What are your super powerful programs/processors doing?

Logic and Proofs!

Induction

≡

Recursion.

What can computers do?

Work with discrete objects.

Discrete Math

⇒

immense application.

Computers learn and interact with the world?

E.g. machine learning, data analysis, robotics, ...

Probability!
Programming + Microprocessors ≡ Superpower!

What are your super powerful programs/processors doing?
  Logic and Proofs!
  Induction ≡ Recursion.

What can computers do?
  Work with discrete objects.
  **Discrete Math** → immense application.

Computers learn and interact with the world?
  E.g. machine learning, data analysis, robotics, ...
  **Probability!**
The truth: My hopes and dreams.

You learn to think more clearly and more powerfully.
The truth: My hopes and dreams.

You learn to think more clearly and more powerfully.

..And to deal clearly with uncertainty itself.
Probability Unit

• How can we predict unknown future events (e.g., gambling profit, next week rainfall, traffic congestion, ...)?
  – Constructive Models: Model the overall system (including the sources of uncertainty).
    ▪ For modeling uncertainty, we’ll develop probabilistic models and techniques for analyzing them.
  – Deductive Models: Extract the “trend” from the previous outcomes (e.g., linear regression).
Veritassium on Khan
Learning.

Veritassium on Khan
Confusion is the sweat of learning.
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Confusion is the sweat of learning.
Confusion is the sweat of discovery.
Fig. 1. Final recall (a) after repeatedly studying a text in four study periods (SSSS condition), reading a text in three study periods and then recalling it in one retrieval period (SSSR condition), or reading a text in one study period and then repeatedly recalling it in three retrieval periods (SRRR condition). Judgments of learning (b) were made on a 7-point scale, where 7 indicated that students believed they would remember material very well. The data presented in these graphs are adapted from Experiment 2 of Roediger and Karpicke (2006b). The pattern of students' metacognitive judgments of learning (predicted recall) was exactly the opposite of the pattern of students' actual long-term retention.
Learning styles.

How to search google.
Learning styles.

How to search google.
“Learning styles”
Learning styles.

How to search google.
“Learning styles”
“Learning styles debunked.”
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CS70: Notes, lectures, discussions, vitamins, homeworks.
An effective student is...

Smart, rich,
An effective student is...

Smart, rich,
and beautiful.
An effective student is...

Smart, rich,
and beautiful.

All memes. The last one is not a meme.
First one, learning is inherent.
An effective student is...

Smart, rich,
and beautiful.

All memes. The last one is not a meme.
First one, learning is inherent. You are all capable.
Second, background, background, etc.
An effective student is...

Smart, rich,
and beautiful.

All memes. The last one is not a meme.
First one, learning is inherent. You are all capable.
Second, background, background, etc. The material is doable.
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Smart, rich,

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All memes. The last one is not a meme.
First one, learning is inherent. You are all capable.
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What I think.
An effective student is...

Smart, rich,
and beautiful.

All memes. The last one is not a meme.
   First one, learning is inherent. You are all capable.
   Second, background, background, etc. The material is doable.

What I think.

   Confident, motivated,
An effective student is...

Smart, rich,
and beautiful.

All memes. The last one is not a meme.
First one, learning is inherent. You are all capable.
Second, background, background, etc. The material is doable.

What I think.

Confident, motivated,
has integrity.
There are the known knowns, known unknowns, and unknown unknowns.
Known knowns..

There are the known knowns, known unknowns, and unknown unknowns.
The last one is what **always gets you.**
Known knowns..

There are the known knowns, known unknowns, and unknown unknowns.

The last one is what always gets you.

In learning, one goes from unknown unknowns, to known unknowns, to known knowns.
Known knowns..

There are the known knowns, known unknowns, and unknown unknowns. The last one is what always gets you. In learning, one goes from unknown unknowns, to known unknowns, to known knowns. The middle one is stressful and where most of the time is spent.
There are the known knowns, known unknowns, and unknown unknowns.

The last one is what **always gets you**.

In learning, one goes from unknown unknowns, to known unknowns, to known knowns.

The middle one is **stressful** and where most of the time is spent.

Confidence is not pretending you know.
There are the known knowns, known unknowns, and unknown unknowns.

The last one is what **always gets you**.

In learning, one goes from unknown unknowns, to known unknowns, to known knowns.

The middle one is **stressful** and where most of the time is spent.

Confidence is not pretending you know. Its being comfortable with what you don’t know.
There are the known knowns, known unknowns, and unknown unknowns.

The last one is what *always gets you*.

In learning, one goes from unknown unknowns, to known unknowns, to known knowns.

The middle one is *stressful* and where most of the time is spent.

Confidence is not pretending you know.
Its being comfortable with what you don’t know.
In order to get there.
Dogs don’t have rights cuz..
Dogs don’t have rights cuz...

They don’t know infinity.
First grade

1, 2, 3, 4, …, 120
First grade

1, 2, 3, 4, …, 120

Peano’s axioms. There is always a successor.
First grade

1, 2, 3, 4, ..., 120

Peano’s axioms. There is always a successor.

