Counting and Probability

Second half of the semester: Probability.
Second half of the semester: Probability.

A bag contains a set of colored balls:
Second half of the semester: Probability.

A bag contains a set of colored balls:

- Red
- Blue
- Yellow
- Blue
- Red
- Red
- Red
- Blue
Second half of the semester: Probability.

A bag contains a set of colored balls:

What is the chance that a ball taken from the bag is blue?
Second half of the semester: Probability.

A bag contains a set of colored balls:

![Image of colored balls]

What is the chance that a ball taken from the bag is blue?
Count blue.
Second half of the semester: Probability.

A bag contains a set of colored balls:

What is the chance that a ball taken from the bag is blue?
Count blue. Count total.
Second half of the semester: Probability.

A bag contains a set of colored balls:

- Red
- Blue
- Yellow
- Blue
- Red
- Red
- Red
- Blue

What is the chance that a ball taken from the bag is blue?

Second half of the semester: Probability.

A bag contains a set of colored balls:

What is the chance that a ball taken from the bag is blue?

Today:
Second half of the semester: Probability.

A bag contains a set of colored balls:

What is the chance that a ball taken from the bag is blue?
Today: Counting!
Second half of the semester: Probability.

A bag contains a set of colored balls:

What is the chance that a ball taken from the bag is blue?


Today: Counting!

After the Midterm: Probability.
Second half of the semester: Probability.

A bag contains a set of colored balls:

What is the chance that a ball taken from the bag is blue?

Today: Counting!

After the Midterm: Probability. Professor Sinclair.
1. Counting.

2. Tree

3. Rules of Counting

4. Sample with/without replacement where order does/doesn’t matter.
Count?

How many outcomes possible for $k$ coin tosses?
How many handshakes for $n$ people?
How many 10 digit numbers?
How many 10 digit numbers without repeating digits?
Using a tree of possibilities...

How many 3-bit strings?
Using a tree of possibilities...

How many 3-bit strings?
How many different sequences of three bits from \( \{0, 1\} \)?
Using a tree of possibilities...

How many 3-bit strings?
How many different sequences of three bits from \{0, 1\}? How would you make one sequence?

8 leaves which is $2 \times 2 \times 2$. One leaf for each string. 8 3-bit strings!
Using a tree of possibilities...

How many 3-bit strings?
How many different sequences of three bits from \{0, 1\}? 
How would you make one sequence?
How many different ways to do that making?
Using a tree of possibilities...

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  How many different sequences of three bits from \( \{0, 1\} \)?
  How would you make one sequence?
  How many different ways to do that making?

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One leaf for each string.

8 3-bit strings!
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1
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How would you make one sequence?
How many different ways to do that making?

8 leaves which is \(2 \times 2 \times 2\).
One leaf for each string.
8 3-bit strings!
First Rule of Counting: Product Rule

Objects made by choosing from \( n_1 \), then \( n_2 \), \ldots, then \( n_k \) the number of objects is \( n_1 \times n_2 \times \cdots \times n_k \).
First Rule of Counting: Product Rule

Objects made by choosing from $n_1$, then $n_2$, ..., then $n_k$ the number of objects is $n_1 \times n_2 \cdots \times n_k$. 

$2 \times 2 \times 3 = 12$
First Rule of Counting: Product Rule

Objects made by choosing from $n_1$, then $n_2$, ..., then $n_k$ the number of objects is $n_1 \times n_2 \cdots \times n_k$. 

\[ n_1 \times n_2 \times \cdots \times n_k = 12 \]
First Rule of Counting: Product Rule

Objects made by choosing from \( n_1 \), then \( n_2 \), ..., then \( n_k \)
the number of objects is \( n_1 \times n_2 \cdots \times n_k \).
First Rule of Counting: Product Rule

Objects made by choosing from $n_1$, then $n_2$, ..., then $n_k$, the number of objects is $n_1 \times n_2 \cdots \times n_k$. 
First Rule of Counting: Product Rule

Objects made by choosing from $n_1$, then $n_2$, ..., then $n_k$ the number of objects is $n_1 \times n_2 \cdots \times n_k$.

In picture, $2 \times 2 \times 3 = 12$
First Rule of Counting: Product Rule

Objects made by choosing from $n_1$, then $n_2$, ..., then $n_k$ the number of objects is $n_1 \times n_2 \cdots \times n_k$.

In picture, $2 \times 2 \times 3 = 12$
Using the first rule..

How many outcomes possible for \(k\) coin tosses?
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice,
How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2^k$
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2$
Using the first rule..

How many outcomes possible for \( k \) coin tosses?

2 ways for first choice, 2 ways for second choice, ...

\[ 2 \times 2 \times \ldots \]
Using the first rule..

How many outcomes possible for $k$ coin tosses?

2 ways for first choice, 2 ways for second choice, ...

$2 \times 2 \cdots \times 2$
Using the first rule...

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \dots \times 2 = 2^k$
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \times \cdots \times 2 = 2^k$

How many 10 digit numbers (leading zeroes are OK)?
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \times \cdots \times 2 = 2^k$

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice,
Using the first rule..

