( P \) allows you
Implication and English.

\[ P \implies Q \]

Poll.

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  
  Just reversing the order.
- \( P \) only if \( Q \).
  
  Remember if \( P \) is true then \( Q \) must be true.
  
  this suggests that \( P \) can only be true if \( Q \) is true.
  
  since if \( Q \) is false \( P \) must have been false.

- \( P \) is sufficient for \( Q \).
  
  This means that proving \( P \) allows you
  
  to conclude that \( Q \) is true.
Implication and English.

\[ P \implies Q \]

Poll.

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  Just reversing the order.
- \( P \) only if \( Q \).
  Remember if \( P \) is true then \( Q \) must be true.
  this suggests that \( P \) can only be true if \( Q \) is true.
  since if \( Q \) is false \( P \) must have been false.
- \( P \) is sufficient for \( Q \).
  This means that proving \( P \) allows you
  to conclude that \( Q \) is true.
  Example: Showing \( n > 4 \) is sufficient for showing \( n > 3 \).
Implication and English.

\[ P \Rightarrow Q \]

Poll.

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  
  Just reversing the order.

- \( P \) only if \( Q \).
  
  Remember if \( P \) is true then \( Q \) must be true. this suggests that \( P \) can only be true if \( Q \) is true.
  
  since if \( Q \) is false \( P \) must have been false.

- \( P \) is sufficient for \( Q \).
  
  This means that proving \( P \) allows you to conclude that \( Q \) is true.
  
  Example: Showing \( n > 4 \) is sufficient for showing \( n > 3 \).

- \( Q \) is necessary for \( P \).
  
  For \( P \) to be true it is necessary that \( Q \) is true.
  
  Or if \( Q \) is false then we know that \( P \) is false.
Implication and English.

\[ P \implies Q \]

Poll.

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  
  Just reversing the order.

- \( P \) only if \( Q \).
  
  Remember if \( P \) is true then \( Q \) must be true.
  
  this suggests that \( P \) can only be true if \( Q \) is true.
  
  since if \( Q \) is false \( P \) must have been false.

- \( P \) is sufficient for \( Q \).
  
  This means that proving \( P \) allows you to conclude that \( Q \) is true.

  Example: Showing \( n > 4 \) is sufficient for showing \( n > 3 \).

- \( Q \) is necessary for \( P \).
  
  For \( P \) to be true it is necessary that \( Q \) is true.
  
  Or if \( Q \) is false then we know that \( P \) is false.

  Example: It is necessary that \( n > 3 \) for \( n > 4 \).
Truth Table: implication.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \implies Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
Truth Table: implication.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \implies Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

These two propositional forms are logically equivalent!
Truth Table: implication.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \implies Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
Truth Table: implication.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \implies Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

These two propositional forms are logically equivalent!
Truth Table: implication.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \implies Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
Truth Table: implication.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \implies Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

$P \implies Q$ is logically equivalent to $\neg P \lor Q$. 

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
Truth Table: implication.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \implies Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
Truth Table: implication.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \implies Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
Truth Table: implication.

\[
\begin{array}{c|c|c}
P & Q & P \implies Q \\
\hline
T & T & T \\
T & F & F \\
F & T & T \\
F & F & T \\
\end{array}
\]

\[
\begin{array}{c|c|c}
P & Q & \neg P \lor Q \\
\hline
T & T & T \\
T & F & F \\
F & T & T \\
F & F & T \\
\end{array}
\]

These two propositional forms are logically equivalent!
Truth Table: implication.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \Rightarrow Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

$\neg P \lor Q \equiv P \Rightarrow Q$.  

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
Truth Table: implication.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \implies Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg P \lor Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

$\neg P \lor Q \equiv P \implies Q$.

These two propositional forms are logically equivalent!
Contraposition, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$. 

  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - Converse of $P \implies Q$ is $Q \implies P$.
  - If fish die the plant pollutes. Not logically equivalent!

- **Definition:** If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$ or $P \iff Q$. (Logically Equivalent: \iff )
Contrapositive, Converse

- **Contrapositive** of \( P \implies Q \) is \( \neg Q \implies \neg P \).
  - If the plant pollutes, fish die.

- If fish die the plant pollutes.
  - Not logically equivalent!

**Definition:** If \( P \implies Q \) and \( Q \implies P \) is \( P \) if and only if \( Q \) or \( P \iff Q \).
  - (Logically Equivalent: \( \iff \).)
Contrapositive, Converse

Contrapositive of \( P \implies Q \) is \( \neg Q \implies \neg P \).

- If the plant pollutes, fish die.
- If the fish don’t die, the plant does not pollute.
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute. (contrapositive)

- **Converse** of $P \implies Q$ is $Q \implies P$.
  - If fish die the plant pollutes.
  - Not logically equivalent!

Definition:
If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$.
(Logically Equivalent: $\iff$.)
Contrapositive, Converse

- **Contrapositive** of \( P \implies Q \) is \( \neg Q \implies \neg P \).
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute.
    (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.

- **Converse** of $P \implies Q$ is $Q \implies P$.
  - If fish die the plant pollutes.
  - Not logically equivalent!

- **Definition**: If $P \implies Q$ and $Q \implies P$ is $P \iff Q$.
  (Logically Equivalent: $\iff$.)
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute.
    (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.
    (not contrapositive!)

- **Converse** of $P \implies Q$ is $Q \implies P$.
  - If fish die the plant pollutes.
  - Not logically equivalent!

Definition:
If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$ or $P \iff Q$. (Logically Equivalent: $\iff$.)
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - If you did not get wet, you did not stand in the rain.
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute.
    (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.
    (not contrapositive!)
  - If you did not get wet, you did not stand in the rain.
    (contrapositive.)
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute. *(contrapositive)*

- If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. *(not contrapositive!)*
  - If you did not get wet, you did not stand in the rain. *(contrapositive.)*

Logically equivalent! Notation: $\equiv$. 
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute.
    (contrapositive)
  
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.
    (not contrapositive!)
  - If you did not get wet, you did not stand in the rain.
    (contrapositive.)