+3 means move to successor and another and another, or 3 times.
First grade

1, 2, 3, 4, …, 120

Peano’s axioms. There is always a successor.

+3 means move to successor and another and another, or 3 times.

Metric. (distance.) There is mapping \( f : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N} \)

Obey triangle inequality: \( f(i, j) + f(j, k) \geq f(i, k) \)
First grade

1, 2, 3, 4, ..., 120

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11 is one ten, and one one.
First grade

1, 2, 3, 4, \ldots, 120

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Computer science: efficient representation of a number.
1, 2, 3, 4, ..., 120

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Algorithms: how to add.
First grade

1, 2, 3, 4, …, 120

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Algorithms: how to add.

Place value: democratizes arithmetic.
First grade

1, 2, 3, 4, ..., 120

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$3 \times 5$?
First grade

1, 2, 3, 4, ... , 120

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11 is one ten, and one one.

Computer science: efficient representation of a number.
Algorithms: how to add.
Place value: democratizes arithmetic.

3 × 5?

× means add 3 times.
5 + 5 + 5
10 is moving over 5 from 5
The next number one can use the one’s place.
Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?
(2) You have them! Use the slides to guide you.

Sufficient: understand the slides → mostly understand the course.

Understand the last slide, understand the lecture.

It is easier to present more. "More" is repetition, examples, connection, some jokes (breaks), the details.

Risk: Students get frustrated at not understanding everything. E.g. professor is unclear.

The truth: Students don't understand everything. I don't. It is ok: many levels to grok.

Lecture is one pass. Notes cover material. Discussion. Vitamins. Homework. Study.
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Notes cover material. Discussion. Vitamins. Homework. Study.
How to interact with staff..

My advice to TA’s.
My advice to TA’s.
When a student asks questions, probe to see where they are.
How to interact with staff..

My advice to TA’s.

When a student asks questions, probe to see where they are. And then move them forward.
How to interact with staff..

My advice to TA’s.

When a student asks questions, probe to see where they are. And then move them forward.

E.g., Avoid long explanations with nodding students. You must check in meaningfully.
My advice to TA’s.

When a student asks questions, probe to see where they are. And then move them forward.

E.g., Avoid long explanations with nodding students. You must check in meaningfully.

What should you do?
How to interact with staff..

My advice to TA’s.

When a student asks questions, probe to see where they are. And then move them forward.

E.g., Avoid long explanations with nodding students. You must checkin meaningfully.

What should you do?
   Where does your understanding get iffy?
How to interact with staff..

My advice to TA’s.

When a student asks questions, probe to see where they are. And then move them forward.

E.g., Avoid long explanations with nodding students. You must checkin meaningfully.

What should you do?

   Where does your understanding get iffy?
   Explain what you understand, then say what you don’t.
Advice from (former) TA’s

Distinguished Alumnus (DA) Megan:
Advice from (former) TA’s

Distinguished Alumnus (DA) Megan:
   I read the notes until I could reproduce the proofs myself.
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DA Lili:
Distinguished Alumnus (DA) Megan:
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Head TA Richard:
“carefully review the homework solutions after they are released and understand them to the point of being able to replicate them without needing to reference them.”
Known knowns..

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Known knowns..

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The last one is what get’s you.
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Mini-vitamins.

1) Mini-vitamins. Do before lecture. But, it's before it's taught! Read the notes. You do it in English class! or should maybe? Rao lectures follow them closely. Ask any professor: watching after you know something is far more useful.

2) Mini-vitamins. Do before section. TAs: they don't even know the basic definitions to do the worksheet.

3) Discussion. Will not cover everything on sheet. May not present any solutions. Opportunity for guided practice. Warning: you might not like it. But you will learn more. See this paper, for example and a good discussion. Please do not take it out on your TA's.
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   Ya do it in English class! or should maybe?
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Please do not take it out on your TA’s.
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Announcements, logistics, critical advice.
Wason’s experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.

Which cards must you flip to test the theory?

Answer: (A), (B), (C), (D).
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Alice
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Charlie
Chicago

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Today: Note 1.
CS70: Lecture 1. Outline.

Today: Note 1. Note 0 is background.
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The language of proofs!
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The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan’s Laws
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
\[ 2+2 = 4 \]
\[ 2+2 = 3 \]
\[ 826\text{th digit of pi is 4} \]
John\nny Depp is a good actor
\[ \text{Any even } > 2 \text{ is sum of 2 primes} \]
\[ 4 + 5 \]
\[ x + x \]
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- Any even > 2 is sum of 2 primes
- \( 4 + 5 \)
- \( x + x \)
- Alice travelled to Chicago

*Proposition*
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]  \hspace{1cm} \text{Proposition}  \hspace{1cm} \text{True}

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- Any even $> 2$ is sum of 2 primes  
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- $4 + 5$  
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- Alice travelled to Chicago  
  - Proposition.  
  - False
- I love you.  
  - Hmm.  
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Again: “value” of a proposition is ...
Propositions: Statements that are true or false.