How many outcomes possible for \( k \) coin tosses?
2 ways for first choice, 2 ways for second choice, ...
\[ 2 \times 2 \cdots \times 2 = 2^k \]

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
\[ 10 \times 10 \cdots \times 10 = 10^{10} \]

How many \( n \) digit base \( m \) numbers?
m ways for first, m ways for second, ...
\[ m \times m \cdots \times m = m^n \]
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
10
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
$10 \times$
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots$
Using the first rule..

How many outcomes possible for \( k \) coin tosses?
2 ways for first choice, 2 ways for second choice, ...
\[ 2 \times 2 \cdots \times 2 = 2^k \]

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
\[ 10 \times 10 \cdots \times 10 \]
Using the first rule..

How many outcomes possible for $k$ coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers (leading zeroes are OK)?

10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10 = 10^{10}$
Using the first rule..

How many outcomes possible for \( k \) coin tosses?
2 ways for first choice, 2 ways for second choice, ...
\[ 2 \times 2 \cdots \times 2 = 2^k \]

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
\[ 10 \times 10 \cdots \times 10 = 10^{10} \]

How many 10 digit numbers (no leading zeroes)?
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10 = 10^{10}$

How many 10 digit numbers (no leading zeroes)?
9 ways for first choice,
Using the first rule..

How many outcomes possible for \( k \) coin tosses?
2 ways for first choice, 2 ways for second choice, ...
\[ 2 \times 2 \times \cdots \times 2 = 2^k \]

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
\[ 10 \times 10 \times \cdots \times 10 = 10^{10} \]

How many 10 digit numbers (no leading zeroes)?
9 ways for first choice, 10 ways for second choice, ...
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \times \cdots \times 2 = 2^k$

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \times \cdots \times 10 = 10^{10}$

How many 10 digit numbers (no leading zeroes)?
9 ways for first choice, 10 ways for second choice, ...
$9 \times$
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10 = 10^{10}$

How many 10 digit numbers (no leading zeroes)?
9 ways for first choice, 10 ways for second choice, ...
$9 \times 10 \cdots$
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10 = 10^{10}$

How many 10 digit numbers (no leading zeroes)?
9 ways for first choice, 10 ways for second choice, ...
$9 \times 10 \cdots \times 10$
Using the first rule.

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10 = 10^{10}$

How many 10 digit numbers (no leading zeroes)?
9 ways for first choice, 10 ways for second choice, ...
$9 \times 10 \cdots \times 10 = 9 \times 10^9$
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10 = 10^{10}$

How many 10 digit numbers (no leading zeroes)?
9 ways for first choice, 10 ways for second choice, ...
$9 \times 10 \cdots \times 10 = 9 \times 10^9$

How many $n$ digit base $m$ numbers?
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10 = 10^{10}$

How many 10 digit numbers (no leading zeroes)?
9 ways for first choice, 10 ways for second choice, ...
$9 \times 10 \cdots \times 10 = 9 \times 10^9$

How many $n$ digit base $m$ numbers?
$m$ ways for first,
Using the first rule..

How many outcomes possible for \( k \) coin tosses?
2 ways for first choice, 2 ways for second choice, ...
\[
2 \times 2 \cdots \times 2 = 2^k
\]

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
\[
10 \times 10 \cdots \times 10 = 10^{10}
\]

How many 10 digit numbers (no leading zeroes)?
9 ways for first choice, 10 ways for second choice, ...
\[
9 \times 10 \cdots \times 10 = 9 \times 10^9
\]

How many \( n \) digit base \( m \) numbers?
\( m \) ways for first, \( m \) ways for second, ...
Using the first rule..

How many outcomes possible for \( k \) coin tosses?
2 ways for first choice, 2 ways for second choice, ...
\[ 2 \times 2 \cdots \times 2 = 2^k \]

How many 10 digit numbers (leading zeroes are OK)?
10 ways for first choice, 10 ways for second choice, ...
\[ 10 \times 10 \cdots \times 10 = 10^{10} \]

How many 10 digit numbers (no leading zeroes)?
9 ways for first choice, 10 ways for second choice, ...
\[ 9 \times 10 \cdots \times 10 = 9 \times 10^9 \]

How many \( n \) digit base \( m \) numbers?
m ways for first, \( m \) ways for second, ...
\[ m^n \]
Functions, polynomials.

How many functions $f$ mapping $S$ to $T$?
How many functions $f$ mapping $S$ to $T$?

$|T|$ ways to choose for $f(s_1)$,
How many functions $f$ mapping $S$ to $T$?

$|T|$ ways to choose for $f(s_1)$, $|T|$ ways to choose for $f(s_2)$, ...
How many functions \( f \) mapping \( S \) to \( T \)?

\(|T|\) ways to choose for \( f(s_1) \), \(|T|\) ways to choose for \( f(s_2) \), ...

\(\ldots |T|^{|S|}\)
Functions, polynomials.

How many functions $f$ mapping $S$ to $T$?
$|T|$ ways to choose for $f(s_1)$, $|T|$ ways to choose for $f(s_2)$, ...
....$|T|^{|S|}$

How many polynomials of degree at most $d$ modulo $p$?
Functions, polynomials.

How many functions $f$ mapping $S$ to $T$?
$|T|$ ways to choose for $f(s_1)$, $|T|$ ways to choose for $f(s_2)$, ...
....$|T|^{|S|}$

How many polynomials of degree at most $d$ modulo $p$?
$p$ ways to choose for first coefficient,
Functions, polynomials.