Logically equivalent! Notation: $\equiv$.

$P \implies Q$
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute.  
    (contrapositive)

- If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.  
    (not contrapositive!)
  - If you did not get wet, you did not stand in the rain.  
    (contrapositive.)

Logically equivalent! Notation: $\equiv$.

$P \implies Q \equiv \neg P \lor Q$
Contrapositive, Converse

- **Contrapositive** of \( P \implies Q \) is \( \neg Q \implies \neg P \).
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - If you did not get wet, you did not stand in the rain. (contrapositive.)

Logically equivalent! Notation: \( \equiv \).
\[
P \implies Q \equiv \neg P \vee Q \equiv \neg (\neg Q) \vee \neg P
\]
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.

  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute. *(contrapositive)*

  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. *(not contrapositive!)*
  - If you did not get wet, you did not stand in the rain. *(contrapositive.)*

Logically equivalent! Notation: $\equiv$.

\[
P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P \equiv \neg Q \implies \neg P.
\]
Contrapositive, Converse

- **Contrapositive** of \( P \implies Q \) is \( \neg Q \implies \neg P \).
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute.
    (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.
    (not contrapositive!)
  - If you did not get wet, you did not stand in the rain.
    (contrapositive.)

Logically equivalent! Notation: \( \equiv \).
\[
P \implies Q \equiv \neg P \lor Q \equiv \neg(\neg Q) \lor \neg P \equiv \neg Q \implies \neg P.
\]

- **Converse** of \( P \implies Q \) is \( Q \implies P \).
Contrapositive, Converse

- **Contrapositive** of \( P \implies Q \) is \( \neg Q \implies \neg P \).
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute.  
    (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.  
    (not contrapositive!)
  - If you did not get wet, you did not stand in the rain.  
    (contrapositive.)

Logically equivalent! Notation: \( \equiv \).
\[
P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P \equiv \neg Q \implies \neg P.
\]

- **Converse** of \( P \implies Q \) is \( Q \implies P \).
  - If fish die the plant pollutes.
**Contrapositive, Converse**

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!) converse!
  - If you did not get wet, you did not stand in the rain. (contrapositive.)

Logically equivalent! Notation: $\equiv$.

$$P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P \equiv \neg Q \implies \neg P.$$ 

- **Converse** of $P \implies Q$ is $Q \implies P$.
  - If fish die the plant pollutes.
Contrapositive, Converse

Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.

- If the plant pollutes, fish die.
- If the fish don’t die, the plant does not pollute.
    (contrapositive)

- If you stand in the rain, you get wet.
- If you did not stand in the rain, you did not get wet.
    (not contrapositive!) converse!
- If you did not get wet, you did not stand in the rain.
    (contrapositive.)

Logically equivalent! Notation: $\equiv$.

\[ P \implies Q \equiv \neg P \lor Q \equiv \neg(\neg Q) \lor \neg P \equiv \neg Q \implies \neg P. \]

Converse of $P \implies Q$ is $Q \implies P$.

If fish die the plant pollutes.
Not logically equivalent!
Contrapositive, Converse

- **Contrapositive** of \( P \implies Q \) is \( \neg Q \implies \neg P \).
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!) converse!
  - If you did not get wet, you did not stand in the rain. (contrapositive.)

Logically equivalent! Notation: \( \equiv \).
\[
P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P \equiv \neg Q \implies \neg P.
\]

- **Converse** of \( P \implies Q \) is \( Q \implies P \).
  - If fish die the plant pollutes.
  - Not logically equivalent!

- **Definition:** If \( P \implies Q \) and \( Q \implies P \) is \( P \) if and only if \( Q \) or \( P \iff Q \).
  (Logically Equivalent: \( \iff \).)
Variables.

Propositions?

\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
Variables.

Propositions?

\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]

\[ x > 2 \]
Variables.

Propositions?

- \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).
- \( x > 2 \)
- \( n \) is even and the sum of two primes
Variables.
Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes

No.
Variables.

Propositions?

- \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).
- \( x > 2 \)
- \( n \) is even and the sum of two primes

No. They have a free variable.
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) = "x$ is even"
Variables.

Propositions?

\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]

\[ x > 2 \]

\[ n \text{ is even and the sum of two primes} \]

No. They have a free variable.

We call them predicates, e.g., \( Q(x) = \text{“x is even”} \)

Same as boolean valued functions from 61A!
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $x > 2$
- $n$ is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) = \text{“}x \text{ is even}\text{”}$

Same as boolean valued functions from 61A!

- $P(n) = \text{“}\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\text{”}$
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) = \text{“} x \text{ is even} \text{”}$

Same as boolean valued functions from 61A!

- $P(n) = \text{“} \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \text{”}$
- $R(x) = \text{“} x > 2 \text{”}$
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) = \text{“}x \text{ is even}\text{”}$

Same as boolean valued functions from 61A!

- $P(n) = \text{“}\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\text{”}$
- $R(x) = \text{“}x > 2\text{”}$
- $G(n) = \text{“}n \text{ is even and the sum of two primes}\text{”}$
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) = \text{“}x \text{ is even}\text{”}$

Same as boolean valued functions from 61A!

- $P(n) = \text{“}\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\text{”}$
- $R(x) = \text{“}x > 2\text{”}$
- $G(n) = \text{“}n \text{ is even and the sum of two primes}\text{”}$
- Remember Wason’s experiment!
  - $F(x) = \text{“}Person \ x \text{ flew}\text{”}$
Variables.

Propositions?

- \[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
- \( x > 2 \)
- \( n \) is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., \( Q(x) = "x \text{ is even}" \)

Same as boolean valued functions from 61A!

- \( P(n) = "\sum_{i=1}^{n} i = \frac{n(n+1)}{2}." \)
- \( R(x) = "x > 2" \)
- \( G(n) = "n \text{ is even and the sum of two primes}" \)
- Remember Wason’s experiment!
  \( F(x) = "\text{Person } x \text{ flew.}" \)
  \( C(x) = "\text{Person } x \text{ went to Chicago}" \)
Variables.