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</tr>
<tr>
<td>826th digit of pi is 4</td>
<td>False</td>
</tr>
<tr>
<td>Johnny Depp is a good actor</td>
<td>False</td>
</tr>
<tr>
<td>Any even $\geq 2$ is sum of 2 primes</td>
<td>False</td>
</tr>
<tr>
<td>$4 + 5$</td>
<td></td>
</tr>
<tr>
<td>$x + x$</td>
<td></td>
</tr>
<tr>
<td>Alice travelled to Chicago</td>
<td>False</td>
</tr>
<tr>
<td>I love you.</td>
<td>Its complicated.</td>
</tr>
<tr>
<td>Again: “value” of a proposition is ...</td>
<td>True or False</td>
</tr>
</tbody>
</table>
Propositional Forms.

Put propositions together to make another...
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

“$P \land Q$” is True if both $P$ and $Q$ are True.
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"$2 + 2 = 4$" – a proposition that is True.

"$2 + 2 = 3$" $\land$ "$2 + 2 = 4$" – a proposition that is False.

"$2 + 2 = 3$" $\lor$ "$2 + 2 = 4$" – a proposition that is True.
Propositional Forms.

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Examples:

\( \neg (2 + 2 = 4) \) – a proposition that is False.

\( (2 + 2 = 3) \land (2 + 2 = 4) \) – a proposition that is False.

\( (2 + 2 = 3) \lor (2 + 2 = 4) \) – a proposition that is True.
Propositional Forms.

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Put them together.

Propositions:

$P_1$ - Person 1 rides the bus.
Put them together..

Propositions:

$P_1$ - Person 1 rides the bus.
$P_2$ - Person 2 rides the bus.

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

Propositional Form:

$\neg((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5))$

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?
This seems complicated.
We can program!!!!
We need a way to keep track!
Put them together..

Propositions:

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\[
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Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

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<thead>
<tr>
<th>$P$</th>
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Other operators: 

$P \lor Q$ is True if at least one of $P$ or $Q$ is True.

De Morgan’s Law:

$\neg(P \land Q)$ is equivalent to $\neg P \lor \neg Q$.

$\neg(P \lor Q)$ is equivalent to $\neg P \land \neg Q$. 

Check: $\land$ and $\lor$ are commutative. One use for truth tables:

Logical Equivalence of propositional forms!”

Example:

$\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$.

Same Truth Table!
Truth Tables for Propositional Forms.

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Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example:

$\neg (P \land Q)$ logically equivalent to $\neg P \lor \neg Q$.

DeMorgan’s Law’s for Negation: distribute and flip!

$\neg (P \land Q) \equiv \neg P \lor \neg Q$

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Truth Tables for Propositional Forms.

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“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

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Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

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“\( P \land Q \)” is True if both \( P \) and \( Q \) are True.

\[
\begin{array}{ccc}
P & Q & P \land Q \\
T & T & T \\
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Check: \( \land \) and \( \lor \) are commutative.

“\( P \lor Q \)” is True if \( \geq \) one of \( P \) or \( Q \) is True.

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T & T & T \\
T & F & T \\
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\end{array}
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Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

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“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

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Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!
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One use for truth tables: Logical Equivalence of propositional forms!

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Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!
Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!

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Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

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“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

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DeMorgan’s Law’s for Negation: distribute and flip!

$\neg(P \land Q)$
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

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One use for truth tables: Logical Equivalence of propositional forms!

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DeMorgan’s Law’s for Negation: distribute and flip!

$\neg(P \land Q) \equiv \neg P \lor \neg Q$
Truth Tables for Propositional Forms.

“\( P \land Q \) is True if both \( P \) and \( Q \) are True.

\[
\begin{array}{|c|c|c|}
\hline
P & Q & P \land Q \\
\hline
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F \\
\hline
\end{array}
\]

Check: \( \land \) and \( \lor \) are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: \( \neg(P \land Q) \) logically equivalent to \( \neg P \lor \neg Q \). Same Truth Table!

\[
\begin{array}{|c|c|c|c|}
\hline
P & Q & \neg(P \lor Q) & \neg P \land \neg Q \\
\hline
T & T & F & F \\
T & F & F & F \\
F & T & F & F \\
F & F & T & T \\
\hline
\end{array}
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DeMorgan’s Law’s for Negation: distribute and flip!

\[ \neg(P \land Q) \equiv \neg P \lor \neg Q \]

\[ \neg(P \lor Q) \]
**Truth Tables for Propositional Forms.**

“$P \land Q$” is True if both $P$ and $Q$ are True.

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DeMorgan’s Law’s for Negation: distribute and flip!

$\neg(P \land Q) \equiv \neg P \lor \neg Q$  \hspace{1cm} $\neg(P \lor Q) \equiv \neg P \land \neg Q$
Quick Questions

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Is $(T \land Q) \equiv Q$?
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Is $(T \land Q) \equiv Q$? Yes?
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Is $(T \land Q) \equiv Q$? Yes? No?
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Is $(T \land Q) \equiv Q$? Yes? No?
Yes!
Quick Questions

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Is $(T \land Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$. 
Quick Questions

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Is $(T \land Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \land Q)$?
Quick Questions

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Is \((T ∧ Q) \equiv Q\)? Yes? No?

