What are our super powerful programs/processors doing?

Logic and Proofs!

Induction ≡ Recursion.

What can computers do?

Work with discrete objects.

Discrete Math ⇒ immense application.

Computers learn and interact with the world?

E.g. AI/machine learning, cyber-physical systems/robotics, networking/wireless communications, ...

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Probability!
Programming + Data Structures/Algorithms
Programming + Data Structures/Algorithms + Microprocessors
Discrete Math and Probability Theory

Programming + Data Structures/Algorithms + Microprocessors ≡ Superpower!

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Instructor: Sanjit Seshia.
Instructors

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Professor of EECS (office: 566 Cory)
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19th year on the faculty at Berkeley!
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Research: Formal Methods
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Taught: CS 70, EECS 149/249A, CS 172, EECS 144/244, EECS 219C, EECS149.1x on edX, ...
Instructors

• Alistair Sinclair

• Professor of CS (office 677 Soda)

• @ Berkeley since pre-history (1994)

• Originally from the UK: undergrad @ Cambridge, PhD @ Edinburgh

• Research: CS Theory, esp. algorithms, randomness, statistical physics, stochastic processes…

• Teaching: CS70, CS170, CS172, CS174 + various grad classes
Course Webpage: http://eecs70.org/
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Explains policies, has office hours, schedule, homework, exam dates, etc.
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One midterm, final.
Admin

Course Webpage: http://eecs70.org/

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One midterm, final.
   midterm on March 6
Course Webpage:  [http://eecs70.org/](http://eecs70.org/)

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  - midterm on March 6
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Course Webpage:  

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Questions/Announcements ⇒ Ed Discussion

Grading – see course webpage
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One midterm, final.
    midterm on March 6
    final on May 10

Questions/Announcements ⇒ Ed Discussion

Grading – see course webpage
    homework/no-homework option continues
Learning and Teaching

Variety of Background Knowledge on the Topics of CS70

"mini-vitamins" before lecture can help

Variety of Learning Styles

take "notes" during lecture?

Variety of Teaching Styles

slides vs. no slides

Learn by Doing (Mathematical Modeling/Problem Solving)
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Learn by Doing (Mathematical Modeling/Problem Solving)
Today: Note 1.

The language of proofs!

Mathematical Logic!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws
Today: Note 1.  (Note 0 is background. Do read/skim it.)
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The language of proofs!
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The language of proofs! Mathematical Logic!
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The language of proofs! Mathematical Logic!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan’s Laws
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
\[ 2 + 2 = 4 \]
\[ 2 + 2 = 3 \]
\[ 826\text{th digit of } \pi \text{ is 4} \]
\[ \text{Stephen Curry is a good basketball player} \]
\[ \text{All evens } > 2 \text{ are unique sums of 2 primes} \]
\[ 4 + 5 \]
\[ x + x \]
Propositions: Statements that are true or false.

\[
\sqrt{2} \text{ is irrational} \\
2 + 2 = 4 \\
2 + 2 = 3 \\
826\text{th digit of pi is 4} \\
\text{Stephen Curry is a good basketball player} \\
\text{All evens} > 2 \text{ are unique sums of 2 primes} \\
4 + 5 \\
x + x
\]
Propositions: Statements that are true or false.

- \( \sqrt{2} \) is irrational  \hspace{2cm} \text{Proposition} \hspace{2cm} \text{True}
- \( 2 + 2 = 4 \)
- \( 2 + 2 = 3 \)
- 826th digit of pi is 4
- Stephen Curry is a good basketball player
- All evens \( > 2 \) are unique sums of 2 primes
- \( 4 + 5 \)
- \( x + x \)
Propositions: Statements that are true or false.

