What are your super powerful programs/processors doing?

Logic and Proofs!

Induction ≡ Recursion.

What can computers do?

Work with discrete objects.

Discrete Math =⇒ immense application.

Computers learn and interact with the world?

E.g. machine learning, data analysis, robotics, ...

Probability!
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   E.g. machine learning, data analysis, robotics, ...
   **Probability!**
My hopes and dreams.

You learn to think more clearly and more powerfully.
My hopes and dreams.

You learn to think more clearly and more powerfully.

..And to deal clearly with uncertainty itself.
Probability Unit

• How can we predict unknown future events (e.g., gambling profit, next week rainfall, traffic congestion, ...)?
  – Constructive Models: Model the overall system (including the sources of uncertainty).
    ▪ For modeling uncertainty, we’ll develop probabilistic models and techniques for analyzing them.
  – Deductive Models: Extract the “trend” from the previous outcomes (e.g., linear regression).
Learning.

Veritassium on Khan
Veritassium on Khan

Confusion is the sweat of learning.
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Confusion is the sweat of learning.
Confusion is the sweat of discovery.
Fig. 1. Final recall (a) after repeatedly studying a text in four study periods (SSSS condition), reading a text in three study periods, and then recalling it in one retrieval period (SSSR condition), or reading a text in one study period and then repeatedly recalling it in three retrieval periods (SRRR condition). Judgments of learning (b) were made on a 7-point scale, where 7 indicated that students believed they would remember material very well. The data presented in these graphs are adapted from Experiment 2 of Roediger and Karpicke (2006b). The pattern of students’ metacognitive judgments of learning (predicted recall) was exactly the opposite of the pattern of students’ actual long-term retention.
Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

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(2) You have them! Use the slides to guide you. Sufficient: understand the slides → mostly understand the course. Understand the last slide, understand the lecture.

It is easier to present more. "More" is repetition, examples, connection, some jokes (breaks), the details.

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How to interact with staff.

My advice to TA's.
How to interact with staff..

My advice to TA’s.
When a student asks questions, probe to see where they are.

And then move them forward.
E.g., Avoid long explanations with nodding students. You must check in meaningfully.

What should you do?
Where does your understanding get iffy?
Explain what you understand, then say what you don’t.
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Distinguished Alumnus (DA) Megan:

I read the notes until I could reproduce the proofs myself.

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When I took the course, I tried my best to attend every discussion and ask questions whenever I was confused!

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Announcements, logistics, critical advice.
Wason’s experiment: 1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
Wason’s experiment:1

Suppose we have four cards on a table:

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- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

```
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<th>Charlie</th>
<th>Donna</th>
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Which cards must you flip to test the theory?
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Answer: (A), (B), (C), (D).
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Answer: (A), (B), (C), (D). Later.
CS70: Lecture 1. Outline.

Today: Note 1.
Today: Note 1. Note 0 is background.
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Today: Note 1.  Note 0 is background. Do read it.
The language of proofs!
Today: Note 1. Note 0 is background. Do read it.

The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan’s Laws
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
\[ 2 + 2 = 4 \]
\[ 2 + 2 = 3 \]
\[ 826\text{th digit of pi is 4} \]
Johnny Depp is a good actor
Any even \( > 2 \) is sum of 2 primes
\[ 4 + 5 \]
\[ x + x \]
Alice travelled to Chicago

I love you. Hmmm. Its complicated. Again: "value" of a proposition is ... True or False
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4 + 5 & & \text{Not Proposition.} & \\
x + x & & \text{Not a Proposition.} & \\
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\end{align*}
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\[ \text{Any even } \geq 2 \text{ is sum of 2 primes} \quad \text{Proposition} \quad \text{False} \]
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I love you.
Hmmm.
It's complicated.
Again: "value" of a proposition is...

True or False
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Propositions: Statements that are true or false.

√2 is irrational  Proposition  True
2+2 = 4  Proposition  True
2+2 = 3  Proposition  False
826th digit of pi is 4  Proposition  False
Johnny Depp is a good actor  Not Proposition  False
Any even > 2 is sum of 2 primes  Proposition  False
4 + 5  Not Proposition.
x + x  Not a Proposition.
Alice travelled to Chicago
Propositions: Statements that are true or false.

- \( \sqrt{2} \) is irrational
- \( 2+2 = 4 \)
- \( 2+2 = 3 \)
- 826th digit of pi is 4
- Johnny Depp is a good actor
- Any even \( n > 2 \) is sum of 2 primes
- \( 4 + 5 \)
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\[ \text{Any even} > 2 \text{ is sum of 2 primes} \]
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\[ x + x \]
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- Proposition 
- True
- Proposition 
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- Proposition 
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### Propositions: Statements that are true or false.

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Again: “value” of a proposition is ...
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
\[ 2 + 2 = 4 \]
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\[ 826^{\text{th}} \text{ digit of pi is 4} \]
\[ \text{Johnny Depp is a good actor} \]
\[ \text{Any even } > 2 \text{ is sum of 2 primes} \]
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\begin{align*}
\sqrt{2} \text{ is irrational} & : \text{Proposition} & \text{True} \\
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4 + 5 & : \text{Not Proposition.} & \text{False} \\
x + x & : \text{Not a Proposition.} & \text{False} \\
\text{Alice travelled to Chicago} & : \text{Proposition.} & \text{False} \\
\text{I love you.} & : \text{Hmmm.} & \text{Its complicated.} \\
\end{align*}

Again: “value” of a proposition is ... \text{True or False}
Propositional Forms.

Put propositions together to make another...
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): \( P \land Q \)

"\( P \land Q \)" is True if both \( P \) and \( Q \) are True.
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

"$P \land Q$" is True if both $P$ and $Q$ are True. Else False.
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True if both $P$ and $Q$ are True. Else False.

Disjunction ("or"): $P \vee Q$
Propositional Forms.

Put propositions together to make another...

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"$P \lor Q$" is True if at least one $P$ or $Q$ is True. Else False.
Propositional Forms.

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Examples:

$\neg \ " (2 + 2 = 4) "$ – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

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Examples:

$\neg "(2 + 2 = 4)"$ – a proposition that is ... False

"$2 + 2 = 3$" $\land$ "$2 + 2 = 4$" – a proposition that is ...

"$2 + 2 = 4$" – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): \( P \land Q \)

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\( \neg "(2 + 2 = 4)" \) – a proposition that is ... False

"2 + 2 = 3" \( \land \) "2 + 2 = 4" – a proposition that is ... False
Propositional Forms.

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Examples:

\( \neg \) "(2 + 2 = 4)" – a proposition that is ... False

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"2 + 2 = 3" \( \lor \) "2 + 2 = 4" – a proposition that is ...
Propositional Forms.

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$\neg "(2 + 2 = 4)"$ – a proposition that is ... False

"$2 + 2 = 3" \land "2 + 2 = 4"$ – a proposition that is ... False

"$2 + 2 = 3" \lor "2 + 2 = 4"$ – a proposition that is ... True
Put them together..

Propositions:

\[ P_1 \text{ - Person 1 rides the bus.} \]
Put them together..

Propositions:

$P_1$ - Person 1 rides the bus.

$P_2$ - Person 2 rides the bus.

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

Propositional Form:

$\neg (((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5)))$

Can person 3 ride the bus?

Can person 3 and person 4 ride the bus together?

This seems complicated.

We can program!!!!

We need a way to keep track!
Put them together..

Propositions:

\[ P_1 \] - Person 1 rides the bus.
\[ P_2 \] - Person 2 rides the bus.

....
Put them together.

Propositions:
$P_1$ - Person 1 rides the bus.
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Propositional Form:

\[ \neg \left( \left( \left( P_1 \lor P_2 \right) \land \left( P_3 \lor P_4 \right) \right) \lor \left( \left( P_2 \lor P_3 \right) \land \left( P_4 \lor \neg P_5 \right) \right) \right) \]
Put them together..

Propositions:
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....

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We need a way to keep track!
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

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DeMorgan's Laws for Negation: distribute and flip!

$\neg(P \land Q)$ is logically equivalent to $\neg P \lor \neg Q$.

$\neg(P \lor Q)$ is logically equivalent to $\neg P \land \neg Q$. 

Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!
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“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

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Check: $\land$ and $\lor$ are commutative.

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

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Truth Tables for Propositional Forms.

“$P \land Q$” is **True** if both $P$ and $Q$ are **True**.

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“$P \lor Q$” is **True** if at least one of $P$ or $Q$ is **True**.

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One use for truth tables: Logical Equivalence of propositional forms!
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“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

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One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. 

DeMorgan’s Law’s for Negation: distribute and flip!

$\neg(P \land Q) \equiv \neg P \lor \neg Q$

$\neg(P \lor Q) \equiv \neg P \land \neg Q$
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One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same
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One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!
Truth Tables for Propositional Forms.

“\( P \land Q \)” is True if both \( P \) and \( Q \) are True.

\[
\begin{array}{|c|c|c|}
\hline
P & Q & P \land Q \\
\hline
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F \\
\hline
\end{array}
\]

“\( P \lor Q \)” is True if \( \geq \) one of \( P \) or \( Q \) is True.

\[
\begin{array}{|c|c|c|}
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One use for truth tables: Logical Equivalence of propositional forms!

Example: \( \neg (P \land Q) \) logically equivalent to \( \neg P \lor \neg Q \). Same Truth Table!

\[
\begin{array}{|c|c|c|c|}
\hline
P & Q & \neg (P \lor Q) & \neg P \land \neg Q \\
\hline
T & T & F & \\
T & F & F & \\
F & T & F & \\
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\hline
\end{array}
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DeMorgan’s Law’s for Negation: distribute and flip!
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Check: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!
Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$. Same Truth Table!