Yes! Look at rows in truth table for \(P = T\).

What is \((F ∧ Q)\)? F or False.
Quick Questions

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Is $(T \land Q) \equiv Q$? Yes? No?
Yes! Look at rows in truth table for $P = T$.

What is $(F \land Q)$? F or False.
What is $(T \lor Q)$?
### Quick Questions

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Is $(T \land Q) \equiv Q$? Yes? No?
Yes! Look at rows in truth table for $P = T$.

What is $(F \land Q)$? F or False.
What is $(T \lor Q)$? $T$
Quick Questions

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Foil 1:

\[(A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D)\]?
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Implication.

\[ P \implies Q \text{ interpreted as} \]

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Examples:

Statement: If you stand in the rain, then you'll get wet. 
\( P = \text{"you stand in the rain"}, Q = \text{"you will get wet"} \).

Statement: "Stand in the rain"
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Statement:
If a right triangle has sidelengths \( a \leq b \leq c \), then \( a^2 + b^2 = c^2 \).
\( P = \text{"a right triangle has sidelengths } a \leq b \leq c \), Q = \text{"} a^2 + b^2 = c^2 \).
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The statement “$P \implies Q$”
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only is False if $P$ is True and $Q$ is False.
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing.

If chemical plant pollutes river, fish die.

If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and $Q$ are True does not mean $P$ is True.

Be careful!

Instead we have:

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The chemical plant pollutes river.

Can we conclude fish die?
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\( P \implies Q \) and \( Q \) are True does not mean \( P \) is True

Be careful!

Instead we have:
\( P \implies Q \) and \( P \) are True does mean \( Q \) is True.
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
$P\text{ False}$ means $Q$ can be True or False
Anything implies true.
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The chemical plant pollutes river. Can we conclude fish die?
Implication and English.

\[ P \implies Q \]

Poll.

- If \( P \), then \( Q \).

Remember if \( P \) is true then \( Q \) must be true. This suggests that \( P \) can only be true if \( Q \) is true. Since if \( Q \) is false \( P \) must have been false.

\( P \) is sufficient for \( Q \). This means that proving \( P \) allows you to conclude that \( Q \) is true.

Example: Showing \( n > 4 \) is sufficient for showing \( n > 3 \).

\( Q \) is necessary for \( P \). For \( P \) to be true it is necessary that \( Q \) is true. Or if \( Q \) is false then we know that \( P \) is false.

Example: It is necessary that \( n > 3 \) for \( n > 4 \).
Implication and English.

\[ P \implies Q \]
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- If \( P \), then \( Q \).
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Implication and English.

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\( P \implies Q \)  
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\[ P \implies Q \]

Poll.

- If \( P \), then \( Q \).
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- Remember if \( P \) is true then \( Q \) must be true.

\[ \text{this suggests that} \]
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\begin{array}{c|c|c}
P & Q & P \implies Q \\
\hline
T & T & T \\
T & F & F \\
F & T & T \\
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\end{array}
$$

\(\neg P \lor Q \equiv P \implies Q.\)

$$
\begin{array}{c|c|c}
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\[-P \lor Q \equiv P \implies Q.\]

These two propositional forms are logically equivalent!
Contraposition, Converse

- **Contrapositive** of \( P \implies Q \) is \( \neg Q \implies \neg P \).
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.

- If fish die, the plant pollutes.
  - Not logically equivalent!

- **Definition:** If $P \implies Q$ and $Q \implies P$ is $P \iff Q$.
  - Logically Equivalent: $\iff$. 

- Let $P = \implies Q$.
  - $P \iff Q \equiv \neg (\neg Q) \lor \neg P \equiv \neg Q \implies \neg P$. 

- **Converse** of $P \implies Q$ is $Q \implies P$.
  - If you did not stand in the rain, you did not get wet.
**Contrapositive, Converse**

- **Contrapositive** of \( P \implies Q \) is \( \neg Q \implies \neg P \).
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute.
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
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  - If the fish don’t die, the plant does not pollute. 
    (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.
    (not contrapositive!)

- **Converse** of $P \implies Q$ is $Q \implies P$.
  - If fish die the plant pollutes.
  - Not logically equivalent!

- **Definition:** If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$ or $P$ $\iff Q$.
  (Logically Equivalent: $\iff$.)
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute. (contrapositive)

- If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - If you did not get wet, you did not stand in the rain.

Recall:

$$(X \implies Y) \equiv (\neg X \lor Y)$$

$P \implies Q \equiv \neg P \lor Q \equiv Q \lor \neg P \equiv \neg (\neg Q) \lor \neg P \equiv \neg Q \implies \neg P.$
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.

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  - If the fish don’t die, the plant does not pollute.
    (contrapositive)

  - If you stand in the rain, you get wet.
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    (not contrapositive!)