- $\sqrt{2}$ is irrational
- $2 + 2 = 4$ (Proposition, True)
- $2 + 2 = 3$ (Proposition, False)
- 826th digit of pi is 4
- Stephen Curry is a good basketball player (Not a Proposition)
- All evens $> 2$ are unique sums of 2 primes
- $4 + 5$
- $x + x$

Again: "value" of a proposition is True or False.
Propositions: Statements that are true or false.

\[
\sqrt{2} \text{ is irrational} \\
2 + 2 = 4 \\
2 + 2 = 3 \\
826\text{th digit of pi is 4} \\
\text{Stephen Curry is a good basketball player} \\
\text{All evens} > 2 \text{ are unique sums of 2 primes} \\
4 + 5 \\
x + x
\]
Propositions: Statements that are true or false.

<table>
<thead>
<tr>
<th>Proposition</th>
<th>True/False</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{2}$ is irrational</td>
<td>True</td>
</tr>
<tr>
<td>$2 + 2 = 4$</td>
<td>True</td>
</tr>
<tr>
<td>$2 + 2 = 3$</td>
<td>False</td>
</tr>
<tr>
<td>826th digit of pi is 4</td>
<td>False</td>
</tr>
<tr>
<td>Stephen Curry is a good basketball player</td>
<td>False</td>
</tr>
<tr>
<td>All evens $&gt; 2$ are unique sums of 2 primes</td>
<td>False</td>
</tr>
<tr>
<td>$4 + 5$</td>
<td></td>
</tr>
<tr>
<td>$x + x$</td>
<td></td>
</tr>
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</table>
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\[ \sqrt{2} \text{ is irrational} \]
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\[ \sqrt{2} \text{ is irrational} \]
\[ 2 + 2 = 4 \] Proposition True
\[ 2 + 2 = 3 \] Proposition False
\[ 826\text{th digit of pi is 4} \] Proposition False
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\[ 4 + 5 \] Proposition False
\[ x + x \] Proposition False
Propositions: Statements that are true or false.

\[
\begin{aligned}
\sqrt{2} &\text{ is irrational} & \text{Proposition} & \text{True} \\
2 + 2 &\text{ = 4} & \text{Proposition} & \text{True} \\
2 + 2 &\text{ = 3} & \text{Proposition} & \text{False} \\
826\text{th digit of } \pi &\text{ is 4} & \text{Proposition} & \text{False} \\
\text{Stephen Curry is a good basketball player} & & \text{Not a Proposition} & \\
\text{All evens } > 2 \text{ are unique sums of 2 primes} & & \text{Not a Proposition} & \\
4 + 5 & & \text{Not a Proposition} & \\
x + x & & \text{Not a Proposition} &
\end{aligned}
\]
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
2+2 = 4
2+2 = 3
826th digit of pi is 4
Stephen Curry is a good basketball player
All evens > 2 are unique sums of 2 primes
4 + 5
x + x
Propositions: Statements that are true or false.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Proposition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{2}$ is irrational</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>$2+2 = 4$</td>
<td>Proposition</td>
<td>True</td>
</tr>
<tr>
<td>$2+2 = 3$</td>
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</tr>
<tr>
<td>826th digit of pi is 4</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>Stephen Curry is a good basketball player</td>
<td>Not a Proposition</td>
<td></td>
</tr>
<tr>
<td>All evens $&gt; 2$ are unique sums of 2 primes</td>
<td>Proposition</td>
<td>False</td>
</tr>
<tr>
<td>$4 + 5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x + x$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Propositions: Statements that are true or false.

<table>
<thead>
<tr>
<th>Proposition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sqrt{2}) is irrational</td>
<td>True</td>
</tr>
<tr>
<td>(2 + 2 = 4)</td>
<td>True</td>
</tr>
<tr>
<td>(2 + 2 = 3)</td>
<td>False</td>
</tr>
<tr>
<td>826th digit of pi is 4</td>
<td>False</td>
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<td>Stephen Curry is a good basketball player</td>
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<tr>
<td>All evens (&gt; 2) are unique sums of 2 primes</td>
<td>False</td>
</tr>
<tr>
<td>(4 + 5)</td>
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</tr>
<tr>
<td>(x + x)</td>
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Propositions: Statements that are true or false.