How many functions $f$ mapping $S$ to $T$?

$|T|$ ways to choose for $f(s_1)$, $|T|$ ways to choose for $f(s_2)$, ...

$\ldots |T|^{|S|}$

How many polynomials of degree at most $d$ modulo $p$?

$p$ ways to choose for first coefficient, $p$ ways for second, $\ldots$
How many functions $f$ mapping $S$ to $T$?
$|T|$ ways to choose for $f(s_1)$, $|T|$ ways to choose for $f(s_2)$, ...
....$|T|^{|S|}$

How many polynomials of degree at most $d$ modulo $p$?
$p$ ways to choose for first coefficient, $p$ ways for second, ...
...$p^{d+1}$
Functions, polynomials.

How many functions $f$ mapping $S$ to $T$?

$|T|$ ways to choose for $f(s_1)$, $|T|$ ways to choose for $f(s_2)$, ...

$\ldots |T|^{|S|}$

How many polynomials of degree at most $d$ modulo $p$?

$p$ ways to choose for first coefficient, $p$ ways for second, ...

$\ldots p^{d+1}$

$p$ values for first point,
Functions, polynomials.

How many functions \( f \) mapping \( S \) to \( T \)?
| \( T \) | ways to choose for \( f(s_1) \), | \( T \) | ways to choose for \( f(s_2) \), ...
....| \( T \) |\(^{|S|}\)

How many polynomials of degree at most \( d \) modulo \( p \)?
\( p \) ways to choose for first coefficient, \( p \) ways for second, ...
...\( p^{d+1} \)

\( p \) values for first point, \( p \) values for second, ...
Functions, polynomials.

How many functions $f$ mapping $S$ to $T$?
| $T$ | ways to choose for $f(s_1)$, | $T$ | ways to choose for $f(s_2)$, ...
.... | $T$ | $|S|$ |

How many polynomials of degree at most $d$ modulo $p$?
$p$ ways to choose for first coefficient, $p$ ways for second, ...
... $p^{d+1}$

$p$ values for first point, $p$ values for second, ...
... $p^{d+1}$
Permutations.

1. How many 10 digit numbers without repeating a digit?
   \[10 \times 9 \times 8 \times \cdots \times 1 = 10!\]

2. How many different samples of size \(k\) from \(n\) numbers without replacement?
   \[n \times (n-1) \times (n-2) \times \cdots \times (n-k+1) = \frac{n!}{(n-k)!}\]

3. How many orderings of \(n\) objects are there?
   \[n \times (n-1) \times (n-2) \times \cdots \times 1 = n!\]

\(^1\) By definition: \(0! = 1\). \(n! = n(n-1)(n-2)\ldots 1\).
Permutations.

How many 10 digit numbers **without repeating a digit**?

\[ \text{10 ways for first, 9 ways for second, 8 ways for third, ... 1 way} = 10! \]

How many different samples of size \( k \) from \( n \) numbers without replacement.

\[ \text{n ways for first choice, } n-1 \text{ ways for second, } n-2 \text{ choices for third, ... } n-(k-1) \text{ ways} = \frac{n!}{(n-k)!} \]

How many orderings of \( n \) objects are there?

Permutations of \( n \) objects.

\[ \text{n ways for first, } n-1 \text{ ways for second, } n-2 \text{ ways for third, ... 1 way} = n! \]

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\(^1\)By definition: 0! = 1. \( n! = n(n-1)(n-2) \ldots 1 \).
Permutations.

How many 10 digit numbers without repeating a digit?
10 ways for first,

\[10 \times 9 \times 8 \times \ldots \times 1 = 10!.
\]

How many different samples of size \(k\) from \(n\) numbers without replacement.
\(n\) ways for first choice,
\(n - 1\) ways for second,
\(n - 2\) choices for third,
\(\ldots\)
\(n \times (n - 1) \times (n - 2) \times \ldots \times (n - k + 1) = \frac{n!}{(n-k)!}.
\]

How many orderings of \(n\) objects are there?
Permutations of \(n\) objects.
\(n\) ways for first,
\(n - 1\) ways for second,
\(n - 2\) ways for third,
\(\ldots\)
\(n \times (n - 1) \times (n - 2) \times \ldots \times 1 = n!.
\]

\(^1\)By definition: 0! = 1. \(n! = n(n-1)(n-2)\ldots1.\)
Permutations.

How many 10 digit numbers **without repeating a digit**?
10 ways for first, 9 ways for second,

\[10 \times 9 \times 8 \times \cdots \times 1 = 10!\]

How many different samples of size \(k\) from \(n\) numbers without replacement.
\(n\) ways for first choice, \(n-1\) ways for second, \(n-2\) choices for third, ...

\[n \times (n-1) \times (n-2) \cdots (n-k+1) = \frac{n!}{(n-k)!}\]

How many orderings of \(n\) objects are there?
Permutations of \(n\) objects.
\(n\) ways for first, \(n-1\) ways for second, \(n-2\) ways for third, ...

\[n \times (n-1) \times (n-2) \cdots 1 = n!\]

\(^1\)By definition: \(0! = 1\). \(n! = n(n-1)(n-2)\ldots 1\).
Permutations.