```
<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>\neg(P \lor Q)</th>
<th>\neg P \land \neg Q</th>
</tr>
</thead>
<tbody>
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DeMorgan’s Law’s for Negation: distribute and flip!

$\neg(P \land Q) \equiv \neg P \lor \neg Q$  
$\neg(P \lor Q) \equiv \neg P \land \neg Q$
Truth Tables for Propositional Forms.

“$P \land Q$” is True if both $P$ and $Q$ are True.

<table>
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<tr>
<th>$P$</th>
<th>$Q$</th>
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</table>

“$P \lor Q$” is True if $\geq$ one of $P$ or $Q$ is True.

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<th>$P$</th>
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Truth Tables for Propositional Forms.

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DeMorgan’s Law’s for Negation: distribute and flip!

$\neg(P \land Q)$
Truth Tables for Propositional Forms.

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DeMorgan’s Law’s for Negation: distribute and flip!

$\neg(P \land Q) \equiv \neg P \lor \neg Q \quad \neg(P \lor Q)$
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DeMorgan’s Law’s for Negation: distribute and flip!

$\neg(P \land Q) \equiv \neg P \lor \neg Q \quad \neg(P \lor Q) \equiv \neg P \land \neg Q$
Quick Questions

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Is $(T \land Q) \equiv Q$?
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Is $(T \land Q) \equiv Q$? Yes?
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Is $(T \land Q) \equiv Q$? Yes? No?
Quick Questions

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Is $(T \land Q) \equiv Q$? Yes? No?
Yes!
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Is $(T \land Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$. 
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Is $(T \land Q) \equiv Q$? Yes? No?
Yes! Look at rows in truth table for $P = T$.
What is $(F \land Q)$?
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Is \((T ∧ Q) \equiv Q\)? Yes? No?

Yes! Look at rows in truth table for \(P = T\).

What is \((F ∧ Q)\)? F or False.
Quick Questions

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Is $(T \land Q) \equiv Q$? Yes? No?
Yes! Look at rows in truth table for $P = T$.

What is $(F \land Q)$? F or False.
What is $(T \lor Q)$?
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Is $(T \land Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \land Q)$? F or False.

What is $(T \lor Q)$? T
Quick Questions

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Is \((T \land Q) \equiv Q\)? Yes? No?
Yes! Look at rows in truth table for \( P = T \).

What is \((F \land Q)\)? F or False.
What is \((T \lor Q)\)? T
What is \((F \lor Q)\)?
Quick Questions

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Is $(T \land Q) \equiv Q$? Yes? No?
Yes! Look at rows in truth table for $P = T$.

What is $(F \land Q)$? F or False.

What is $(T \lor Q)$? T

What is $(F \lor Q)$? Q
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \(( T \land Q) \equiv Q,\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

Cases:

\(P\) is True .

LHS: \(T \land (Q \lor R)\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

- **P is True**.
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:
- \(P\) is \textbf{True}.
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
  - RHS: \((T \land Q) \lor (T \land R)\)
Distributive?

\( P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) \) ?

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

Cases:

- \( P \) is True .

  LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).
  RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R) \).
Distributive?

\( P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) \)

Simplify: \( (T \land Q) \equiv Q, \ (F \land Q) \equiv F \).

Cases:
- \( P \) is True.
  - LHS: \( T \land (Q \lor R) \equiv (Q \lor R). \)
  - RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R). \)
- \( P \) is False.
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

\(P\) is True.

LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)

RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)

\(P\) is False.

LHS: \(F \land (Q \lor R)\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)？ \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F. \)

Cases:
- \( P \) is True.
  - LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).
  - RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R) \).
- \( P \) is False.
  - LHS: \( F \land (Q \lor R) \equiv F. \)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

Cases:

\(P\) is True.

LHS: \(T \land (Q \lor R) \equiv (Q \lor R)\).

RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R)\).

\(P\) is False.

LHS: \(F \land (Q \lor R) \equiv F\).

RHS: \((F \land Q) \lor (F \land R)\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q\), \((F \land Q) \equiv F\).

Cases:

\(P\) is True.

LHS: \(T \land (Q \lor R) \equiv (Q \lor R)\).
RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R)\).

\(P\) is False.

LHS: \(F \land (Q \lor R) \equiv F\).
RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F)\).
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F. \)

Cases:

- \( P \) is True.
  - LHS: \( T \land (Q \lor R) \equiv (Q \lor R). \)
  - RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R). \)

- \( P \) is False.
  - LHS: \( F \land (Q \lor R) \equiv F. \)
  - RHS: \( (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F. \)
Distributive?

Does $P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$ hold?

Simplify: $(T \land Q) \equiv Q$, $(F \land Q) \equiv F$.

Cases:
- $P$ is True.
  - LHS: $T \land (Q \lor R) \equiv (Q \lor R)$.
  - RHS: $(T \land Q) \lor (T \land R) \equiv (Q \lor R)$.
- $P$ is False.
  - LHS: $F \land (Q \lor R) \equiv F$.
  - RHS: $(F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F$. 

Foil 1:

$(A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D)$?

Foil 2:

$(A \land B) \lor (C \land D) \equiv (A \lor C) \land (A \lor D) \land (B \lor C) \land (B \lor D)$?
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, \ (F \land Q) \equiv F.\)

Cases:

- \(P\) is True .
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)

- \(P\) is False .
  - LHS: \(F \land (Q \lor R) \equiv F.\)
  - RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F.\)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

\(P\) is True .

LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)

\(P\) is False .

LHS: \(F \land (Q \lor R) \equiv F.\)
RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F.\)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \(T \lor Q \equiv T,\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

\(P\) is True .

LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)

RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)

\(P\) is False .

LHS: \(F \land (Q \lor R) \equiv F.\)

RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F.\)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \(T \lor Q \equiv T, F \lor Q \equiv Q. \ldots \)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) \, ? \]

Simplify: \( (T \land Q) \equiv Q \), \( (F \land Q) \equiv F \).

Cases:

\( P \) is \textit{True}.

\begin{align*}
\text{LHS: } T \land (Q \lor R) & \equiv (Q \lor R). \\
\text{RHS: } (T \land Q) \lor (T \land R) & \equiv (Q \lor R).
\end{align*}

\( P \) is \textit{False}.

\begin{align*}
\text{LHS: } F \land (Q \lor R) & \equiv F. \\
\text{RHS: } (F \land Q) \lor (F \land R) & \equiv (F \lor F) \equiv F.
\end{align*}

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R) \, ? \]

Simplify: \( T \lor Q \equiv T \), \( F \lor Q \equiv Q \). ...

Foil 1:
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

- \(P\) is True.
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R)\).
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R)\).

- \(P\) is False.
  - LHS: \(F \land (Q \lor R) \equiv F\).
  - RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F\).

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R) ? \]

Simplify: \(T \lor Q \equiv T, F \lor Q \equiv Q. \ldots\)

Foil 1:

\[(A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D)?\]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q\), \((F \land Q) \equiv F\).

**Cases:**

- **\(P\) is True.**
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R)\).
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R)\).

- **\(P\) is False.**
  - LHS: \(F \land (Q \lor R) \equiv F\).
  - RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F\).

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \(T \lor Q \equiv T\), \(F \lor Q \equiv Q\). ...

Foil 1:

\[(A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D)?\]

Foil 2:
Distributive?

\[ P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)? \]
Simplify: \( (T \wedge Q) \equiv Q, (F \wedge Q) \equiv F. \)

Cases:
\( P \) is True .

LHS: \( T \wedge (Q \vee R) \equiv (Q \vee R). \)
RHS: \( (T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R). \)

\( P \) is False .

LHS: \( F \wedge (Q \vee R) \equiv F. \)
RHS: \( (F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F. \)

\[ P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)? \]
Simplify: \( T \vee Q \equiv T, F \vee Q \equiv Q. \) ...

Foil 1:
\( (A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)? \)

Foil 2:
\( (A \wedge B) \vee (C \wedge D) \equiv (A \vee C) \wedge (A \vee D) \wedge (B \vee C) \wedge (B \vee D)? \)
Implication.

\[ P \implies Q \text{ interpreted as} \]
Implication.

\[ P \implies Q \text{ interpreted as } \\
\text{If } P, \text{ then } Q. \]
Implication.

\[ P \implies Q \text{ interpreted as } \]

If \( P \), then \( Q \).
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).

Examples:

Statement: If you stand in the rain, then you'll get wet.
\[ P = \text{"you stand in the rain"}, \quad Q = \text{"you will get wet"} \]

Statement: "Stand in the rain"
Can conclude: "you'll get wet."

Statement: If a right triangle has sidelengths \( a \leq b \leq c \), then \( a^2 + b^2 = c^2 \).
\[ P = \text{"a right triangle has sidelengths } a \leq b \leq c\text{"}, \quad Q = \text{"} a^2 + b^2 = c^2 \text{"}. \]
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.
Implication.

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Non-Consequences/consequences of Implication

The statement “$P \implies Q$”
Non-Consequences/consequences of Implication

The statement "\( P \implies Q \)"

only is False if \( P \) is True and \( Q \) is False.
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The statement “$P \implies Q$"

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False implies nothing
The statement “$P \implies Q$”
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False implies nothing
P False means

---

Non-Consequences/consequences of Implication

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?
Not necessarily.

$P \implies Q$ and $Q$ are True does not mean $P$ is True

Be careful!
Instead we have:
$P \implies Q$ and $P$ are True does mean $Q$ is True.