Propositions?

- \[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
- \( x > 2 \)
- \( n \) is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., \( Q(x) = "x \) is even”

Same as boolean valued functions from 61A!

- \( P(n) = \"\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.\" \)
- \( R(x) = "x > 2" \)
- \( G(n) = "n \) is even and the sum of two primes” \)
- Remember Wason’s experiment!
  - \( F(x) = "Person x flew.\"
  - \( C(x) = "Person x went to Chicago\"
- \( C(x) \implies F(x). \)
Variables.

Propositions?

\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]

\[ x > 2 \]

\[ n \text{ is even and the sum of two primes} \]

No. They have a free variable.

We call them **predicates**, e.g., \( Q(x) = \text{“x is even”} \)

Same as boolean valued functions from 61A!

\[ P(n) = \text{“} \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \text{“} \]

\[ R(x) = \text{“} x > 2 \text{”} \]

\[ G(n) = \text{“} n \text{ is even and the sum of two primes} \text{”} \]

Remember Wason’s experiment!

\[ F(x) = \text{“Person x flew.”} \]

\[ C(x) = \text{“Person x went to Chicago} \]

\[ C(x) \implies F(x). \text{ Theory from Wason’s.} \]
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) = \text{"x is even"}$
Same as boolean valued functions from 61A!

- $P(n) = \text{"$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$"}$
- $R(x) = \text{"$x > 2$"}$
- $G(n) = \text{"$n$ is even and the sum of two primes"}$

Remember Wason’s experiment!

$F(x) = \text{"Person } x \text{ flew."
}$
$C(x) = \text{"Person } x \text{ went to Chicago"}$

- $C(x) \implies F(x)$. Theory from Wason’s.
  If person $x$ goes to Chicago then person $x$ flew.
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) = “x$ is even”

Same as boolean valued functions from 61A!

- $P(n) = “\sum_{i=1}^{n} i = \frac{n(n+1)}{2}”$
- $R(x) = “x > 2”$
- $G(n) = “n$ is even and the sum of two primes”

Remember Wason’s experiment!

- $F(x) = “$Person $x$ flew.”
- $C(x) = “$Person $x$ went to Chicago

- $C(x) \implies F(x)$. Theory from Wason’s.

If person $x$ goes to Chicago then person $x$ flew.

Next:
Variables.

Propositions?

- \[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
- \( x > 2 \)
- \( n \) is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., \( Q(x) = \text{"x is even"} \)

Same as boolean valued functions from 61A!

- \( P(n) = \text{"} \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \text{"} \)
- \( R(x) = \text{"} x > 2 \text{"} \)
- \( G(n) = \text{"} n \) is even and the sum of two primes" \)
- Remember Wason’s experiment!
  - \( F(x) = \text{"} \text{Person } x \text{ flew.} \text{"} \)
  - \( C(x) = \text{"} \text{Person } x \text{ went to Chicago} \text{"} \)

- \( C(x) \implies F(x) \). Theory from Wason’s.
  If person \( x \) goes to Chicago then person \( x \) flew.

Next: Statements about boolean valued functions!!
Quantifiers...

There exists quantifier:

\((\exists x \in S)(P(x))\)

means "There exists an \(x\) in \(S\) where \(P(x)\) is true."

For example:

\((\exists x \in \mathbb{N})(x^2 = x)\)

Equivalent to "\((0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots\)"

Much shorter to use a quantifier!

For all quantifier;

\((\forall x \in S)(P(x))\)

means "For all \(x\) in \(S\), \(P(x)\) is True."

Examples:

"Adding 1 makes a bigger number."

\((\forall x \in \mathbb{N})(x + 1 > x)\)

"the square of a number is always non-negative"

\((\forall x \in \mathbb{N})(x^2 \geq 0)\)

Wait!

What is \(\mathbb{N}\)?
Quantifiers

There exists quantifier:

$$(\exists x \in S)(P(x))$$ means “There exists an $x$ in $S$ where $P(x)$ is true.”
Quantifiers.

There exists quantifier:

$$\exists x \in S)(P(x))$$ means “There exists an $$x$$ in $$S$$ where $$P(x)$$ is true.”

For example:

$$\exists x \in \mathbb{N})(x = x^2)$$
Quantifiers..

There exists quantifier:

\((\exists x \in S)(P(x))\) means “There exists an \(x\) in \(S\) where \(P(x)\) is true.”

For example:

\((\exists x \in \mathbb{N})(x = x^2)\)

Equivalent to “\((0 = 0)\)”
Quantifiers..

There exists quantifier:

$$(\exists x \in S)(P(x))$$ means “There exists an $x$ in $S$ where $P(x)$ is true.”

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to “$0 = 0 \lor 1 = 1$”
Quantifiers..

There exists quantifier:

\((\exists x \in S)(P(x))\) means “There exists an \(x\) in \(S\) where \(P(x)\) is true.”

For example:

\((\exists x \in \mathbb{N})(x = x^2)\)

Equivalent to “\((0 = 0) \lor (1 = 1) \lor (2 = 4)\)”
Quantifiers.

There exists quantifier:

\[(\exists x \in S)(P(x))\] means “There exists an \(x\) in \(S\) where \(P(x)\) is true.”

For example:

\[(\exists x \in \mathbb{N})(x = x^2)\]

Equivalent to “\((0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots\)"
There exists quantifier:

\[(\exists x \in S)(P(x))\] means “There exists an \(x\) in \(S\) where \(P(x)\) is true.”

For example:

\[(\exists x \in \mathbb{N})(x = x^2)\]

Equivalent to “\((0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots\)”

Much shorter to use a quantifier!
There exists quantifier:

$(\exists x \in S)(P(x))$ means “There exists an $x$ in $S$ where $P(x)$ is true.”

For example:

$(\exists x \in \mathbb{N})(x = x^2)$

Equivalent to “$(0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots$”

Much shorter to use a quantifier!

For all quantifier;

$(\forall x \in S) (P(x))$. means “For all $x$ in $S$, $P(x)$ is True.”
Quantifiers.