How many 10 digit numbers **without repeating a digit**?
10 ways for first, 9 ways for second, 8 ways for third,

\[10 \cdot 9 \cdot 8 \cdots 1 = 10! \]

How many different samples of size \(k\) from \(n\) numbers **without replacement**.

\[n \text{ ways for first choice, } n-1 \text{ ways for second, } n-2 \text{ choices for third, } \ldots \]

\[n \cdot (n-1) \cdot (n-2) \cdots (n-k+1) = \frac{n!}{(n-k)!}\]

How many orderings of \(n\) objects are there?

**Permutations of** \(n\) **objects**.

\[n \text{ ways for first, } n-1 \text{ ways for second, } n-2 \text{ ways for third, } \ldots \]

\[n \cdot (n-1) \cdot (n-2) \cdots 1 = n!\]

\(^1\)By definition: 0! = 1. \(n! = n(n-1)(n-2)\ldots 1\).
Permutations.

How many 10 digit numbers **without repeating a digit**?
10 ways for first, 9 ways for second, 8 ways for third, ...

\[ n! = n(n-1)(n-2)\ldots 1. \]

\(^1\)By definition: 0! = 1.
Permutations.

How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second, 8 ways for third, ...

... $10 \times 9 \times 8 \cdots \times 1 = 10!$.\(^1\)

\(^1\)By definition: $0! = 1$. $n! = n(n-1)(n-2)\cdots 1$. 
Permutations.

How many 10 digit numbers without repeating a digit?  
10 ways for first, 9 ways for second, 8 ways for third, ... 
... $10 \times 9 \times 8 \times \ldots \times 1 = 10!$.\(^1\)

How many different samples of size $k$ from $n$ numbers without replacement.

\(^1\)By definition: $0! = 1$. $n! = n(n-1)(n-2)\ldots1$. 
Permutations.

How many 10 digit numbers **without repeating a digit**?
10 ways for first, 9 ways for second, 8 ways for third, ...
... $10 \times 9 \times 8 \cdots 1 = 10!$.\(^1\)

How many different samples of size $k$ from $n$ numbers **without replacement**.

$n$ ways for first choice,

\[^1\text{By definition: } 0! = 1. \ n! = n(n-1)(n-2)\cdots 1.\]
Permutations.

How many 10 digit numbers \textbf{without repeating a digit}? 
10 ways for first, 9 ways for second, 8 ways for third, ...
... $10 \times 9 \times 8 \cdots \times 1 = 10!$.\(^1\)

How many different samples of size $k$ from $n$ numbers \textbf{without replacement}.

$n$ ways for first choice, $n-1$ ways for second,
Permutations.

How many 10 digit numbers **without repeating a digit**?  
10 ways for first, 9 ways for second, 8 ways for third, ...  
... $10 \times 9 \times 8 \cdots 1 = 10!$.

How many different samples of size $k$ from $n$ numbers **without replacement**.  
$n$ ways for first choice, $n – 1$ ways for second,  
$n – 2$ choices for third,

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$^1$By definition: $0! = 1$. $n! = n(n – 1)(n – 2) \ldots 1$. 
Permutations.

How many 10 digit numbers **without repeating a digit**?
10 ways for first, 9 ways for second, 8 ways for third, ...
... $10 \times 9 \times 8 \cdots \times 1 = 10!$.\(^1\)

How many different samples of size $k$ from $n$ numbers **without replacement**.

$n$ ways for first choice, $n - 1$ ways for second, $n - 2$ choices for third, ...

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Permutations.

How many 10 digit numbers **without repeating a digit**?
10 ways for first, 9 ways for second, 8 ways for third, ...
... $10 \times 9 \times 8 \times \cdots \times 1 = 10!$.\(^1\)

How many different samples of size $k$ from $n$ numbers **without replacement**.

$n$ ways for first choice, $n-1$ ways for second,
$n-2$ choices for third, ...
... $n \times (n-1) \times (n-2) \times \cdots \times (n-k+1) = \frac{n!}{(n-k)!}$.

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\(^1\)By definition: $0! = 1$. $n! = n(n-1)(n-2)\ldots 1$. 
Permutations.

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10 ways for first, 9 ways for second, 8 ways for third, ...
... $10 \times 9 \times 8 \cdots \times 1 = 10!$.\(^1\)

How many different samples of size $k$ from $n$ numbers without replacement.

$n$ ways for first choice, $n - 1$ ways for second, $n - 2$ choices for third, ...
... $n \times (n - 1) \times (n - 2) \times \cdots \times (n - k + 1) = \frac{n!}{(n-k)!}$.

How many orderings of $n$ objects are there? Permutations of $n$ objects.

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\(^1\) By definition: $0! = 1$. $n! = n(n-1)(n-2)\ldots 1$. 
Permutations.

How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second, 8 ways for third, ...

... $10 \times 9 \times 8 \cdots \times 1 = 10!$.\(^1\)

How many different samples of size $k$ from $n$ numbers **without replacement**.

$n$ ways for first choice, $n-1$ ways for second, $n-2$ choices for third, ...

... $n \times (n-1) \times (n-2) \cdots \times (n-k+1) = \frac{n!}{(n-k)!}$.