- $\sqrt{2}$ is irrational  
  Proposition: True

- $2+2 = 4$  
  Proposition: True

- $2+2 = 3$  
  Proposition: False

- 826th digit of pi is 4  
  Proposition: False

- Stephen Curry is a good basketball player  
  Not a Proposition

- All evens > 2 are unique sums of 2 primes  
  Proposition: False

- $4 + 5$  
  Not a Proposition

- $x + x$  
  Not a Proposition.
Propositions: Statements that are true or false.

\( \sqrt{2} \) is irrational
2+2 = 4
2+2 = 3
826th digit of pi is 4
Stephen Curry is a good basketball player
All evens > 2 are unique sums of 2 primes
4 + 5
\( x + x \)

Again: “value” of a proposition is ...

Proposition: True
Proposition: True
Proposition: False
Proposition: False
Not a Proposition
Proposition: False
Not a Proposition
Not a Proposition
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \quad \text{Proposition} \quad \text{True} \]
\[ 2+2 = 4 \quad \text{Proposition} \quad \text{True} \]
\[ 2+2 = 3 \quad \text{Proposition} \quad \text{False} \]
\[ \text{826th digit of } \pi \text{ is 4} \quad \text{Proposition} \quad \text{False} \]
\[ \text{Stephen Curry is a good basketball player} \quad \text{Not a Proposition} \]
\[ \text{All evens } > 2 \text{ are unique sums of 2 primes} \quad \text{Proposition} \quad \text{False} \]
\[ 4 + 5 \quad \text{Not a Proposition.} \]
\[ x + x \quad \text{Not a Proposition.} \]

Again: “value” of a proposition is ... True or False
Propositional Forms.

Put propositions together to make another...

Examples:

¬(2 + 2 = 4) – a proposition that is False

2 + 2 = 3 ∧ 2 + 2 = 4 – a proposition that is False

2 + 2 = 3 ∨ 2 + 2 = 4 – a proposition that is True
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

“$P \land Q$” is True when both $P$ and $Q$ are True.
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): \( P \land Q \)

"\( P \land Q \)" is True when both \( P \) and \( Q \) are True. Else False.
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Negation (“not”): \( \neg P \)
Propositional Forms.

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Examples:

\( \neg \) "\((2 + 2 = 4)\)" – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

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"\( P \land Q \)" is True when both \( P \) and \( Q \) are True. Else False.

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"\( \neg P \)" is True when \( P \) is False. Else False.

Examples:

\( \neg "(2 + 2 = 4)" \) – a proposition that is ... False
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True when both $P$ and $Q$ are True. Else False.

Disjunction ("or"): $P \lor Q$

"$P \lor Q$" is True when at least one $P$ or $Q$ is True. Else False.

Negation ("not"): $\neg P$

"$\neg P$" is True when $P$ is False. Else False.

Examples:

$\neg "(2 + 2 = 4)"$ – a proposition that is ... False

"$2 + 2 = 3" \land "2 + 2 = 4$" – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): \( P \land Q \)

"\( P \land Q \)" is True when both \( P \) and \( Q \) are True. Else False.

Disjunction ("or"): \( P \lor Q \)

"\( P \lor Q \)" is True when at least one \( P \) or \( Q \) is True. Else False.

Negation ("not"): \( \neg P \)

"\( \neg P \)" is True when \( P \) is False. Else False.

Examples:

\( \neg \) "\((2 + 2 = 4)\)" – a proposition that is ... False

"\( 2 + 2 = 3 \) \land \) "\( 2 + 2 = 4 \)" – a proposition that is ... False
Propositional Forms.

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Examples:

\( \neg "(2 + 2 = 4)" \) – a proposition that is ... False

"2 + 2 = 3" \( \land "2 + 2 = 4" \) – a proposition that is ... False

"2 + 2 = 3" \( \lor "2 + 2 = 4" \) – a proposition that is ...
Propositional Forms.

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Conjunction ("and"): $P \land Q$

"$P \land Q$" is True when both $P$ and $Q$ are True. Else False.