The chemical plant pollutes river.
Can we conclude fish die?
The statement “\( P \implies Q \)” only is False if \( P \) is True and \( Q \) is False.

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The chemical plant pollutes river. Can we conclude fish die?
Implication and English.

\[ P \implies Q \]

Poll.

- If \( P \), then \( Q \).

Just reversing the order.

\( P \) only if \( Q \).

Remember if \( P \) is true then \( Q \) must be true.

This suggests that \( P \) can only be true if \( Q \) is true.

since if \( Q \) is false \( P \) must have been false.

\( P \) is sufficient for \( Q \).

This means that proving \( P \) allows you to conclude that \( Q \) is true.

Example: Showing \( n > 4 \) is sufficient for showing \( n > 3 \).

\( Q \) is necessary for \( P \).

For \( P \) to be true it is necessary that \( Q \) is true.

Or if \( Q \) is false then we know that \( P \) is false.

Example: It is necessary that \( n > 3 \) for \( n > 4 \).
Implication and English.

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\[ \neg P \lor Q \equiv P \implies Q. \]

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$\neg P \lor Q \equiv P \implies Q$.

These two propositional forms are logically equivalent!
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.

- **Converse** of $P \implies Q$ is $Q \implies P$.
  - If fish die the plant pollutes.

Not logically equivalent!
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute.
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
- If the plant pollutes, fish die.
- If the fish don’t die, the plant does not pollute.
  (contrapositive)
Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute.
    (contrapositive)
  
  - If you stand in the rain, you get wet.

If fish die the plant pollutes.
Not logically equivalent!

- **Definition:** If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$.
  (Logically Equivalent: $\iff$.)
**Contrapositive, Converse**

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
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- **Converse** of $P \implies Q$ is $Q \implies P$.
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Logically equivalent! Notation: \( \equiv \).
Contrapositive, Converse

- **Contrapositive** of \( P \implies Q \) is \( \neg Q \implies \neg P \).

  - If the plant pollutes, fish die.
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- If you did not get wet, you did not stand in the rain.
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Logically equivalent! Notation: \( \equiv \). Recall: \( (X \implies Y) \equiv (\neg X \lor Y) \)
Contrapositive, Converse

- **Contrapositive** of \( P \implies Q \) is \( \neg Q \implies \neg P \).
  - If the plant pollutes, fish die.
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\( P \implies Q \)
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Logically equivalent! Notation: $\equiv$. Recall: $(X \implies Y) \equiv (\neg X \lor Y)$

$P \implies Q \equiv \neg P \lor Q$
Contrapositive, Converse

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Logically equivalent! Notation: $\equiv$. Recall: $(X \implies Y) \equiv (\neg X \lor Y)$

$$P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P$$
Contrapositive, Converse

- **Contrapositive** of \( P \implies Q \) is \( \neg Q \implies \neg P \).
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P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P \equiv \neg Q \implies \neg P.
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$P \implies Q \equiv \neg P \lor Q \equiv \neg(\neg Q) \lor \neg P \equiv \neg Q \implies \neg P$.

- **Converse** of $P \implies Q$ is $Q \implies P$. 
Contrapositive, Converse

- **Contrapositive** of \( P \implies Q \) is \( \neg Q \implies \neg P \).
  - If the plant pollutes, fish die.
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  - If fish die the plant pollutes.
  - Not logically equivalent!

- **Definition**: If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$ or $P \iff Q$.
  (Logically Equivalent: $\iff$. )
Variables.
Propositions?

\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
Variables.
Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes

No.
Variables.

Propositions?

- \[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
- \[ x > 2 \]
- \[ n \text{ is even and the sum of two primes} \]

No. They have a free variable.
Variables.
Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) = \text{"x is even"}$
Variables.
Propositions?

- \[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \]
- \( x > 2 \)
- \( n \) is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., \( Q(x) = \text{"x is even"} \)
Same as boolean valued functions from 61A!
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $x > 2$
- $n$ is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) = "x$ is even”

Same as boolean valued functions from 61A!

- $P(n) = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. “
Variables.

Propositions?

- \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).
- \( x > 2 \).
- \( n \) is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., \( Q(x) = "x \) is even”

Same as boolean valued functions from 61A!

- \( P(n) = \left( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \right) \).
- \( R(x) = "x > 2" \)

Remember Wason's experiment!
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) = \text{“x is even”}$

Same as boolean valued functions from 61A!

- $P(n) = \text{“$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.”}$
- $R(x) = \text{“x > 2”}$
- $G(n) = \text{“n is even and the sum of two primes”}$
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
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We call them **predicates**, e.g., $Q(x) = “x$ is even”

Same as boolean valued functions from 61A!

- $P(n) = “\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.”$
- $R(x) = “x > 2”$
- $G(n) = “n$ is even and the sum of two primes”
- Remember Wason’s experiment!
  $F(x) = “Person x flew.”$
Variables.

Propositions?

- \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).
- \( x > 2 \)
- \( n \) is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., \( Q(x) = \text{“}x \text{ is even}\)”

Same as boolean valued functions from 61A!

- \( P(n) = \text{“}\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\)”\)
- \( R(x) = \text{“}x > 2\)”
- \( G(n) = \text{“}n \text{ is even and the sum of two primes}\)”
- Remember Wason’s experiment!
  - \( F(x) = \text{“}Person \ x \ \text{flew}\)”
  - \( C(x) = \text{“}Person \ x \ \text{went to Chicago}\)
Variables.

Propositions?

- \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).
- \( x > 2 \)
- \( n \) is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., \( Q(x) = \text{“x is even”} \)

Same as boolean valued functions from 61A!

- \( P(n) = \text{“} \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \text{”} \)
- \( R(x) = \text{“} x > 2 \text{”} \)
- \( G(n) = \text{“} n \) is even and the sum of two primes\text{”} \)
- Remember Wason’s experiment!
  \( F(x) = \text{“} \text{Person x flew.} \text{”} \)
  \( C(x) = \text{“} \text{Person x went to Chicago} \text{”} \)
  \( C(x) \implies F(x). \)
Variables.

Propositions?

\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]
\[ x > 2 \]
\[ n \text{ is even and the sum of two primes} \]

No. They have a free variable.