How many orderings of $n$ objects are there? **Permutations of $n$ objects**.

$n$ ways for first,

---

\(^1\)By definition: $0! = 1$. $n! = n(n-1)(n-2)\cdots 1$. 
Permutations.

How many 10 digit numbers **without repeating a digit**?
10 ways for first, 9 ways for second, 8 ways for third, ...
... $10 \times 9 \times 8 \cdots \times 1 = 10!$.\(^1\)

How many different samples of size $k$ from $n$ numbers **without replacement**.

$n$ ways for first choice, $n - 1$ ways for second, $n - 2$ choices for third, ...
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One-to-One Functions.

How many one-to-one functions from $S$ to $S$?

$|S|$ choices for $f(s_1)$, $|S| - 1$ choices for $f(s_2)$, ...

So total number is $|S| \times (|S| - 1) \cdots 1 = |S|!$

A one-to-one function (from $S$ to $S$) is a permutation!
One-to-One Functions.

How many one-to-one functions from $S$ to $S$?
One-to-One Functions.

How many one-to-one functions from \( S \) to \( S \)?

\(|S| \) choices for \( f(s_1) \),
One-to-One Functions.

How many one-to-one functions from $S$ to $S$?

$|S|$ choices for $f(s_1)$, $|S| - 1$ choices for $f(s_2)$, ...
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Counting sets..when order doesn’t matter.

How many sets of 5 playing cards ("poker hands")?

\[ 52 \times 51 \times 50 \times 49 \times 48 \div 5! \]

\[ \binom{52}{5} \]

\(^2\text{When each unordered object corresponds equal numbers of ordered objects.}\)
Counting sets...when order doesn’t matter.

How many sets of 5 playing cards (“poker hands”)?

\[ 52 \times 51 \times 50 \times 49 \times 48 \]

\[ \binom{52}{5} = \frac{52!}{5! \times 47!} \]

\[ \text{Generic: ways to choose 5 out of 52 possibilities.} \]

\[ ^2 \text{When each unordered object corresponds equal numbers of ordered objects.} \]
Counting sets..when order doesn’t matter.

How many sets of 5 playing cards ("poker hands")?

$$52 \times 51 \times 50 \times 49 \times 48$$

2 When each unordered object corresponds equal numbers of ordered objects.
Counting sets when order doesn’t matter.

How many sets of 5 playing cards (“poker hands”)?

\[52 \times 51 \times 50 \times 49 \times 48 \ ???\]

Are A, K, Q, 10, J of spades and 10, J, Q, K, A of spades the same?

\[\frac{\text{Number of orderings for a poker hand}}{5!} \]

Can write as...

\[\frac{52!}{5! \times 47!} \]

Generic: ways to choose 5 out of 52 possibilities.

\[\text{Each unordered object corresponds equal numbers of ordered objects.} \]

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Are A, K, Q, 10, J of spades and 10, J, Q, K, A of spades the same?

**Second Rule of Counting:** If order doesn’t matter count ordered objects and then divide by number of orderings.\(^2\)

\(^2\)When each unordered object corresponds equal numbers of ordered objects.
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Number of orderings for a poker hand: 5!

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Number of orderings for a poker hand: $5!$

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

---

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When order doesn’t matter.
When order doesn’t matter.

Choose 2 out of $n$?
When order doesn’t matter.

Choose 2 out of \( n \)?

\[
n \times (n - 1)
\]
When order doesn’t matter.

Choose 2 out of \( n \)?

\[
\frac{n \times (n-1)}{2}
\]
When order doesn’t matter.

Choose 2 out of \( n \)?

\[
\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}
\]
When order doesn’t matter.

Choose 2 out of $n$?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

Choose 3 out of $n$?
When order doesn’t matter.

Choose 2 out of $n$?

\[
\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}
\]

Choose 3 out of $n$?

\[
\frac{n \times (n-1) \times (n-2)}{(n-1) \times (n-2)}
\]
When order doesn’t matter.

Choose 2 out of $n$?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

Choose 3 out of $n$?

$$\frac{n \times (n-1) \times (n-2)}{3!}$$

Notation: $n \choose k$ and pronounced "$n$ choose $k$."
When order doesn’t matter.

Choose 2 out of \( n \)?

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Notation: $\binom{n}{k}$ and pronounced “$n$ choose $k$.”
Simple Practice.

How many orderings of letters of CAT?

3 ways to choose first letter, 2 ways to choose second, 1 for last.

$3 \times 2 \times 1 = 3!$ orderings.

How many orderings of the letters in ANAGRAM?

Ordered, except for A!

Total orderings of 7 letters: $7!$

Total "extra counts" or orderings of two A's: $3!$

Total orderings: $7! / 3!$

How many orderings of letters in MISSISSIPPI?

4 S's, 4 I's, 2 P's.

11 letters total!

$11!$ ordered objects!

$11! / 4! \times 4! \times 2!$ ordered objects per "unordered object".

$= 11! / (4! \times 4! \times 2!)$. 


Simple Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways to choose second, 1 for last.
\[3 \times 2 \times 1 = 3! \text{ orderings}\]
Simple Practice.

How many orderings of letters of \textbf{CAT}?

3 ways to choose first letter, 2 ways to choose second, 1 for last.