There exists quantifier:

$$(\exists x \in S)(P(x))$$ means “There exists an $x$ in $S$ where $P(x)$ is true.”

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to “$(0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots$”

Much shorter to use a quantifier!

For all quantifier;

$$(\forall x \in S)(P(x))$$ means “For all $x$ in $S$, $P(x)$ is True.”

Examples:
Quantifiers..

There exists quantifier:

\[(\exists x \in S)(P(x))\] means “There exists an \(x\) in \(S\) where \(P(x)\) is true.”

For example:

\[(\exists x \in \mathbb{N})(x = x^2)\]

Equivalent to “\((0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots\)”

Much shorter to use a quantifier!

For all quantifier;

\[(\forall x \in S)\ (P(x))\] means “For all \(x\) in \(S\), \(P(x)\) is True.”

Examples:

“Adding 1 makes a bigger number.”
Quantifiers.

There exists quantifier:

\((\exists x \in S)(P(x))\) means “There exists an \(x\) in \(S\) where \(P(x)\) is true.”

For example:

\[(\exists x \in \mathbb{N})(x = x^2)\]

Equivalent to “\((0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots\)”

Much shorter to use a quantifier!

For all quantifier;

\((\forall x \in S) (P(x))\). means “For all \(x\) in \(S\), \(P(x)\) is True .”

Examples:

“Adding 1 makes a bigger number.”

\[(\forall x \in \mathbb{N}) (x + 1 > x)\]
Quantifiers...

There exists quantifier:

\((\exists x \in S)(P(x))\) means “There exists an \(x\) in \(S\) where \(P(x)\) is true.”

For example:

\((\exists x \in \mathbb{N})(x = x^2)\)

Equivalent to “\((0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots\)”

Much shorter to use a quantifier!

For all quantifier:

\((\forall x \in S) (P(x))\). means “For all \(x\) in \(S\), \(P(x)\) is True.”

Examples:

“Adding 1 makes a bigger number.”

\((\forall x \in \mathbb{N}) (x + 1 > x)\)

“the square of a number is always non-negative”
Quantifiers..

There exists quantifier:

\[(\exists x \in S)(P(x))\] means “There exists an \(x\) in \(S\) where \(P(x)\) is true.”

For example:

\[(\exists x \in \mathbb{N})(x = x^2)\]

Equivalent to “\((0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots\)”

Much shorter to use a quantifier!

For all quantifier;

\[(\forall x \in S) (P(x))\] means “For all \(x\) in \(S\), \(P(x)\) is True.”

Examples:

“Adding 1 makes a bigger number.”

\[(\forall x \in \mathbb{N}) (x + 1 > x)\]

”the square of a number is always non-negative”

\[(\forall x \in \mathbb{N})(x^2 \geq 0)\]
Quantifiers..

There exists quantifier:

\((\exists x \in S)(P(x))\) means “There exists an \(x\) in \(S\) where \(P(x)\) is true.”

For example:

\((\exists x \in \mathbb{N})(x = x^2)\)

Equivalent to “\((0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots\)”

Much shorter to use a quantifier!

For all quantifier;

\((\forall x \in S)(P(x))\) means “For all \(x\) in \(S\), \(P(x)\) is True.”

Examples:

“Adding 1 makes a bigger number.”

\((\forall x \in \mathbb{N})(x + 1 > x)\)

”the square of a number is always non-negative”

\((\forall x \in \mathbb{N})(x^2 \geq 0)\)

Wait!
Quantifiers..

There exists quantifier:

$$(\exists x \in S)(P(x))$$ means “There exists an $x$ in $S$ where $P(x)$ is true.”

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to “$(0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots$”

Much shorter to use a quantifier!

For all quantifier;

$$(\forall x \in S)(P(x)).$$ means “For all $x$ in $S$, $P(x)$ is True.”

Examples:

“Adding 1 makes a bigger number.”

$$(\forall x \in \mathbb{N})(x + 1 > x)$$

”the square of a number is always non-negative”

$$(\forall x \in \mathbb{N})(x^2 \geq 0)$$

Wait! What is $\mathbb{N}$?
Quantifiers: universes.

Proposition: “For all natural numbers $n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.”

Proposition has universe:
Quantifiers: universes.

Proposition: “For all natural numbers \( n \), \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).”

Proposition has universe: “the natural numbers”.

Other proposition notation (for discussion):

- "\( d \mid n \)" means \( d \) divides \( n \) or \( \exists k \in \mathbb{Z}, n = kd \)."}

- \( 2 \mid 4 \)? True.

- \( 4 \mid 2 \)? False.
Quantifiers: universes.

Proposition: “For all natural numbers $n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.”

Proposition has universe: “the natural numbers”.

Universe examples include..

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
- $\mathbb{Z}^+$ (positive integers)
- $\mathbb{R}$ (real numbers)
- Any set: $S = \{Alice, Bob, Charlie, Donna\}$.
- See note 0 for more!

Other proposition notation (for discussion):

“$d|n$” means $d$ divides $n$
Quantifiers: universes.

Proposition: “For all natural numbers \( n \), \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).”

Proposition has universe: “the natural numbers”.

Universe examples include:
- \( \mathbb{N} = \{0, 1, \ldots\} \) (natural numbers).
- \( \mathbb{Z} = \{\ldots, -1, 0, \ldots\} \) (integers)
- \( \mathbb{Z}^+ \) (positive integers)
- \( \mathbb{R} \) (real numbers)
- Any set: \( S = \{Alice, Bob, Charlie, Donna\} \).
- See note 0 for more!

Other proposition notation (for discussion):
“\( d \mid n \)” means \( d \) divides \( n \)

or \( \exists k \in \mathbb{Z}, n = kd \).

2|4?
Quantifiers: universes.

Proposition: “For all natural numbers \( n \), \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).”

Proposition has universe: “the natural numbers”.