  - If you did not get wet, you did not stand in the rain.
    (contrapositive.)

\[ P \implies Q \equiv \neg P \lor Q \equiv Q \lor \neg P \equiv \neg (\neg Q) \implies \neg P. \]
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
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Logically equivalent! Notation: $\equiv$. 

Contrapositive, Converse

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  - If you did not stand in the rain, you did not get wet.  
    (not contrapositive!)
  - If you did not get wet, you did not stand in the rain.  
    (contrapositive.)

Logically equivalent! Notation: \( \equiv \). Recall: \( (X \implies Y) \equiv (\neg X \lor Y) \)
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute. \((\text{contrapositive})\)
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- **Converse** of $P \implies Q$ is $Q \implies P$.
  - If fish die the plant pollutes.
Contrapositive, Converse

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- **Converse** of $P \implies Q$ is $Q \implies P$.
  - If fish die the plant pollutes.
  - Not logically equivalent!

- **Definition**: If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$ or $P \iff Q$.
  (Logically Equivalent: $\iff$.)
Variables.

Propositions?

\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
Variables.

Propositions?

- \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).
- \( x > 2 \)
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
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No.
Variables.

Propositions?

- \[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
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No. They have a free variable.
Variables.

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No. They have a free variable.

We call them **predicates**, e.g., \( Q(x) = "x \text{ is even}" \)
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- $P(n) = “\sum_{i=1}^{n} i = \frac{n(n+1)}{2}”.$
- $R(x) = “x > 2”$
Variables.

Propositions?

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- \( G(n) = \"n \) is even and the sum of two primes\" \)
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- $G(n) =$ “$n$ is even and the sum of two primes”
- Remember Wason’s experiment!
  $F(x) =$ “Person $x$ flew.”
Variables.

Propositions?

\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]

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Remember Wason’s experiment!

\[ F(x) = "\text{Person } x \text{ flew.}" \]

\[ C(x) = "\text{Person } x \text{ went to Chicago}" \]
Variables.

Propositions?

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\( C(x) \implies F(x) \).
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Next:
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If person \( x \) goes to Chicago then person \( x \) flew.

Next: Statements about boolean valued functions!!
Quantifiers..

There exists quantifier:

\[ \exists x \in S \] \( P(x) \)

means "There exists an \( x \) in \( S \) where \( P(x) \) is true."

For example:

\[ \exists x \in \mathbb{N} \] \( x = x^2 \)

Equivalent to "

\( 0 = 0 \) \lor \( 1 = 1 \) \lor \( 2 = 4 \) \lor ... \)

Much shorter to use a quantifier!

For all quantifier:

\[ \forall x \in S \] \( P(x) \)

means "For all \( x \) in \( S \), \( P(x) \) is True."

Examples:

"Adding 1 makes a bigger number."

\[ \forall x \in \mathbb{N} \] \( x + 1 > x \)

"the square of a number is always non-negative"

\[ \forall x \in \mathbb{N} \] \( x^2 \geq 0 \)

Wait!

What is \( \mathbb{N} \)?
Quantifiers.

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\((\exists x \in S)(P(x))\) means “There exists an \(x\) in \(S\) where \(P(x)\) is true.”

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Equivalent to “\((0 = 0)\)"
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Equivalent to “$(0 = 0) \lor (1 = 1)$”
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Equivalent to “\(0 = 0\) \(\lor\) \(1 = 1\) \(\lor\) \(2 = 4\)”

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For example:

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Examples:
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\((\forall x \in \mathbb{N})(x^2 >= 0)\)
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\((\forall x \in \mathbb{N})(x^2 \geq 0)\)

Wait!
Quantifiers...

There exists quantifier:

\((\exists x \in S)(\neg(x))\) means “There exists an \(x\) in \(S\) where \(P(x)\) is true.”

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”the square of a number is always non-negative”

\((\forall x \in \mathbb{N})(x^2 \geq 0)\)

Wait! What is \(\mathbb{N}\)?
Quantifiers: universes.

Proposition: “For all natural numbers \( n \), \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).”

Proposition has universe:
Quantifiers: universes.

Proposition: “For all natural numbers \( n \), \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).”

Proposition has **universe**: “the natural numbers”.

Universe examples include:
- \( \mathbb{N} = \{0, 1, ..., \} \) (natural numbers).
- \( \mathbb{Z} = \{..., -1, 0, 1, ...\} \) (integers).
- \( \mathbb{Z}^+ \) (positive integers).
- \( \mathbb{R} \) (real numbers).
- Any set: \( S = \{Alice, Bob, Charlie, Donna\} \).

See note 0 for more!

Other proposition notation (for discussion):
- \( d | n \) means \( d \) divides \( n \) or \( \exists k \in \mathbb{Z}, n = kd \).
- \( 2 | 4 \)? True.
- \( 4 | 2 \)? False.
Quantifiers: universes.

Proposition: “For all natural numbers $n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.”

Proposition has universe: “the natural numbers”.

Universe examples include:

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
- $\mathbb{Z}^+$ (positive integers)
- $\mathbb{R}$ (real numbers)
- Any set: $S = \{Alice, Bob, Charlie, Donna\}$.