We call them \textbf{predicates}, e.g., \( Q(x) = "x \text{ is even}" \)

Same as boolean valued functions from 61A!

\[ P(n) = \"\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\." \]
\[ R(x) = \"x > 2\" \]
\[ G(n) = \"n \text{ is even and the sum of two primes}\" \]

Remember Wason’s experiment!

\( F(x) = \"\text{Person } x \text{ flew}\." \)
\( C(x) = \"\text{Person } x \text{ went to Chicago}\)
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- $n$ is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) = “x$ is even”

Same as boolean valued functions from 61A!

- $P(n) = “\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.”$
- $R(x) = “x > 2”$
- $G(n) = “n$ is even and the sum of two primes”

Remember Wason’s experiment!

- $F(x) = “$Person $x$ flew.”
- $C(x) = “$Person $x$ went to Chicago

$C(x) \implies F(x)$. Theory from Wason’s.

If person $x$ goes to Chicago then person $x$ flew.
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
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No. They have a free variable.

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- $P(n) = \text{“}\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\text{”}$
- $R(x) = \text{“}x > 2\text{”}$
- $G(n) = \text{“}n \text{ is even and the sum of two primes}\text{”}$
- Remember Wason’s experiment!
  - $F(x) = \text{“}\text{Person } x \text{ flew}\text{”}$
  - $C(x) = \text{“}\text{Person } x \text{ went to Chicago}\text{”}$

- $C(x) \implies F(x)$. Theory from Wason’s.
  If person $x$ goes to Chicago then person $x$ flew.

Next:
Variables.

Propositions?

- \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).
- \( x > 2 \)
- \( n \) is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., \( Q(x) = \text{"x is even"} \)

Same as boolean valued functions from 61A!

- \( P(n) = \text{"} \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \text{"} \)
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  - \( F(x) = \text{"} \text{Person } x \text{ flew.} \)
  - \( C(x) = \text{"} \text{Person } x \text{ went to Chicago} \)
- \( C(x) \implies F(x) \). Theory from Wason’s.
  If person \( x \) goes to Chicago then person \( x \) flew.

Next: Statements about boolean valued functions!!
Quantifiers.

There exists quantifier:

\[ \exists x \in S \ (P(x)) \]

means "There exists an \( x \) in \( S \) where \( P(x) \) is true."

For example:

\[ (\exists x \in \mathbb{N}) \ (x = x^2) \]

Equivalent to "\((0 = 0) \lor (1 = 1) \lor (2 = 4) \lor ...\)"

Much shorter to use a quantifier!

For all quantifier:

\[ (\forall x \in S) \ (P(x)) \]

means "For all \( x \) in \( S \), \( P(x) \) is True."

Examples:

"Adding 1 makes a bigger number."

\[ (\forall x \in \mathbb{N}) \ (x + 1 > x) \]

"the square of a number is always non-negative"

\[ (\forall x \in \mathbb{N}) \ (x^2 \geq 0) \]

Wait!

What is \( \mathbb{N} \)?
Quantifiers..

There exists quantifier:

\[(\exists x \in S)(P(x))\] means “There exists an \(x\) in \(S\) where \(P(x)\) is true.”
Quantifiers:

There exists quantifier:

$$(\exists x \in S)(P(x))$$ means "There exists an $x$ in $S$ where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$
Quantifiers.

There exists quantifier:

\[(\exists x \in S)(P(x))\] means “There exists an x in S where P(x) is true.”

For example:

\[(\exists x \in \mathbb{N})(x = x^2)\]

Equivalent to “(0 = 0)”
Quantifiers:

There exists quantifier:

$$(\exists x \in S)(P(x))$$ means “There exists an $x$ in $S$ where $P(x)$ is true.”

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to “$(0 = 0) \lor (1 = 1)$”

There exists quantifier:

$(\exists x \in S)(P(x))$ means “There exists an $x$ in $S$ where $P(x)$ is true.”

For example:

$(\exists x \in \mathbb{N})(x = x^2)$

Equivalent to “$(0 = 0) \lor (1 = 1) \lor (2 = 4)$”
There exists quantifier:

\((\exists x \in S)(P(x))\) means “There exists an \(x\) in \(S\) where \(P(x)\) is true.”

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$$(\forall x \in \mathbb{N}) (x + 1 > x)$$
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”the square of a number is always non-negative”
Quantifiers..

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Equivalent to “\(0 = 0\) \(\lor\) \((1 = 1) \lor (2 = 4) \lor \ldots\)”

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**For all quantifier;**

\[(\forall x \in S) (P(x))\] means “For all \(x\) in \(S\), \(P(x)\) is True .”

Examples:

“Adding 1 makes a bigger number.”

\[(\forall x \in \mathbb{N}) (x + 1 > x)\]

”the square of a number is always non-negative”

\[(\forall x \in \mathbb{N})(x^2 >= 0)\]
Quantifiers.

There exists quantifier:

$$(\exists x \in S)(P(x))$$ means “There exists an $x$ in $S$ where $P(x)$ is true.”

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to “$(0 = 0) \lor (1 = 1) \lor (2 = 4) \lor \ldots$”

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$$(\forall x \in \mathbb{N}) (x + 1 > x)$$

”the square of a number is always non-negative”

$$(\forall x \in \mathbb{N})(x^2 \geq 0)$$

Wait!
Quantifiers.

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\[(\forall x \in \mathbb{N}) (x + 1 > x)\]

”the square of a number is always non-negative”

\[(\forall x \in \mathbb{N})(x^2 \geq 0)\]

Wait! What is \(\mathbb{N}\)?
Quantifiers: universes.

Proposition: “For all natural numbers \( n \), \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).”

Proposition has universe:
Quantifiers: universes.

Proposition: “For all natural numbers $n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.”

Proposition has universe: “the natural numbers”.

Other proposition notation (for discussion):

$d \mid n$ means $d$ divides $n$ or $\exists k \in \mathbb{Z}, n = kd$.

$2 \mid 4$? True.

$4 \mid 2$? False.
Quantifiers: universes.

Proposition: “For all natural numbers $n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.”

Proposition has universe: “the natural numbers”.

Universe examples include:

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
- $\mathbb{Z}^+$ (positive integers)
- $\mathbb{R}$ (real numbers)
- Any set: $S = \{Alice, Bob, Charlie, Donna\}$.
- See note 0 for more!

Other proposition notation (for discussion):

“$d|n$” means $d$ divides $n$
Quantifiers: universes.

Proposition: “For all natural numbers \( n \), \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).”

Proposition has universe: “the natural numbers”.