Universe examples include:

- \( \mathbb{N} = \{0, 1, \ldots\} \) (natural numbers).
- \( \mathbb{Z} = \{\ldots, -1, 0, \ldots\} \) (integers)
- \( \mathbb{Z}^+ \) (positive integers)
- \( \mathbb{R} \) (real numbers)
- Any set: \( S = \{Alice, Bob, Charlie, Donna\} \).
- See note 0 for more!

Other proposition notation (for discussion):

“\( d \mid n \)” means \( d \) divides \( n \)

or \( \exists k \in \mathbb{Z}, n = kd \).

\( 2 \mid 4? \) True.
Quantifiers: universes.

Proposition: “For all natural numbers $n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.”

Proposition has universe: “the natural numbers”.

Universe examples include:

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
- $\mathbb{Z}^+$ (positive integers)
- $\mathbb{R}$ (real numbers)
- Any set: $S = \{Alice, Bob, Charlie, Donna\}$.
- See note 0 for more!

Other proposition notation (for discussion):
“$d|n$” means $d$ divides $n$
or $\exists k \in \mathbb{Z}, n = kd$.

2|4? True.
4|2?
Quantifiers: universes.

Proposition: “For all natural numbers \( n \), \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).”

Proposition has universe: “the natural numbers”.

Universe examples include:

- \( \mathbb{N} = \{0, 1, \ldots\} \) (natural numbers).
- \( \mathbb{Z} = \{\ldots, -1, 0, \ldots\} \) (integers)
- \( \mathbb{Z}^+ \) (positive integers)
- \( \mathbb{R} \) (real numbers)
- Any set: \( S = \{Alice, Bob, Charlie, Donna\}\).
- See note 0 for more!

Other proposition notation (for discussion):

“\( d|n \)” means \( d \) divides \( n \)

or \( \exists k \in \mathbb{Z}, n = kd \).

2|4? True.
4|2? False.
Theory:

If a person travels to Chicago, he/she flies.

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

Chicago (x) = "x went to Chicago."

Flew (x) = "x flew".

Statement/theory:

∀ x ∈ {A, B, C, D}, Chicago (x) =⇒ Flew (x).

Chicago (A) = False.

Do we care about Flew (A)? No.

Chicago (B) =⇒ Flew (B) is true.

since Chicago (A) is False, Flew (B) = False.

Do we care about Chicago (B)? Yes.

Chicago (B) =⇒ Flew (B) ≡ ¬Flew (B) =⇒ ¬Chicago (B).

So Chicago (Bob) must be False.

Chicago (C) = True.

Do we care about Flew (C)? Yes.

Chicago (C) =⇒ Flew (C) means Flew (C) must be true.

Flew (D) = True.

Do we care about Chicago (D)? No.

Chicago (D) =⇒ Flew (D) is true if Flew (D) is true.

Only have to turn over cards for Bob and Charlie.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”
Theory: “If a person travels to Chicago, he/she flies.”
Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Only have to turn over cards for Bob and Charlie.
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?
Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew. Which cards do you need to flip to test the theory?

\[
\text{Chicago}(x) = \text{“x went to Chicago.”}
\]
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\( Chicago(x) = \text{“} x \text{ went to Chicago.}\) \( Flew(x) = \text{“} x \text{ flew}\)
Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{"x went to Chicago."} \quad \text{Flew}(x) = \text{"x flew"} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \)
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”
Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.
Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“}x\text{ went to Chicago.}\] \quad \text{Flew}(x) = \text{“}x\text{ flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \Rightarrow \text{Flew}(x) \)
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she flies.”
Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.
Which cards do you need to flip to test the theory?

$Chicago(x) = \text{“}x\text{ went to Chicago.}$$ \quad Flew(x) = \text{“}x\text{ flew”}$

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \Rightarrow Flew(x)$

$Chicago(A) = \text{False}$ .
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”
Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.
Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{ A, B, C, D \}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?
No.
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\( \text{Chicago}(x) = \text{“} x \text{ went to Chicago.}\) \quad \text{Flew}(x) = \text{“} x \text{ flew”} \)

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,

Only have to turn over cards for Bob and Charlie.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\textit{Chicago}(A) = \text{False} . Do we care about \textit{Flew}(A)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,

\( \text{Flew}(B) = \text{False} . \)
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[
\text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”}
\]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\text{Chicago}(A) = \text{False} . \text{ Do we care about } \text{Flew}(A) ?

No. \text{Chicago}(A) \implies \text{Flew}(A) \text{ is true.}

since \text{Chicago}(A) \text{ is False } ,

\text{Flew}(B) = \text{False} . \text{ Do we care about } \text{Chicago}(B) ?
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[
\text{Chicago}(x) = \text{“} x \text{ went to Chicago.}\]
\[
\text{Flew}(x) = \text{“} x \text{ flew}\]

Statement/theory: \(\forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x)\)

\(\text{Chicago}(A) = \text{False}\). Do we care about \(\text{Flew}(A)\)?
No. \(\text{Chicago}(A) \implies \text{Flew}(A)\) is true.

since \(\text{Chicago}(A)\) is \text{False} ,

\(\text{Flew}(B) = \text{False}\). Do we care about \(\text{Chicago}(B)\)?
Yes. \(\text{Chicago}(B) \implies \text{Flew}(B)\)
**Back to: Wason’s experiment:1**

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“}x\text{ went to Chicago.”} \quad \text{Flew}(x) = \text{“}x\text{ flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \lnot \text{Flew}(B) \implies \lnot \text{Chicago}(B) \).
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False. 
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ Chicago(x) = “x \text{ went to Chicago.”} \quad Flew(x) = “x \text{ flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x) \)

\( Chicago(A) = \text{False} \). Do we care about \( Flew(A) \)?

No. \( Chicago(A) \implies Flew(A) \) is true.

since \( Chicago(A) \) is False,

\( Flew(B) = \text{False} \). Do we care about \( Chicago(B) \)?

Yes. \( Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B). \)

So \( Chicago(Bob) \) must be False.

\( Chicago(C) = \text{True} \).
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\( \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \)

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False}. \) Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False},

\( \text{Flew}(B) = \text{False}. \) Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B). \)

So \( \text{Chicago}(Bob) \) must be \text{False}.