See note 0 for more!

Other proposition notation (for discussion):
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2\( \mid 4 \)? True.
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Universe examples include:
- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
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- Any set: $S = \{Alice, Bob, Charlie, Donna\}$.
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Other proposition notation (for discussion):
“$d|n$” means $d$ divides $n$
- or $\exists k \in \mathbb{Z}, n = kd$.

2|4? True.
4|2?
Quantifiers: universes.

Proposition: “For all natural numbers $n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.”

Proposition has universe: “the natural numbers”.

Universe examples include:

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
- $\mathbb{Z}^+$ (positive integers)
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- Any set: $S = \{Alice, Bob, Charlie, Donna\}$.
- See note 0 for more!

Other proposition notation (for discussion):

“$d | n$” means $d$ divides $n$

or $\exists k \in \mathbb{Z}, n = kd$.

$2 | 4$? True.
$4 | 2$? False.
Theory:

"If a person travels to Chicago, he/she/they flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew. Which cards do you need to flip to test the theory?

\begin{align*}
\text{Chicago} (x) &= \text{"x went to Chicago."} \\
\text{Flew} (x) &= \text{"x flew"} \\
\end{align*}

Statement/theory:

\begin{align*}
\forall x \in \{A, B, C, D\},  \\
\text{Chicago} (x) &= \Rightarrow \text{Flew} (x) \\
\end{align*}

Chicago (A) = \text{False}.

Do we care about Flew (A)?

No.

Chicago (A) = \Rightarrow Flew (A) is true. since Chicago (A) is False,

Flew (B) = \text{False}.

Do we care about Chicago (B)?

Yes.

Chicago (B) = \Rightarrow Flew (B) \equiv \neg Flew (B) = \Rightarrow \neg Chicago (B). \\

So Chicago (Bob) must be False.

Chicago (C) = \text{True}.

Do we care about Flew (C)?

Yes. Chicago (C) = \Rightarrow Flew (C) means Flew (C) must be true.

Flew (D) = \text{True}.

Do we care about Chicago (D)?

No. Chicago (D) = \Rightarrow Flew (D) is true if Flew (D) is true.

Only have to turn over cards for Bob and Charlie.
Theory: “If a person travels to Chicago, he/she/they flies.”
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.
Which cards do you need to flip to test the theory?
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ Chicago(x) = "x \text{ went to Chicago.}" \]
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“} x \text{ went to Chicago.} \] \quad \text{Flew}(x) = \text{“} x \text{ flew} \]
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”
Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.
Which cards do you need to flip to test the theory?

\[
\text{Chicago}(x) = \text{“} x \text{ went to Chicago.} \]
\[
\text{Flew}(x) = \text{“} x \text{ flew} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \)
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[
\text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”}
\]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\( \text{Chicago}(x) = \text{“} x \text{ went to Chicago.} \) \quad \text{Flew}(x) = \text{“} x \text{ flew} \)

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \).
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

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Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\text{Chicago}(A) = \text{False} . \text{ Do we care about } \text{Flew}(A) ? \quad \text{No.}
Theory: “If a person travels to Chicago, he/she/they flies.”
Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.
Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“} x \text{ went to Chicago.} \]  \[ \text{Flew}(x) = \text{“} x \text{ flew} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?
No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,
Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[
\begin{align*}
\text{Chicago}(x) &= \text{“} x \text{ went to Chicago.}\text{”} \quad \text{Flew}(x) = \text{“} x \text{ flew}\text{”}
\end{align*}
\]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} \).
Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[
\text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”}
\]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

Chicago(\(x\)) = “\(x\) went to Chicago.” \(Flew(x) = “x\) flew”

Statement/theory: \(\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)\)

\(Chicago(A) = False\) . Do we care about \(Flew(A)\) ?

No. \(Chicago(A) \implies Flew(A)\) is true.

since \(Chicago(A)\) is False ,

\(Flew(B) = False\) . Do we care about \(Chicago(B)\) ?

Yes.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“}x\text{ went to Chicago.”} \quad \text{Flew}(x) = \text{“}x\text{ flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \)
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\( \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \)

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

- $Chicago(x) = \text{“}x\text{ went to Chicago.}\text{”}$
- $Flew(x) = \text{“}x\text{ flew}\text{”}$

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A) = \text{False}$. Do we care about $Flew(A)$?

No. $Chicago(A) \implies Flew(A)$ is true.

since $Chicago(A)$ is False,

$Flew(B) = \text{False}$. Do we care about $Chicago(B)$?

Yes. $Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B)$.

So $Chicago(Bob)$ must be False.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ Chicago(x) = \text{“} x \text{ went to Chicago.} \quad Flew(x) = \text{“} x \text{ flew} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \ \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False.

\( \text{Chicago}(C) = \text{True} \).
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = “x \text{ went to Chicago.}” \quad \text{Flew}(x) = “x \text{ flew}” \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\text{Chicago}(A) = \text{False} . \text{ Do we care about Flew}(A)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\text{Flew}(B) = \text{False} . \text{ Do we care about Chicago}(B)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False .