Universe examples include..

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Other proposition notation (for discussion):

“\( d \mid n \)” means \( d \) divides \( n \)

or \( \exists k \in \mathbb{Z}, n = kd \).

\( 2 \mid 4? \)
Quantifiers: universes.

Proposition: “For all natural numbers \( n \), \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).”

Proposition has **universe**: “the natural numbers”.

Universe examples include..

- \( \mathbb{N} = \{0, 1, \ldots\} \) (natural numbers).
- \( \mathbb{Z} = \{\ldots, -1, 0, \ldots\} \) (integers)
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Other proposition notation(for discussion):

“\( d \mid n \)” means \( d \) divides \( n \)

Or \( \exists k \in \mathbb{Z}, n = kd \).

2|4? True.
Quantifiers: universes.

Proposition: “For all natural numbers \( n \), \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).”

Proposition has universe: “the natural numbers”.

Universe examples include:

- \( \mathbb{N} = \{0, 1, \ldots\} \) (natural numbers).
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Other proposition notation (for discussion):

“\( d \mid n \)” means \( d \) divides \( n \)

or \( \exists k \in \mathbb{Z}, n = kd \).

2\( \mid \)4? True.

4\( \mid \)2?
Quantifiers: universes.

Proposition: “For all natural numbers \( n, \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).”

Proposition has universe: “the natural numbers”.

Universe examples include:

- \( \mathbb{N} = \{0, 1, \ldots\} \) (natural numbers).
- \( \mathbb{Z} = \{\ldots, -1, 0, \ldots\} \) (integers)
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- Any set: \( S = \{Alice, Bob, Charlie, Donna\} \).
- See note 0 for more!

Other proposition notation (for discussion):

“\( d \mid n \)” means \( d \) divides \( n \)

or \( \exists k \in \mathbb{Z}, n = kd \).

2|4? True.
4|2? False.
Back to: Wason’s experiment:1

Theory:

"If a person travels to Chicago, he/she/they flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew. Which cards do you need to flip to test the theory?

Chicago (x) = "x went to Chicago."

Flew (x) = "x flew."

Statement/theory: ∀ x ∈ {A, B, C, D}, Chicago (x) ⇒ Flew (x)

Chicago (A) = False. Do we care about Flew (A)? No.

Chicago (B) = ⇒ Flew (B) is true. since Chicago (B) is False, Flew (B) = False. Do we care about Chicago (B)? Yes.

Chicago (C) = True. Do we care about Flew (C)? Yes. Chicago (C) = ⇒ Flew (C) means Flew (C) must be true.

Flew (D) = True. Do we care about Chicago (D)? No. Chicago (D) = ⇒ Flew (D) is true if Flew (D) is true. Only have to turn over cards for Bob and Charlie.
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ Chicago(x) = \text{“x went to Chicago.”} \]
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

*Chicago*(*x*) = “*x* went to Chicago.”  
*Flew*(*x*) = “*x* flew”
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“} x \text{ went to Chicago.} \quad F\text{lew}(x) = \text{“} x \text{ flew} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \)
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{ A, B, C, D \}, \text{Chicago}(x) \implies \text{Flew}(x) \)
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \).
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

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Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

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Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

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Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False,
Back to: Wason's experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,

\( \text{Flew}(B) = \text{False} \).
Back to: Wason's experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

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\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?
Back to: Wason’s experiment:1

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\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \)
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ Chicago(x) = \text{“x went to Chicago.”} \quad Flew(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x) \)

\( Chicago(A) = \text{False}. \) Do we care about \( Flew(A) \)?

No. \( Chicago(A) \implies Flew(A) \) is true.

since \( Chicago(A) \) is \text{False},

\( Flew(B) = \text{False}. \) Do we care about \( Chicago(B) \)?

Yes. \( Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B). \)
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“} x \text{ went to Chicago.} \text{”} \quad \text{Flew}(x) = \text{“} x \text{ flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False .
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\( \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \)

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is \text{False} ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be \text{False} .

\( \text{Chicago}(C) = \text{True} \).
Back to: Wason's experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\( Chicago(x) = “x \text{ went to Chicago.”} \quad Flew(x) = “x \text{ flew”} \)

Statement/theory: \( \forall x \in \{A, B, C, D\}, \quad Chicago(x) \implies Flew(x) \)

\( Chicago(A) = \text{False} \). Do we care about \( Flew(A) \)?

No. \( Chicago(A) \implies Flew(A) \) is true.

since \( Chicago(A) \) is \( \text{False} \),

\( Flew(B) = \text{False} \). Do we care about \( Chicago(B) \)?

Yes. \( Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B) \).

So \( Chicago(Bob) \) must be \( \text{False} \).

\( Chicago(C) = \text{True} \). Do we care about \( Flew(C) \)?
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\[ \text{Chicago}(A) = \text{False} . \text{Do we care about Flew}(A) ? \]

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False , \( \text{Flew}(B) = \text{False} . \text{Do we care about Chicago}(B) ? \)

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B). \)

So \( \text{Chicago}(Bob) \) must be False .

\[ \text{Chicago}(C) = \text{True} . \text{Do we care about Flew}(C) ? \]

Yes.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”  
Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

Chicago(x) = “x went to Chicago.”  Flew(x) = “x flew”

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

- **Chicago(A) = False**. Do we care about **Flew(A)**?
  - No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

- since \( \text{Chicago}(A) \) is False ,

- \( \text{Flew}(B) = \text{False} \). Do we care about **Chicago(B)**?
  - Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

  - So **Chicago(Bob)** must be False  .

- \( \text{Chicago}(C) = \text{True} \). Do we care about **Flew(C)**?
  - Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means **Flew(C)** must be true.
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{ A, B, C, D \}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\text{Chicago}(A) = \text{False} . \text{ Do we care about Flew}(A)?

No. \text{Chicago}(A) \implies \text{Flew}(A) \text{ is true.}

since \text{Chicago}(A) \text{ is False} ,

\text{Flew}(B) = \text{False} . \text{ Do we care about Chicago}(B)?

Yes. \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B).

So \text{Chicago}(Bob) \text{ must be False} .

\text{Chicago}(C) = \text{True} . \text{ Do we care about Flew}(C)?

Yes. \text{Chicago}(C) \implies \text{Flew}(C) \text{ means Flew}(C) \text{ must be true.}

\text{Flew}(D) = \text{True} .
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False .

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} \). Do we care about \( \text{Chicago}(D) \)?
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False .