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"$\neg P$" is True when $P$ is False. Else False.

Examples:

$\neg "(2 + 2 = 4)"$ – a proposition that is ... False

"$2 + 2 = 3$" $\land$ "$2 + 2 = 4$" – a proposition that is ... False

"$2 + 2 = 3$" $\lor$ "$2 + 2 = 4$" – a proposition that is ... True
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True when both $P$ and $Q$ are True. Else False.

Disjunction ("or"): $P \lor Q$

"$P \lor Q$" is True when at least one $P$ or $Q$ is True. Else False.

Negation ("not"): $\neg P$

"$\neg P$" is True when $P$ is False. Else False.

Examples:

$\neg (2 + 2 = 4)$ – a proposition that is ... False

"2 + 2 = 3" \ $\land$ "2 + 2 = 4" – a proposition that is ... False

"2 + 2 = 3" \ $\lor$ "2 + 2 = 4" – a proposition that is ... True
Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational”} \]
Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational} \text{”} \]
\[ Q = \text{“} 826\text{th digit of pi is 2} \text{”} \]
Propositional Forms: quick check!

$P = \text{“} \sqrt{2} \text{ is rational”}$
$Q = \text{“} 826\text{th digit of pi is 2”}$
Propositional Forms: quick check!

\[ P = \text{“}\sqrt{2} \text{ is rational”} \]
\[ Q = \text{“826th digit of pi is 2”} \]

\[ P \text{ is } \ldots \]
\[ Q \text{ is } \ldots \]

\[ P \land Q \text{ } \ldots \]
\[ P \lor Q \text{ } \ldots \]

\[ \neg P \text{ } \ldots \]
Propositional Forms: quick check!

\[ P = \text{"} \sqrt{2} \text{ is rational} \]
\[ Q = \text{"} 826\text{th digit of } \pi \text{ is 2} \]

\[ P \text{ is } \ldots \text{False } . \]
Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational”} \]
\[ Q = \text{“} 826 \text{th digit of pi is 2”} \]

\[ P \text{ is } \ldots \text{False} \]
\[ Q \text{ is } \ldots \]
Propositional Forms: quick check!

\[ P = \text{"} \sqrt{2} \text{ is rational"} \]
\[ Q = \text{"} 826\text{th digit of pi is 2"} \]

\[ P \text{ is ...} \text{False} \text{ .} \]
\[ Q \text{ is ...} \text{True} \text{ .} \]
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\[ P \] is ... \text{False} .
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\[ P \land Q \] ...
Propositional Forms: quick check!

\[ P = \text{“}\sqrt{2} \text{ is rational”} \]
\[ Q = \text{“}826\text{th digit of pi is 2”} \]

- \( P \) is ... \text{False}.
- \( Q \) is ... \text{True}.

\( P \land Q \) ... \text{False}
Propositional Forms: quick check!

\[ P = \text{“\(\sqrt{2}\) is rational”} \]
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- \(P\) is \underline{False}.
- \(Q\) is \underline{True}.

\[ P \land Q \ldots \underline{False} \]
\[ P \lor Q \ldots \]
Propositional Forms: quick check!

\[ P = "\sqrt{2} \text{ is rational}" \]
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\[ P \text{ is } ... \text{False} . \]
\[ Q \text{ is } ... \text{True} . \]

\[ P \land Q \text{ ... False} \]
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\[ P \text{ is ...False .} \]
\[ Q \text{ is ...True .} \]

\[ P \land Q \text{ ... False} \]
\[ P \lor Q \text{ ... True} \]
\[ \neg P \text{ ...} \]
Propositional Forms: quick check!

$P = "\sqrt{2} \text{ is rational}"

$Q = "826\text{th digit of pi is 2}"

$P$ is ...False .