\text{Chicago}(C) = \text{True} . \text{ Do we care about Flew}(C)?
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”
Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.
Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?
   No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.
   since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?
   Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).
   So \( \text{Chicago}(Bob) \) must be False .

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?
   Yes.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(\text{Bob}) \) must be False .

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False.

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} \).
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False .

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} \). Do we care about \( \text{Chicago}(D) \)?
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“}x\text{ went to Chicago.”} \quad \text{Flew}(x) = \text{“}x\text{ flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?
  No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.
  since \( \text{Chicago}(A) \) is \text{False} ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?
  Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).
  So \( \text{Chicago}(Bob) \) must be \text{False}.

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?
  Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} \). Do we care about \( \text{Chicago}(D) \)?
  No.
Back to: Wason’s experiment

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{"x went to Chicago."} \quad \text{Flew}(x) = \text{"x flew"} \]

Statement/theory: \( \forall x \in \{ A, B, C, D \}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False.

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} \). Do we care about \( \text{Chicago}(D) \)?

No. \( \text{Chicago}(D) \implies \text{Flew}(D) \) is true if \( \text{Flew}(D) \) is true.
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[
\begin{align*}
\text{Chicago}(x) &= “x \text{ went to Chicago.”} \\
\text{Flew}(x) &= “x \text{ flew”}
\end{align*}
\]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be \text{False} .

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} \). Do we care about \( \text{Chicago}(D) \)?

No. \( \text{Chicago}(D) \implies \text{Flew}(D) \) is true if \( \text{Flew}(D) \) is true.

Only have to turn over cards for Bob and Charlie.
More for all quantifiers examples.

"doubling a number always makes it larger"
\[
(\forall x \in \mathbb{N}) (2x > x)
\]
False

Consider \( x = 0 \)
Can fix statement...

\[
(\forall x \in \mathbb{N}) (2x \geq x)
\]
True

"Square of any natural number greater than 5 is greater than 25."
\[
(\forall x \in \mathbb{N}) (x > 5 \implies x^2 > 25)
\]

Idea alert: Restrict domain using implication.
Later we may omit universe if clear from context.
More for all quantifiers examples.

- “doubling a number always makes it larger”
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x)\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in N) (2x > x) \quad \text{False}\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\((\forall x \in \mathbb{N}) (2x > x)\) \hspace{1cm} \text{False} \hspace{1cm} \text{Consider} \ x = 0
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Can fix statement...
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- “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in \mathbb{N})\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N})(2x \geq x) \quad \text{False} \quad \text{Consider} \quad x = 0\]

Can fix statement...

\[(\forall x \in \mathbb{N})(2x \geq x) \quad \text{True}\]

- “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in \mathbb{N})(x > 5)\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N})(2x > x)\] False Consider \(x = 0\)

Can fix statement...

\[(\forall x \in \mathbb{N})(2x \geq x)\] True

- “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in \mathbb{N})(x > 5 \implies \)
More for all quantifiers examples.

- “doubling a number always makes it larger”

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Can fix statement...

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- “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in \mathbb{N})(x > 5 \implies x^2 > 25).\]
More for all quantifiers examples.

► “doubling a number always makes it larger”

$$\forall x \in N \ (2x > x) \quad \text{False} \quad \text{Consider } x = 0$$

Can fix statement...

$$\forall x \in N \ (2x \geq x) \quad \text{True}$$

► “Square of any natural number greater than 5 is greater than 25.”

$$\forall x \in N \ (x > 5 \implies x^2 > 25)$$

Idea alert:

Later we may omit universe if clear from context.
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x) \text{ False } \text{ Consider } x = 0\]

Can fix statement...

\[(\forall x \in \mathbb{N}) (2x \geq x) \text{ True}\]

- “Square of any natural number greater than 5 is greater than 25.”

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Idea alert: Restrict domain using implication.
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\[(\forall x \in \mathbb{N})(2x \geq x) \quad \text{True}\]

▶ “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in \mathbb{N})(x > 5 \implies x^2 > 25)\]

Idea alert: Restrict domain using implication.

Later we may omit universe if clear from context.
Quantifiers...not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[
\exists y \in \mathbb{N} \quad \forall x \in \mathbb{N} \quad (y = x^2)
\]

False

- In English: “the square of every natural number is a natural number.”

\[
\forall x \in \mathbb{N} \quad \exists y \in \mathbb{N} \quad (y = x^2)
\]

True
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

$(\exists y \in \mathbb{N})$
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in \mathbb{N}) (\forall x \in \mathbb{N})$$
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2)\]
Quantifiers..not commutative.

▶ In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2)\] False
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- In English: “there is a natural number that is the square of every natural number”.
  \[(\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2)\] False

- In English: “the square of every natural number is a natural number.”
  \[(\forall x \in \mathbb{N})\]
Quantifiers...not commutative.

► In English: “there is a natural number that is the square of every natural number”.

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\[ (\forall x \in \mathbb{N}) (\exists y \in \mathbb{N}) \]
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\[ (\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2) \] False

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Quantifiers...not commutative.