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} \). Do we care about \( \text{Chicago}(D) \)?

No.
Back to: Wason’s experiment:1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ \text{Chicago}(x) = \text{“x went to Chicago.”} \quad \text{Flew}(x) = \text{“x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, \text{Chicago}(x) \implies \text{Flew}(x) \)

\( \text{Chicago}(A) = \text{False} \). Do we care about \( \text{Flew}(A) \)?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} \). Do we care about \( \text{Chicago}(B) \)?

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) \).

So \( \text{Chicago}(Bob) \) must be False .

\( \text{Chicago}(C) = \text{True} \). Do we care about \( \text{Flew}(C) \)?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} \). Do we care about \( \text{Chicago}(D) \)?

No. \( \text{Chicago}(D) \implies \text{Flew}(D) \) is true if \( \text{Flew}(D) \) is true.
Back to: Wason’s experiment: 1

Theory: “If a person travels to Chicago, he/she/they flies.”

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[
\text{Chicago}(x) = \text{“} x \text{ went to Chicago.}\quad \text{Flew}(x) = \text{“} x \text{ flew}\]

Statement/theory: \( \forall x \in \{A, B, C, D\},\text{Chicago}(x) \implies \text{Flew}(x) \)

\text{Chicago}(A) = \text{False} . \text{ Do we care about } \text{Flew}(A) ?

No. \( \text{Chicago}(A) \implies \text{Flew}(A) \) is true.

since \( \text{Chicago}(A) \) is False ,

\( \text{Flew}(B) = \text{False} . \text{ Do we care about } \text{Chicago}(B) ? \)

Yes. \( \text{Chicago}(B) \implies \text{Flew}(B) \equiv \neg \text{Flew}(B) \implies \neg \text{Chicago}(B) . \)

So \( \text{Chicago}(Bob) \) must be False .

\text{Chicago}(C) = \text{True} . \text{ Do we care about } \text{Flew}(C) ?

Yes. \( \text{Chicago}(C) \implies \text{Flew}(C) \) means \( \text{Flew}(C) \) must be true.

\( \text{Flew}(D) = \text{True} . \text{ Do we care about } \text{Chicago}(D) ? \)

No. \( \text{Chicago}(D) \implies \text{Flew}(D) \) is true if \( \text{Flew}(D) \) is true.

Only have to turn over cards for Bob and Charlie.
More for all quantifiers examples.

- "Doubling a number always makes it larger":
  \[(\forall x \in \mathbb{N}) (2x > x)\] is false, because for example, when \(x = 0\).

- "Square of any natural number greater than 5 is greater than 25."
  \[(\forall x \in \mathbb{N}) (x > 5 \Rightarrow x^2 > 25)\] is true.

Idea alert: Restrict the domain using implication. Later we may omit the universe if clear from context.
More for all quantifiers examples.

- “doubling a number always makes it larger”
More for all quantifiers examples.

▶ “doubling a number always makes it larger”

\[(\forall x \in N) (2x > x)\]
More for all quantifiers examples.

▶ “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x) \quad \text{False}\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

  \((\forall x \in \mathbb{N}) (2x > x)\)  \text{ False}  \text{ Consider } x = 0
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x)\]  \textbf{False} \quad \text{Consider} \ x = 0

Can fix statement...
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N})(2x > x) \quad \text{False} \quad \text{Consider} \quad x = 0\]

Can fix statement...

\[(\forall x \in \mathbb{N})(2x \geq x)\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[
\forall x \in \mathbb{N} \ (2x > x) \quad \text{False} \quad \text{Consider } x = 0
\]

Can fix statement...

\[
\forall x \in \mathbb{N} \ (2x \geq x) \quad \text{True}
\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in N) (2x > x)\]  \text{False}  \text{ Consider } x = 0

Can fix statement...

\[(\forall x \in N) (2x \geq x)\]  \text{True}

- “Square of any natural number greater than 5 is greater than 25.”
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[ \forall x \in \mathbb{N} \ (2x > x) \quad \text{False} \quad \text{Consider } x = 0 \]

Can fix statement...

\[ \forall x \in \mathbb{N} \ (2x \geq x) \quad \text{True} \]

- “Square of any natural number greater than 5 is greater than 25.”

\[ \forall x \in \mathbb{N} \]

\[ (x > 5) \Rightarrow x^2 > 25 \]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0\]

Can fix statement...

\[(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}\]

- “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in \mathbb{N})(x > 5)\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N})(2x > x) \quad \text{False} \quad \text{Consider } x = 0\]

Can fix statement...

\[(\forall x \in \mathbb{N})(2x \geq x) \quad \text{True}\]

- “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in \mathbb{N})(x > 5) \implies \]
More for all quantifiers examples.

- “doubling a number always makes it larger”

  \[(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0\]

  Can fix statement...

  \[(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}\]

- “Square of any natural number greater than 5 is greater than 25.”

  \[(\forall x \in \mathbb{N})(x > 5 \implies x^2 > 25).\]
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0\]

Can fix statement...

\[(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}\]

- “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in \mathbb{N})(x > 5 \implies x^2 > 25)\]

Idea alert:
More for all quantifiers examples.

- “doubling a number always makes it larger”

\[
(\forall x \in \mathbb{N}) \ (2x > x) \quad \text{False} \quad \text{Consider} \ x = 0
\]

Can fix statement...

\[
(\forall x \in \mathbb{N}) \ (2x \geq x) \quad \text{True}
\]

- “Square of any natural number greater than 5 is greater than 25.”

\[
(\forall x \in \mathbb{N})(x > 5 \implies x^2 > 25).
\]

Idea alert: Restrict domain using implication.
More for all quantifiers examples.

- “doubling a number always makes it larger”
  
  \[(\forall x \in N)(2x > x) \quad \text{False} \quad \text{Consider} \ x = 0\]

  Can fix statement...

  \[(\forall x \in N)(2x \geq x) \quad \text{True}\]

- “Square of any natural number greater than 5 is greater than 25.”

  \[(\forall x \in N)(x > 5 \implies x^2 > 25)\]

  Idea alert: Restrict domain using implication.

  Later we may omit universe if clear from context.
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.
Quantifiers..not commutative.

► In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N)\]
Quantifiers...not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) (\forall x \in N)\]
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) \ (\forall x \in N) \ (y = x^2)\]
Quantifiers..not commutative.