\( \text{Chicago}(C) = \text{True}. \) Do we care about \( \text{Flew}(C) \)?
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“} x \text{ went to Chicago.}\] \quad \text{Flew}(x) = \text{“} x \text{ flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False.

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \lnot \text{Flew}(B) \implies \lnot \text{Chicago}(B) \).

So \( \text{Chicago}(\text{Bob}) \) must be \text{False} .

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“}x \text{ went to Chicago.} \quad \text{Flew}(x) = \text{“}x \text{ flew} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False .

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} \).
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False .

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} \). Do we care about \( \text{Chicago}(D) \)?
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False .

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} \). Do we care about \( \text{Chicago}(D) \)?

No.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[
\text{Chicago}(x) = \text{“} x \text{ went to Chicago.} \quad \text{Flew}(x) = \text{“} x \text{ flew}\]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be \text{False} .

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} \). Do we care about \( \text{Chicago}(D) \)?

No. \( \text{Chicago}(D) \implies \text{Flew}(D) \) is true if \( \text{Flew}(D) \) is true.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[
\text{Chicago}(x) = \text{“} x \text{ went to Chicago.}\quad \text{Flew}(x) = \text{“} x \text{ flew}\]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} . \) Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} . \) Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) . \)

So \( \text{Chicago}(Bob) \) must be False .

\( \text{Chicago}(C) = \text{True} . \) Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} . \) Do we care about \( \text{Chicago}(D) \)?

No. \( \text{Chicago}(D) \implies \text{Flew}(D) \) is true if \( \text{Flew}(D) \) is true.

Only have to turn over cards for Bob and Charlie.
More for all quantifiers examples.

"doubling a number always makes it larger"

\[(\forall x \in \mathbb{N}) (2x > x)\]

False

Consider \(x = 0\)

Can fix statement...

\[(\forall x \in \mathbb{N}) (2x \geq x)\]

True

"Square of any natural number greater than 5 is greater than 25."

\[(\forall x \in \mathbb{N}) (x > 5 \Rightarrow x^2 > 25)\]

Idea alert: Restrict domain using implication.

Later we may omit universe if clear from context.
More for all quantifiers examples.

- “doubling a number always makes it larger”
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x)\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N})(2x > x)\] False
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x)\]

False  Consider \(x = 0\)
More for all quantifiers examples.

▶ “doubling a number always makes it larger”

$$(\forall x \in \mathbb{N})(2x > x) \quad \text{False} \quad \text{Consider } x = 0$$

Can fix statement...
More for all quantifiers examples.

▶ “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x)\] False  Consider \(x = 0\)

Can fix statement...

\[(\forall x \in \mathbb{N}) (2x \geq x)\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in N) (2x > x)\]  **False**  
Consider \(x = 0\)

Can fix statement...

\[(\forall x \in N) (2x \geq x)\]  **True**
More for all quantifiers examples.

▶ “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider} \quad x = 0\]

Can fix statement...

\[(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}\]

▶ “Square of any natural number greater than 5 is greater than 25.”
More for all quantifiers examples.

- “doubling a number always makes it larger”
  \[(\forall x \in \mathbb{N}) (2x > x)\] False  Consider \(x = 0\)

Can fix statement...

\[(\forall x \in \mathbb{N}) (2x \geq x)\] True

- “Square of any natural number greater than 5 is greater than 25.”
  \[(\forall x \in \mathbb{N}) \]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in N) (2x > x) \quad \text{False} \quad \text{Consider } x = 0\]

Can fix statement...

\[(\forall x \in N) (2x \geq x) \quad \text{True}\]

- “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in N) (x > 5)\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0\]

Can fix statement...

\[(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}\]

- “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in \mathbb{N})(x > 5 \implies \quad)\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in N) \ (2x > x) \quad \text{False} \quad \text{Consider} \quad x = 0\]

Can fix statement...

\[(\forall x \in N) \ (2x \geq x) \quad \text{True}\]

- “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in N) \ (x > 5 \implies x^2 > 25).\]
More for all quantifiers examples.

- “doubling a number always makes it larger”
  \[(\forall x \in \mathbb{N}) (2x > x)\]  
  \(\text{False}\)  
  Consider \(x = 0\)

  Can fix statement...

  \[(\forall x \in \mathbb{N}) (2x \geq x)\]  
  \(\text{True}\)

- “Square of any natural number greater than 5 is greater than 25.”

  \[(\forall x \in \mathbb{N})(x > 5 \implies x^2 > 25)\].

  Idea alert:
More for all quantifiers examples.

- "doubling a number always makes it larger"

\[(\forall x \in N) (2x > x) \quad \text{False} \quad \text{Consider } x = 0\]

Can fix statement...

\[(\forall x \in N) (2x \geq x) \quad \text{True}\]

- "Square of any natural number greater than 5 is greater than 25."

\[(\forall x \in N)(x > 5 \implies x^2 > 25).\]

Idea alert: Restrict domain using implication.
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0\]

Can fix statement...

\[(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}\]

- “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in \mathbb{N})(x > 5 \implies x^2 > 25).\]

Idea alert: Restrict domain using implication.

Later we may omit universe if clear from context.
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[ \exists y \in \mathbb{N} \ (\forall x \in \mathbb{N}) (y = x^2) \]

False

- In English: “the square of every natural number is a natural number.”

\[ (\forall x \in \mathbb{N}) (\exists y \in \mathbb{N}) (y = x^2) \]

True
Quantifiers...not commutative.

- In English: “there is a natural number that is the square of every natural number”.

  $$(\exists y \in N)$$
Quantifiers...not commutative.

▷ In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) (\forall x \in N)\]
Quantifiers..not commutative.