$Q$ is ...True .

\[ P \wedge Q \ldots \text{False} \]

\[ P \lor Q \ldots \text{True} \]

\[ \neg P \ldots \text{True} \]
Propositional Forms: quick check!

\[ P = \text{"\(\sqrt{2}\) is rational"} \]
\[ Q = \text{"826th digit of pi is 2"} \]

\( P \) is \( \text{False} \).
\( Q \) is \( \text{True} \).

\( P \land Q \) \( \text{False} \)
\( P \lor Q \) \( \text{True} \)
\( \neg P \) \( \text{True} \)
Put them together..

Propositions:

\( P_1 \) - Person 1 rides the bus.
Put them together..

Propositions:

$P_1$ - Person 1 rides the bus.

$P_2$ - Person 2 rides the bus.
Propositions:
\( P_1 \) - Person 1 rides the bus.
\( P_2 \) - Person 2 rides the bus.

Put them together.

Suppose we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 ride the bus or person 5 doesn't.

Propositional Form:
\[
\neg ((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5))
\]

Who can ride the bus?

What combinations of people can ride the bus?

This seems complicated.

We need a way to keep track!
Put them together..

Propositions:

\( P_1 \) - Person 1 rides the bus.
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This seems ...complicated.
We need a way to keep track!
Truth Tables for Propositional Forms.

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One use for truth tables: Logical Equivalence of propositional forms!

Example:

\[ \neg (P \land Q) \text{ logically equivalent to } \neg P \lor \neg Q \]

...because the two propositional forms have the same...

...Truth Table!

DeMorgan’s Law's for Negation: distribute and flip!

\[ \neg (P \land Q) \equiv \neg P \lor \neg Q \]
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One use for truth tables: Logical Equivalence of propositional forms!

Example:

$$\neg (P \land Q)$$ logically equivalent to $$\neg P \lor \neg Q$$...because the two propositional forms have the same...

...Truth Table!

$P$ $Q$ $\neg (P \land Q)$ $\neg P \lor \neg Q$
| T | T | F | F |
| T | F | F | T |
| F | T | F | T |
| F | F | T | T |

DeMorgan's Law's for Negation: distribute and flip!

$$\neg (P \land Q) \equiv \neg P \lor \neg Q$$

$$\neg (P \lor Q) \equiv \neg P \land \neg Q$$
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One use for truth tables: Logical Equivalence of propositional forms!
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DeMorgan’s Law’s for Negation: distribute and flip!
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One use for truth tables: Logical Equivalence of propositional forms! Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$ ...because the two propositional forms have the same...

....Truth Table!

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DeMorgan’s Law’s for Negation: distribute and flip! 

$\neg(P \land Q) \equiv \neg P \lor \neg Q$
Truth Tables for Propositional Forms.

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DeMorgan’s Law’s for Negation: distribute and flip!

$\neg(P \land Q) \equiv \neg P \lor \neg Q$

$\neg(P \lor Q) \equiv \neg P \land \neg Q$
Implication.

\[ P \iff Q \text{ interpreted as} \]

True Statements: \( P, P \iff Q \). Conclude: \( Q \) is true.

Example: Statement: If you stand in the rain, then you'll get wet.

\( P \) = "you stand in the rain"  \( Q \) = "you will get wet"

Statement: "Stand in the rain"

Can conclude: "you'll get wet."
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).
Implication.

\[ P \implies Q \text{ interpreted as} \]

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Implication.

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Non-Consequences/consequences of Implication

The statement “$P \implies Q$”
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.
Non-Consequences/consequences of Implication

The statement “$P \implies Q$” only is False if $P$ is True and $Q$ is False. False implies nothing.

If chemical plant pollutes river, fish die. If fish die, did chemical plant polluted river? Not necessarily.

$P \implies Q$ and $Q$ are True does not mean $P$ is True. Instead we have: $P \implies Q$ and $P$ are True does mean $Q$ is True.

Be careful out there!