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How many orderings of the letters in \textbf{ANAGRAM}?
Simple Practice.

How many orderings of letters of **CAT**?

3 ways to choose first letter, 2 ways to choose second, 1 for last.

$$\Rightarrow 3 \times 2 \times 1 = 3! \text{ orderings}$$

How many orderings of the letters in **ANAGRAM**?

Ordered, except for A!

- total orderings of 7 letters. 7!
- total “extra counts” or orderings of two A’s? 3!

Total orderings? \( \frac{7!}{3!} \)

How many orderings of letters in **MISSISSIPPI**?

4 S’s, 4 I’s, 2 P’s.

11 letters total!

11! ordered objects!

$$\frac{11!}{4! \times 4! \times 2!} \text{ ordered objects per “unordered object”}$$

$$\Rightarrow 11!$$
Simple Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways to choose second, 1 for last.  
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$$4! \times 4! \times 2! \text{ ordered objects per “unordered object”}$$
$$\Rightarrow 11! \div 4! \div 4! \div 2!$$
Simple Practice.

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4 S’s, 4 I’s, 2 P’s.
11 letters total!
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- \( 4! \times 4! \times 2! \) ordered objects per “unordered object”
\[ \Rightarrow \frac{11!}{4!4!2!} \]
Sampling...

Sample $k$ items out of $n$
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Sample $k$ items out of $n$

Without replacement:

Order matters:

$$n \times n \times \ldots \times n = n^k$$

Order does not matter:

Second Rule: divide by number of orders —

$$n^k \div k! = \frac{n!}{(n-k)!}$$
Sample $k$ items out of $n$

Without replacement:
Order matters:

Order does not matter:
Second Rule: divide by number of orders 

With Replacement.
Order matters:
Order does not matter:
Second rule???
Problem: depends on how many of each item we chose!
Set: 1, 2, 3
3! orderings map to it.
Set: 1, 2, 2
3! 2! orderings map to it.
How do we deal with this situation!??!
Sampling...

Sample $k$ items out of $n$

Without replacement:

Order matters: $n \times$ $n \times \cdots \times n = n^k - 1 \times n - 2 \cdots \times n - k + 1 = n! \left( \frac{n - k}{n} \right)!$

Order does not matter: Second rule divided by number of orders $- k! = n! \left( \frac{n - k}{n} \right)! k!$.

With Replacement.

Order matters:

Second rule???

Problem: depends on how many of each item we chose!

Set: 1, 2, 3

3! orderings map to it.

Set: 1, 2, 2

3! 2! orderings map to it.

How do we deal with this situation?!?!
Sampling...

Sample $k$ items out of $n$

Without replacement:
Order matters: $n \times n - 1 \times n - 2 \ldots$
Sampling...

Sample $k$ items out of $n$

Without replacement:
  Order matters: $n \times n - 1 \times n - 2 \ldots \times n - k + 1$
Sampling...

Sample $k$ items out of $n$

Without replacement:
- Order matters: $n \times n - 1 \times n - 2 \ldots \times n - k + 1 = \frac{n!}{(n-k)!}$
- Order does not matter:

Second Rule: divide by number of orders
- $k!$
Sampling...

Sample $k$ items out of $n$

Without replacement:
  Order matters: $n \times n - 1 \times n - 2 \ldots \times n - k + 1 = \frac{n!}{(n-k)!}$
  Order does not matter:
    Second Rule: divide by number of orders

With Replacement.
  Order matters: $n \times n \times n \times \ldots \times n = n^k$
  Order does not matter:
    Second rule???
Sampling...

Sample $k$ items out of $n$

Without replacement:
  Order matters: $n \times n - 1 \times n - 2 \ldots \times n - k + 1 = \frac{n!}{(n-k)!}$
  Order does not matter:
    Second Rule: divide by number of orders – “$k!$”

With Replacement.
  Order matters:
  Order does not matter:
    Second rule...
Sampling...

Sample $k$ items out of $n$

Without replacement:
Order matters: $n \times n - 1 \times n - 2 \ldots \times n - k + 1 = \frac{n!}{(n-k)!}$
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$\implies \frac{n!}{(n-k)!k!}$.
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    “$n$ choose $k$”

With Replacement.
Sampling...

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Order does not matter:
   Second Rule: divide by number of orders – “$k!$”
   $\Rightarrow \frac{n!}{(n-k)!k!} \cdot$

“$n$ choose $k$”

With Replacement.
   Order matters: $n$
Sampling...

Sample $k$ items out of $n$

Without replacement:
Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:
Second Rule: divide by number of orders – “$k!”$
$\Rightarrow \frac{n!}{(n-k)!k!} \cdot$
“$n$ choose $k”$

With Replacement.
Order matters: $n \times n$
Sampling...

Sample $k$ items out of $n$

Without replacement:
- Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$
- Order does not matter:
  - Second Rule: divide by number of orders – “$k!$”

  $\Rightarrow \frac{n!}{(n-k)!k!} \cdot$

  “$n$ choose $k$”

With Replacement.
- Order matters: $n \times n \times \ldots n$
Sampling...