In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2)\] False
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in N) (\forall x \in N) (y = x^2) \quad \text{False}$$

- In English: “the square of every natural number is a natural number.”
Quantifiers...not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) (\forall x \in N) (y = x^2)\] False

- In English: “the square of every natural number is a natural number.”

\[(\forall x \in N)\]
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2)\] False

- In English: “the square of every natural number is a natural number.”

\[(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})\]
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in N) \ (\forall x \in N) \ (y = x^2) \quad \text{False}$$

- In English: “the square of every natural number is a natural number.”

$$(\forall x \in N)(\exists y \in N) \ (y = x^2)$$
Quantifiers...not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) (\forall x \in N) (y = x^2)\] False

- In English: “the square of every natural number is a natural number.”

\[(\forall x \in N)(\exists y \in N) (y = x^2)\] True
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2)\] False

- In English: “the square of every natural number is a natural number.”

\[(\forall x \in \mathbb{N})(\exists y \in \mathbb{N}) (y = x^2)\] True
Consider

\[ \neg(\forall x \in S)(P(x)), \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is, \( \neg(\forall x \in S)(P(x)) \) \( \iff \exists x \in S)(\neg P(x)) \).

What we do in this course! We consider claims.

Claim:

\[ (\forall x) P(x) \]

"For all inputs \( x \) the program works."

For False, find \( x \), where \( \neg P(x) \).

Counterexample.

Bad input. Case that illustrates bug.

For True: prove claim.

Next lectures...
Quantifiers....negation...DeMorgan again.

Consider

$$\neg (\forall x \in S)(P(x)),$$

English: there is an $x$ in $S$ where $P(x)$ does not hold.
Consider

\[ \neg (\forall x \in S)(P(x)), \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is,
Consider
\[ \neg (\forall x \in S)(P(x)), \]
English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
That is,
\[ \neg (\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]
Consider
\[ \neg(\forall x \in S)(P(x)), \]
English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
That is,
\[ \neg(\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]
What we do in this course! We consider claims.
Quantifiers....negation...DeMorgan again.

Consider

\[ \neg(\forall x \in S)(P(x)), \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is,

\[ \neg(\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]

What we do in this course! We consider claims.

Claim: \((\forall x) P(x)\)
Consider
\[ \neg (\forall x \in S)(P(x)), \]
English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
That is,
\[ \neg (\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]

What we do in this course! We consider claims.

**Claim:** \( (\forall x) P(x) \)  “For all inputs \( x \) the program works.”
Consider

\[ \neg (\forall x \in S)(P(x)), \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is,

\[ \neg (\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]

What we do in this course! We consider claims.

**Claim**: \( (\forall x) P(x) \)  “For all inputs \( x \) the program works.”

For False, find \( x \), where \( \neg P(x) \).
Consider
\[-(\forall x \in S)(P(x)),\]
English: there is an $x$ in $S$ where $P(x)$ does not hold.
That is,
\[-(\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)).\]

What we do in this course! We consider claims.

**Claim:** $(\forall x) P(x)$ “For all inputs $x$ the program works.”
For **False**, find $x$, where $\neg P(x)$.
Counterexample.
Consider

\( \neg (\forall x \in S)(P(x)) \),

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is,

\[ \neg (\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)) \].

What we do in this course! We consider claims.

**Claim:** \( (\forall x) P(x) \) “For all inputs \( x \) the program works.”

For **False**, find \( x \), where \( \neg P(x) \).

Counterexample.

Bad input.
Consider
\[ \neg(\forall x \in S)(P(x)), \]
English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
That is,
\[ \neg(\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]

What we do in this course! We consider claims.

**Claim:** \((\forall x) P(x)\)  “For all inputs \( x \) the program works.”

For **False**, find \( x \), where \( \neg P(x) \).
   Counterexample.
   Bad input.
   Case that illustrates bug.
Consider
\[ \neg (\forall x \in S)(P(x)), \]
English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
That is,
\[ \neg (\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]

What we do in this course! We consider claims.

**Claim:** \( (\forall x) P(x) \) “For all inputs \( x \) the program works.”
For **False**, find \( x \), where \( \neg P(x) \).
  - Counterexample.
  - Bad input.
  - Case that illustrates bug.
For **True**: prove claim.
Consider

\( \neg (\forall x \in S)(P(x)) \),

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is,

\( \neg (\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \)

What we do in this course! We consider claims.

**Claim:** \( (\forall x) P(x) \) “For all inputs \( x \) the program works.”

For False, find \( x \), where \( \neg P(x) \).

Counterexample.

Bad input.

Case that illustrates bug.

For True: prove claim. Next lectures...
Negation of exists.

Consider

\[ \neg \left( \exists x \in S \right) \left( P(x) \right) \]

English: means that there is no \( x \in S \) where \( P(x) \) is true.

That is, \( \neg \left( \exists x \in S \right) \left( P(x) \right) \iff \forall \left( x \in S \right) \neg P(x) \).
Negation of exists.

Consider

\[ \neg(\exists x \in S)(P(x)) \]
Negation of exists.

Consider

\[ \neg(\exists x \in S)(P(x)) \]

English: means that there is no \( x \in S \) where \( P(x) \) is true.
Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

English: means that there is no $x \in S$ where $P(x)$ is true. English: means that for all $x \in S$, $P(x)$ does not hold.
Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

English: means that there is no $x \in S$ where $P(x)$ is true. English: means that for all $x \in S$, $P(x)$ does not hold.

That is,

$$\neg(\exists x \in S)(P(x)) \iff \forall(x \in S) \neg P(x).$$
Theorem: \((\forall n \in N)(\neg(\exists a, b, c \in N)(n \geq 3 \implies a^n + b^n = c^n))\)
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Summary.

Propositions are statements that are true or false.
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Propositions are statements that are true or false.
Propositional forms use $\land, \lor, \neg$. 

De Morgan's Laws: "Flip and Distribute negation"

$\neg(P \lor Q) \iff \neg P \land \neg Q$.

$\neg\forall x \ P(x) \iff \exists x \neg P(x)$. 

Next Time: proofs!
Summary.

Propositions are statements that are true or false.

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Implication: $P \Rightarrow Q \iff \neg P \lor Q$.
Contrapositive: $\neg Q \Rightarrow \neg P$.
Converse: $Q \Rightarrow P$.

Predicates: Statements with “free” variables.
Quantifiers: $\forall x P(x), \exists y Q(y)$.

Now can state theorems! And disprove false ones!

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