In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) (\forall x \in N) (y = x^2)\]
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2)\] True
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) \ (\forall x \in N) \ (y = x^2)\] False

- In English: “the square of every natural number is a natural number.”
In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) (\forall x \in N) (y = x^2)\]  False

In English: “the square of every natural number is a natural number.”

\[(\forall x \in N)\]
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in \mathbb{N})(\forall x \in \mathbb{N})(y = x^2)\] False

- In English: “the square of every natural number is a natural number.”

\[(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})\]
Quantifiers not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[ (\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2) \quad \text{False} \]

- In English: “the square of every natural number is a natural number.”

\[ (\forall x \in \mathbb{N}) (\exists y \in \mathbb{N}) (y = x^2) \]
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) (\forall x \in N) (y = x^2) \quad \text{False}\]

- In English: “the square of every natural number is a natural number.”

\[(\forall x \in N)(\exists y \in N) (y = x^2) \quad \text{True}\]
Quantifiers...not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2)\]  \text{False}

- In English: “the square of every natural number is a natural number.”

\[(\forall x \in \mathbb{N})(\exists y \in \mathbb{N}) (y = x^2)\]  \text{True}
Consider

\[ \neg (\forall x \in S)(P(x)), \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is,

\[ \neg (\forall x \in S)(P(x)) \iff \exists x \in S(\neg P(x)) \]

What we do in this course! We consider claims.

Claim:

\[ (\forall x)P(x) \]

"For all inputs \( x \) the program works."

For False, find \( x \), where \( \neg P(x) \).

Counterexample. Bad input. Case that illustrates bug.

For True: prove claim.

Next lectures...
Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an $x$ in $S$ where $P(x)$ does not hold.
Quantifiers....negation...DeMorgan again.

Consider

\neg (\forall x \in S)(P(x)),

English: there is an \(x\) in \(S\) where \(P(x)\) does not hold.

That is,
Consider

\[ \neg(\forall x \in S)(P(x)), \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is,

\[ \neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)). \]
Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an $x$ in $S$ where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim:

$$\forall x (P(x))$$

"For all inputs $x$ the program works."

For False, find $x$, where $\neg P(x)$.

Counterexample.

Bad input.

Case that illustrates bug.

For True: prove claim.

Next lectures...
Consider
\[ \neg (\forall x \in S)(P(x)), \]
English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
That is,
\[ \neg (\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]
What we do in this course! We consider claims.

Claim: \( (\forall x) P(x) \)
Consider

\[ \neg (\forall x \in S)(P(x)), \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is,

\[ \neg (\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]

What we do in this course! We consider claims.

**Claim**: \((\forall x)\ P(x)\)   “For all inputs \( x \) the program works.”
Consider
\[ \neg(\forall x \in S)(P(x)), \]
English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
That is,
\[ \neg(\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]

What we do in this course! We consider claims.

**Claim:** \( (\forall x) P(x) \) “For all inputs \( x \) the program works.”
For *False* , find \( x \), where \( \neg P(x) \).
Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an \(x\) in \(S\) where \(P(x)\) does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: \((\forall x) P(x)\) “For all inputs \(x\) the program works.”
For False, find \(x\), where \(\neg P(x)\).

Counterexample.
Consider
\[ \neg(\forall x \in S)(P(x)), \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is,
\[ \neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)). \]

What we do in this course! We consider claims.

**Claim**: \( \forall x \) \( P(x) \)  “For all inputs \( x \) the program works.”
For **False**, find \( x \), where \( \neg P(x) \).
   Counterexample.
   Bad input.
Consider
\[ \neg (\forall x \in S)(P(x)), \]
English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
That is,
\[ \neg (\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]

What we do in this course! We consider claims.

**Claim:** \((\forall x) P(x)\) “For all inputs \( x \) the program works.”

For False, find \( x \), where \( \neg P(x) \).
Counterexample.
Bad input.
Case that illustrates bug.
Consider

\[ \neg (\forall x \in S)(P(x)), \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is,

\[ \neg (\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]

What we do in this course! We consider claims.

**Claim:** \( (\forall x) \ P(x) \)  
“For all inputs \( x \) the program works.”

For **False**, find \( x \), where \( \neg P(x) \).

- Counterexample.
- Bad input.
- Case that illustrates bug.

For **True**: prove claim.
Consider
\[ \neg (\forall x \in S)(P(x)), \]
English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
That is,
\[ \neg (\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]

What we do in this course! We consider claims.

**Claim**: \( (\forall x) P(x) \) “For all inputs \( x \) the program works.”
For **False**, find \( x \), where \( \neg P(x) \).
  - Counterexample.
  - Bad input.
  - Case that illustrates bug.
For **True**: prove claim. Next lectures...
Negation of exists.

Consider
Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$
Negation of exists.

Consider

\[ \neg (\exists x \in S)(P(x)) \]

English: means that there is no \( x \in S \) where \( P(x) \) is true.
Consider

\[ \neg(\exists x \in S)(P(x)) \]

English: means that there is no \( x \in S \) where \( P(x) \) is true. English: means that for all \( x \in S \), \( P(x) \) does not hold.
Negation of exists.

Consider

\[ \neg (\exists x \in S)(P(x)) \]

English: means that there is no \( x \in S \) where \( P(x) \) is true. English: means that for all \( x \in S \), \( P(x) \) does not hold.

That is,

\[ \neg (\exists x \in S)(P(x)) \iff \forall(x \in S)\neg P(x). \]
Which Theorem?

Theorem: \((\forall n \in N) \neg (\exists a, b, c \in N) (n \geq 3 \implies a^n + b^n = c^n)\)
Theorem: $(\forall n \in \mathbb{N}) \neg(\exists a, b, c \in \mathbb{N}) (n \geq 3 \implies a^n + b^n = c^n)$

Which Theorem?
Which Theorem?

Theorem: \( (\forall n \in \mathbb{N}) \neg (\exists a, b, c \in \mathbb{N}) (n \geq 3 \implies a^n + b^n = c^n) \)
Which Theorem?
Fermat’s Last Theorem!
Theorem: $(\forall n \in N) \neg(\exists a, b, c \in N) (n \geq 3 \implies a^n + b^n = c^n)$

Which Theorem?

Fermat’s Last Theorem!