Sample \( k \) items out of \( n \)

Without replacement:
Order matters: \( n \times n - 1 \times n - 2 \ldots \times n - k + 1 = \frac{n!}{(n-k)!} \)
Order does not matter:
Second Rule: divide by number of orders – “\( k! \)”
\[ \Rightarrow \frac{n!}{(n-k)!k!} \cdot \]
“\( n \) choose \( k \)”

With Replacement.
Order matters: \( n \times n \times \ldots n = n^k \)
Sampling...

Sample $k$ items out of $n$

Without replacement:
  Order matters: $n \times n - 1 \times n - 2 \ldots \times n - k + 1 = \frac{n!}{(n-k)!}$
  Order does not matter:
    Second Rule: divide by number of orders – “$k!$”
    \[
    \frac{n!}{(n-k)!k!} \cdot \text{“}n \text{ choose } k\text{”}
    \]

With Replacement.
  Order matters: $n \times n \times \ldots n = n^k$
  Order does not matter:
Sampling...

Sample $k$ items out of $n$

Without replacement:
- Order matters: $n \times (n-1) \times (n-2) \ldots \times n-k+1 = \frac{n!}{(n-k)!}$
- Order does not matter:
  - Second Rule: divide by number of orders – “$k!$”
    \[ \frac{n!}{(n-k)!k!} \]
  - “$n$ choose $k$”

With Replacement.
- Order matters: $n \times n \times \ldots n = n^k$
- Order does not matter: Second rule
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    Second Rule: divide by number of orders – “$k!$”
    $\implies \frac{n!}{(n-k)!k!}$. \\
    “$n$ choose $k$”

With Replacement.
  Order matters: $n \times n \times \ldots n = n^k$
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Sampling...

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  \[ \frac{n!}{(n-k)!k!} \]
  
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Problem: depends on how many of each item we chose!
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  \[
  \Rightarrow \frac{n!}{(n-k)!k!}.
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  “$n$ choose $k$”

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  Order does not matter:
    Second Rule: divide by number of orders – “$k!$”
    \[ \frac{n!}{(n-k)!k!} \]
    “$n$ choose $k$”

With Replacement.
  Order matters: $n \times n \times \ldots n = n^k$
  Order does not matter: Second rule ??

Problem: depends on how many of each item we chose!

Set: 1,2,3 3! orderings map to it.
Set: 1,2,2
Sampling...

Sample $k$ items out of $n$

Without replacement:
   Order matters: $n \times n - 1 \times n - 2 \ldots \times n - k + 1 = \frac{n!}{(n-k)!}$
   Order does not matter:
      Second Rule: divide by number of orders – “$k!$”
      \[ \Rightarrow \frac{n!}{(n-k)!k!} \cdot \]
      “$n$ choose $k$”

With Replacement.
   Order matters: $n \times n \times \ldots n = n^k$
   Order does not matter: Second rule ???

Problem: depends on how many of each item we chose!

Set: $1,2,3$ \hspace{1cm} $3!$ orderings map to it.
Set: $1,2,2$ \hspace{1cm} $\frac{3!}{2!}$ orderings map to it.
Sampling...

Sample \( k \) items out of \( n \)

Without replacement:
- Order matters: \( n \times (n-1) \times (n-2) \ldots \times (n-k+1) = \frac{n!}{(n-k)!} \)
- Order does not matter:
  - Second Rule: divide by number of orders – “\( k! \)”
    \[ \Rightarrow \frac{n!}{(n-k)!k!} \cdot \]
  - “\( n \) choose \( k \)”

With Replacement.
- Order matters: \( n \times n \times \ldots \times n = n^k \)
- Order does not matter: Second rule ???

Problem: depends on how many of each item we chose!

Set: \( 1, 2, 3 \) \( 3! \) orderings map to it.
Set: \( 1, 2, 2 \) \( \frac{3!}{2!} \) orderings map to it.
Sampling...

Sample \( k \) items out of \( n \)

Without replacement:
- Order matters: \( n \times n - 1 \times n - 2 \ldots \times n - k + 1 = \frac{n!}{(n-k)!} \)
- Order does not matter:
  - Second Rule: divide by number of orders — “\( k! \)”
    \[ \longrightarrow \frac{n!}{(n-k)!k!} \cdot \]
  - “\( n \) choose \( k \)”

With Replacement.
- Order matters: \( n \times n \times \ldots n = n^k \)
- Order does not matter: Second rule ???

Problem: depends on how many of each item we chose!

Set: 1,2,3 \( 3! \) orderings map to it.
Set: 1,2,2 \( \frac{3!}{2!} \) orderings map to it.

How do we deal with this situation?!?!
How many ways can Bob and Alice split 5 dollars?
How many ways can Bob and Alice split 5 dollars?
For each of 5 dollars pick Bob or Alice\( (2^5) \), see what results.
How many ways can Bob and Alice split 5 dollars? For each of 5 dollars pick Bob or Alice ($2^5$), see what results.

5 dollars for Bob and 0 for Alice:
How many ways can Bob and Alice split 5 dollars?
For each of 5 dollars pick Bob or Alice ($2^5$), see what results.

5 dollars for Bob and 0 for Alice:
one ordered set: ($B, B, B, B, B$).
New Technique: Stars and Bars....

How many ways can Bob and Alice split 5 dollars? For each of 5 dollars pick Bob or Alice ($2^5$), see what results.