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  \((\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2)\) False

- In English: “the square of every natural number is a natural number.”
  
  \((\forall x \in \mathbb{N})(\exists y \in \mathbb{N}) (y = x^2)\) True
Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an $x$ in $S$ where $P(x)$ does not hold.

That is, $\neg(\forall x \in S)(P(x)) \iff \exists x \in S \neg P(x)$.

What we do in this course! We consider claims.

Claim: $(\forall x)P(x)$

"For all inputs $x$ the program works."

For False, find $x$ where $\neg P(x)$.

Counterexample. Bad input. Case that illustrates bug.

For True: prove claim. Next lectures...
Quantifiers....negation...DeMorgan again.

Consider

\[ \neg (\forall x \in S)(P(x)), \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

Next lectures...
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What we do in this course! We consider claims.

**Claim:** \((\forall x) P(x)\)
Consider
\[ \neg(\forall x \in S)(P(x)), \]
English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
That is,
\[ \neg(\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]

What we do in this course! We consider claims.

Claim: \( \forall x \) \( P(x) \) \hspace{1cm} \text{"For all inputs } x \text{ the program works."}
Consider
\[ \neg (\forall x \in S)(P(x)), \]
English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
That is,
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What we do in this course! We consider claims.

**Claim:** \( (\forall x) P(x) \)  “For all inputs \( x \) the program works.”
For **False**, find \( x \), where \( \neg P(x) \).
Quantifiers....negation...DeMorgan again.

Consider
\[ \neg (\forall x \in S)(P(x)), \]
English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
That is,
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Counterexample.

Bad input.
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\neg (\forall x \in S)(P(x))

English: there is an x in S where P(x) does not hold.

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  Counterexample.
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English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

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**Claim:** \((\forall x) P(x) \)  “For all inputs \( x \) the program works.”

For **False**, find \( x \), where \( \neg P(x) \).

- Counterexample.
- Bad input.
- Case that illustrates bug.

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For **False**, find \( x \), where \( \neg P(x) \).

  - Counterexample.
  - Bad input.
  - Case that illustrates bug.

For **True**: prove claim. Next lectures...
Negation of exists.

Consider

$\neg \left( \exists x \in S \right) \left( P(x) \right)$

English: means that there is no $x \in S$ where $P(x)$ is true.

English: means that for all $x \in S$, $P(x)$ does not hold.

That is, $\neg \left( \exists x \in S \right) \left( P(x) \right) \iff \forall \left( x \in S \right) \neg P(x)$. 
Negation of exists.

Consider

\[
\neg (\exists x \in S)(P(x))
\]
Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

English: means that there is no $x \in S$ where $P(x)$ is true.
Negation of exists.

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$$\neg(\exists x \in S)(P(x))$$

English: means that there is no $x \in S$ where $P(x)$ is true. English: means that for all $x \in S$, $P(x)$ does not hold.
Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

English: means that there is no $x \in S$ where $P(x)$ is true. English: means that for all $x \in S$, $P(x)$ does not hold.

That is,

$$\neg(\exists x \in S)(P(x)) \iff \forall(x \in S)\neg P(x).$$
Which Theorem?

Theorem: \((\forall n \in \mathbb{N}) n \geq 3 \implies \neg (\exists a, b, c \in \mathbb{N}) (a^n + b^n = c^n)\)
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Which Theorem?
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Which Theorem?

Fermat’s Last Theorem!
Theorem: $(\forall n \in \mathbb{N}) \ n \geq 3 \implies \neg(\exists a, b, c \in \mathbb{N}) \ (a^n + b^n = c^n)$

Which Theorem?

Fermat's Last Theorem!

Remember Special Triangles: for $n = 2$, we have 3,4,5 and 5,7, 12 and ...
Theorem: $(\forall n \in \mathbb{N}) \; n \geq 3 \implies \neg(\exists a, b, c \in \mathbb{N}) \; (a^n + b^n = c^n)$

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1637: Proof doesn’t fit in the margins.
Which Theorem?

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DeMorgan Restatement:
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Theorem: \((\forall n \in \mathbb{N}) n \geq 3 \implies \neg (\exists a, b, c \in \mathbb{N}) (a^n + b^n = c^n)\)

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DeMorgan Restatement:

Theorem: \(\neg (\exists n \in \mathbb{N}) (\exists a, b, c \in \mathbb{N}) (n \geq 3 \implies a^n + b^n = c^n)\)
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land, \lor, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q$
Summary.

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Implication: $P \implies Q \iff \neg P \lor Q$.

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Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x \ P(x)$, $\exists y \ Q(y)$

Now can state theorems!
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$\neg(P \lor Q) \iff$
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$\neg (P \lor Q) \iff (\neg P \land \neg Q)$
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Summary.

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DeMorgan’s Laws: “Flip and Distribute negation”

$\neg (P \lor Q) \iff (\neg P \land \neg Q)$

$\neg \forall x \ P(x) \iff \exists x \ \neg P(x)$.

Next Time: proofs!