Some Fun: use propositional formulas to describe implication? $((P \implies Q) \land P) = \implies Q$. 
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

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False implies nothing
P False means
Non-Consequences/consequences of Implication

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\( P \) False means \( Q \) can be True or False
Non-Consequences/consequences of Implication

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P False means \( Q \) can be \textbf{True} or \textbf{False}
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Some Fun: use propositional formulas to describe implication?
\(( (P \implies Q) \land P ) \implies Q \).
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$((P \implies Q) \land P) \implies Q$. 
Implication and English.

\[ P \implies Q \]

- If \( P \), then \( Q \).
Implication and English.

\[ P \iff Q \]

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
Implication and English.

\[ P \implies Q \]

- If \( P \), then \( Q \).
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Implication and English.

\[ P \implies Q \]

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
- \( P \) only if \( Q \).
- \( P \) is sufficient for \( Q \).
- \( Q \) is necessary for \( P \).
Implication and English.

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$\neg P \lor Q \equiv P \implies Q$. 

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\[ P \quad | \quad Q \quad | \quad P \implies Q \]
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\[ \neg P \lor Q \quad \equiv \quad P \implies Q. \]

These two propositional forms are logically equivalent!
Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$. 

- If the plant pollutes, fish die.
- If the fish don't die, the plant does not pollute. (contrapositive)
- If you stand in the rain, you get wet.
- If you did not stand in the rain, you did not get wet. (not contrapositive! converse!)
- If you did not get wet, you did not stand in the rain. (contrapositive.)

Logically equivalent! Notation: $\equiv$.

- $P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P \equiv \neg Q \implies \neg P$.

- Converse of $P \implies Q$ is $Q \implies P$.
- If fish die the plant pollutes. Not logically equivalent!

- Definition: If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$ or $P \iff Q$.

(Logically Equivalent: $\iff$.)
Contraposition, Converse

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(Logically Equivalent: $\iff$.)
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Contrapositive, Converse

Contrapositive of $P \iff Q$ is $\neg Q \iff \neg P$.

- If the plant pollutes, fish die.
- If the fish don’t die, the plant does not pollute.

(contrapositive)
Contrapositive, Converse

- **Contrapositive of** $P \implies Q$ **is** $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
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  - If you stand in the rain, you get wet.

If fish die the plant pollutes. Not logically equivalent!

Definition: If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$. (Logically Equivalent: $\iff$.)
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$P \implies Q$
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$P \implies Q \equiv \neg P \lor Q$
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Logically equivalent! Notation: $\equiv$.

$$P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P$$
Contrapositive, Converse

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Propositions?

\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
Variables.

Propositions?
- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes.
Variables.

Propositions?

- \[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
- \[ x > 2 \]
- \[ n \text{ is even and the sum of two primes} \]

No. They have a free variable.
Variables.

Propositions?

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We call them predicates, e.g., $Q(x) = \text{“}x$ is even\text{”}$
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Same as boolean valued functions from 61A!
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- $P(n) = \text{“} \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \text{”}$.
Variables.

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- $P(n) = \text{"}\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\text{"}$
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Propositions?

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- $G(n) = \text{“}n \text{ is even and the sum of two primes}\text{”}$

Next:
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
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No. They have a free variable.

We call them predicates, e.g., $Q(x) = "x$ is even"

    Same as boolean valued functions from 61A!

- $P(n) = \"\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\"$.
- $R(x) = \"x > 2\"$
- $G(n) = \"n$ is even and the sum of two primes\"$

Next: Statements about boolean valued functions!!
 Quantifiers..

There exists quantifier:

\[
\exists x \in S \left( P(x) \right)
\]

means "\( P(x) \) is true for some \( x \) in \( S \)".

Wait!

What is \( S \)?

\( S \) is the universe: "the type of \( x \)".

Universe examples include:

- \( \mathbb{N} = \{ 0, 1, 2, ... \} \) (natural numbers).
- \( \mathbb{Z} = \{ ..., -1, 0, 1, ... \} \) (integers).
- \( \mathbb{Z}^+ \) (positive integers).

See note 0 for more!
There exists quantifier:

$$(\exists x \in S)(P(x))$$ means "$P(x)$ is true for some $x$ in $S$"
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\(S\) is the **universe**: “the type of \(x\)”.