Remember Special Triangles: for $n = 2$, we have 3,4,5 and 5,7, 12 and ...
Theorem: $(\forall n \in N) \neg (\exists a, b, c \in N) (n \geq 3 \implies a^n + b^n = c^n)$

Which Theorem?

Fermat’s Last Theorem!

Remember Special Triangles: for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

1637: Proof doesn’t fit in the margins.
Which Theorem?

Theorem: $(\forall n \in \mathbb{N}) \neg(\exists a, b, c \in \mathbb{N})(n \geq 3 \implies a^n + b^n = c^n)$

Which Theorem?

Fermat’s Last Theorem!

Remember Special Triangles: for $n = 2$, we have 3, 4, 5 and 5, 7, 12 and ...

1637: Proof doesn’t fit in the margins.

1993: Wiles ...(based in part on Ribet’s Theorem)
Which Theorem?

Theorem: \((\forall n \in \mathbb{N}) \neg (\exists a, b, c \in \mathbb{N}) (n \geq 3 \implies a^n + b^n = c^n)\)

Which Theorem?

Fermat’s Last Theorem!

Remember Special Triangles: for \(n = 2\), we have 3,4,5 and 5,7, 12 and ...

1637: Proof doesn’t fit in the margins.

1993: Wiles ... (based in part on Ribet’s Theorem)

DeMorgan Restatement:
Theorem: $(\forall n \in \mathbb{N}) \neg (\exists a, b, c \in \mathbb{N}) (n \geq 3 \implies a^n + b^n = c^n)$

Which Theorem?

Fermat’s Last Theorem!

Remember Special Triangles: for $n = 2$, we have 3,4,5 and 5,7,12 and ...

1637: Proof doesn’t fit in the margins.

1993: Wiles ...(based in part on Ribet’s Theorem)

DeMorgan Restatement:

Theorem: $\neg (\exists n \in \mathbb{N}) (\exists a, b, c \in \mathbb{N}) (n \geq 3 \implies a^n + b^n = c^n)$
Summary.

Propositions are statements that are true or false.
Summary.

Propositions are statements that are true or false.
Propositional forms use $\land, \lor, \neg$.

DeMorgan's Laws: "Flip and Distribute negation"

$\neg(P \lor Q) \iff (\neg P \land \neg Q)$

$\neg\forall x P(x) \iff \exists x \neg P(x)$.
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land$, $\lor$, $\neg$.

Propositional forms correspond to truth tables.

DeMorgan's Laws: "Flip and Distribute negation"

$\neg (P \lor Q) \iff (\neg P \land \neg Q)$

$\neg \forall x P(x) \iff \exists x \neg P(x)$.
Summary.

Propositions are statements that are true or false.
Propositional forms use $\land, \lor, \neg$.
Propositional forms correspond to truth tables.
Logical equivalence of forms means same truth tables.
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land$, $\lor$, $\neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q$
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land, \lor, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \lor Q$.
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land$, $\lor$, $\neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \lor Q$.

Contrapositive: $\neg Q \implies \neg P$.
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land, \lor, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \lor Q$.

Contrapositive: $\neg Q \implies \neg P$

Converse: $Q \implies P$
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land, \lor, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \lor Q$.

Contrapositive: $\neg Q \implies \neg P$

Converse: $Q \implies P$

Predicates: Statements with “free” variables.
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land, \lor, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \lor Q$.

Contrapositive: $\neg Q \implies \neg P$

Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x \ P(x), \ \exists y \ Q(y)$
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land, \lor, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \lor Q$.

Contrapositive: $\neg Q \implies \neg P$

Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x \, P(x), \exists y \, Q(y)$

Now can state theorems!
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land, \lor, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \lor Q$.

Contrapositive: $\neg Q \implies \neg P$.

Converse: $Q \implies P$.

Predicates: Statements with “free” variables.

Quantifiers: $\forall x \ P(x), \exists y \ Q(y)$.

Now can state theorems! And disprove false ones!
Propositions are statements that are true or false.
Propositional forms use $\land, \lor, \neg$.
Propositional forms correspond to truth tables.
Logical equivalence of forms means same truth tables.
Implication: $P \implies Q \iff \neg P \lor Q$.
Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$
Predicates: Statements with “free” variables.
Quantifiers: $\forall x \, P(x), \exists y \, Q(y)$
Now can state theorems! And disprove false ones!
DeMorgans Laws: “Flip and Distribute negation”
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land, \lor, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \lor Q$.

Contrapositive: $\neg Q \implies \neg P$

Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x \ P(x), \exists y \ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgans Laws: “Flip and Distribute negation”

$\neg(P \lor Q) \iff$
Summary.

Propositions are statements that are true or false.
Propositional forms use $\land, \lor, \neg$.
Propositional forms correspond to truth tables.
Logical equivalence of forms means same truth tables.
Implication: $P \implies Q \iff \neg P \lor Q$.
Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$
Predicates: Statements with “free” variables.
Quantifiers: $\forall x\ P(x), \exists y\ Q(y)$
Now can state theorems! And disprove false ones!
DeMorgan's Laws: “Flip and Distribute negation”
$\neg (P \lor Q) \iff (\neg P \land \neg Q)$
$\neg \forall x\ P(x) \iff \exists x\ \neg P(x)$
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land, \lor, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \lor Q$.

Contrapositive: $\neg Q \implies \neg P$

Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x \ P(x), \exists y \ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgans Laws: “Flip and Distribute negation”

$\neg(P \lor Q) \iff (\neg P \land \neg Q)$

$\neg\forall x \ P(x) \iff \exists x \ \neg P(x)$. 
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land, \lor, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \lor Q$.

Contrapositive: $\neg Q \implies \neg P$.

Converse: $Q \implies P$.

Predicates: Statements with “free” variables.

Quantifiers: $\forall x \ P(x), \exists y \ Q(y)$.

Now can state theorems! And disprove false ones!

DeMorgans Laws: “Flip and Distribute negation”

$\neg(P \lor Q) \iff (\neg P \land \neg Q)$

$\neg \forall x \ P(x) \iff \exists x \ \neg P(x)$.

Next Time: proofs!