▶ \(N = \{0, 1, 2, ...\}\) (natural numbers).
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See note 0 for more!
Quantifiers..

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(∃x ∈ S)(P(x)) means "P(x) is true for some x in S"

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Universe examples include..
Quantifiers..

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Quantifiers

There exists quantifier:

\((\exists x \in S)(P(x)))\) means "\(P(x)\) is true for some \(x\) in \(S\)"

For example:

\((\exists x \in N)(x = x^2)\)
Quantifiers.

There exists quantifier:

$$(\exists x \in S)(P(x))$$ means "$P(x)$ is true for some $x$ in $S$"

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0)$"
Quantifiers..

There exists quantifier:

\[(\exists x \in S)(P(x))\] means "\(P(x)\) is true for some \(x\) in \(S\)"

For example:

\[(\exists x \in \mathbb{N})(x = x^2)\]

Equivalent to "\((0 = 0) \lor (1 = 1)\)"
There exists quantifier:

$(\exists x \in S)(P(x))$ means "$P(x)$ is true for some $x$ in $S$"

For example:

$(\exists x \in N)(x = x^2)$

Equivalent to "$(0 = 0) \lor (1 = 1) \lor (2 = 4)$"
Quantifiers..

There exists quantifier:  
\( (\exists x \in S)(P(x)) \) means "\( P(x) \) is true for some \( x \) in \( S \)"

For example:

\( (\exists x \in \mathbb{N})(x = x^2) \)

Equivalent to “\( (0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots \)”
Quantifiers..

There exists quantifier:
\((\exists x \in S)(P(x))\) means "\(P(x)\) is true for some \(x\) in \(S\)"

For example:
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Equivalent to “\((0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots\)”

Much shorter to use a quantifier!
There exists quantifier:
\((\exists x \in S)(P(x))\) means "\(P(x)\) is true for some \(x\) in \(S\)"
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Much shorter to use a quantifier!

For all quantifier;
\((\forall x \in S) (P(x))\). means "For all \(x\) in \(S\) \(P(x)\) is True ."
Quantifiers..

There exists quantifier:
\[(\exists x \in S)(P(x))\] means "\(P(x)\) is true for some \(x\) in \(S\)"

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Examples:
Quantifiers.

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\( (\exists x \in S)(P(x)) \) means "\( P(x) \) is true for some \( x \) in \( S \)"

For example:

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Equivalent to "\((0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots \)"

Much shorter to use a quantifier!

For all quantifier;

\( (\forall x \in S) (P(x)) \). means "For all \( x \) in \( S \) \( P(x) \) is True ."

Examples:

"Adding 1 makes a bigger number."
Quantifiers.

There exists quantifier:

$(\exists x \in S)(P(x))$ means "$P(x)$ is true for some $x$ in $S$"

For example:

$(\exists x \in N)(x = x^2)$

Equivalent to "$(0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots$"

Much shorter to use a quantifier!

For all quantifier;

$(\forall x \in S) (P(x))$. means “For all $x$ in $S$ $P(x)$ is True .”

Examples:

“Adding 1 makes a bigger number.”

$(\forall x \in N) (x + 1 > x)$
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"the square of a number is always non-negative"
\((\forall x \in \mathbb{N})(x^2 \geq 0)\)
Quantifiers are not commutative.

Consider this English statement: "there is a natural number that is the square of every natural number", i.e. the square of every natural number is the same number!
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\[(\exists y \in N) (\forall x \in N)\]

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English: there is an \(x\) in \(S\) where \(P(x)\) does not hold.

What we do in this course! We consider claims.

Claim:

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(\forall x) P(x)
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"For all inputs \(x\) the program works."

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Counterexample.

Bad input.

Case that illustrates bug.

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Next lectures...
Quantifiers....negation...DeMorgan again.

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\[ \neg (\exists x \in S) (P(x)) \]

Equivalent to:

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