5 dollars for Bob and 0 for Alice: one ordered set: $(B, B, B, B, B)$.

4 for Bob and 1 for Alice:
How many ways can Bob and Alice split 5 dollars? For each of 5 dollars pick Bob or Alice ($2^5$), see what results.

5 dollars for Bob and 0 for Alice:
one ordered set: $(B, B, B, B, B)$.

4 for Bob and 1 for Alice:
5 ordered sets: $(A, B, B, B, B)$ ; $(B, A, B, B, B)$; ...
How many ways can Bob and Alice split 5 dollars? For each of 5 dollars pick Bob or Alice \(2^5\), see what results.

5 dollars for Bob and 0 for Alice:
one ordered set: \((B, B, B, B, B)\).

4 for Bob and 1 for Alice:
5 ordered sets: \((A, B, B, B, B)\); \((B, A, B, B, B)\); ...

Well, we can list the possibilities.
\[0 + 5, 1 + 4, 2 + 3, 3 + 2, 4 + 1, 5 + 0.\]
New Technique: Stars and Bars....

How many ways can Bob and Alice split 5 dollars?
For each of 5 dollars pick Bob or Alice ($2^5$), see what results.

5 dollars for Bob and 0 for Alice:
one ordered set: ($B, B, B, B, B$).

4 for Bob and 1 for Alice:
5 ordered sets: ($A, B, B, B, B$); ($B, A, B, B, B$); ... 

Well, we can list the possibilities.
$0 + 5, 1 + 4, 2 + 3, 3 + 2, 4 + 1, 5 + 0$.

For 2 numbers adding to $k$, we get $k + 1$. 
How many ways can Bob and Alice split 5 dollars?
For each of 5 dollars pick Bob or Alice ($2^5$), see what results.

5 dollars for Bob and 0 for Alice: one ordered set: $(B, B, B, B, B)$.

4 for Bob and 1 for Alice: 5 ordered sets: $(A, B, B, B, B)$; $(B, A, B, B, B)$; ...

Well, we can list the possibilities.
$0 + 5, 1 + 4, 2 + 3, 3 + 2, 4 + 1, 5 + 0$.

For 2 numbers adding to $k$, we get $k + 1$.

For 3 numbers adding to $k$? More than 3?
Stars and Bars.

How many ways to add up $n$ natural numbers to equal $k$?
Stars and Bars.

How many ways to add up \( n \) natural numbers to equal \( k \)?

Or: \( k \) choices from set of \( n \) possibilities with replacement.

Sample with replacement where order just doesn’t matter.
Stars and Bars.

How many ways to add up $n$ natural numbers to equal $k$?

Or: $k$ choices from set of $n$ possibilities with replacement. 

Sample with replacement where order just doesn’t matter.

How many ways can Alice, Bob, and Eve split 5 dollars.
Stars and Bars.

How many ways to add up $n$ natural numbers to equal $k$?

Or: $k$ choices from set of $n$ possibilities with replacement.

Sample with replacement where order just doesn’t matter.

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: ⭐⭐⭐⭐⭐.
Stars and Bars.

How many ways to add up $n$ natural numbers to equal $k$?

Or: $k$ choices from set of $n$ possibilities with replacement.

Sample with replacement where order just doesn’t matter.

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: ⭐⭐⭐⭐⭐.

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars.

How many ways to add up \( n \) natural numbers to equal \( k \)?

Or: \( k \) choices from set of \( n \) possibilities with replacement. Sample with replacement where order just doesn’t matter.

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: ⭐⭐⭐⭐⭐.

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars: ⭐⭐|
Stars and Bars.

How many ways to add up $n$ natural numbers to equal $k$?

Or: $k$ choices from set of $n$ possibilities with replacement.

Sample with replacement where order just doesn’t matter.

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: ⭐⭐⭐⭐⭐.

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars: ⭐⭐|⭐|
Stars and Bars.

How many ways to add up $n$ natural numbers to equal $k$?

Or: $k$ choices from set of $n$ possibilities with replacement. **Sample with replacement where order just doesn’t matter.**

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: ⋆ ⋆ ⋆ ⋆ ⋆.

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars: ⋆ ⋆ | ⋆ | ⋆ ⋆.
Stars and Bars.

How many ways to add up \( n \) natural numbers to equal \( k \)?

Or: \( k \) choices from set of \( n \) possibilities with replacement.

**Sample with replacement where order just doesn’t matter.**

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: ⋆ ⋆ ⋆ ⋆ ⋆.

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars: ⋆ ⋆ | ⋆ ⋆ ⋆.

Alice: 0, Bob: 1, Eve: 4.
How many ways to add up $n$ natural numbers to equal $k$?

Or: $k$ choices from set of $n$ possibilities with replacement.

Sample with replacement where order just doesn’t matter.

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: ⭐⭐⭐⭐⭐.

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars: ⭐⭐|⭐|⭐⭐.

Alice: 0, Bob: 1, Eve: 4.
Stars and Bars: |
Stars and Bars.

How many ways to add up $n$ natural numbers to equal $k$?

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**Sample with replacement where order just doesn’t matter.**

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: ⭐⭐⭐⭐⭐.

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars: ⭐⭐|⭐|⭐⭐.

Alice: 0, Bob: 1, Eve: 4.
Stars and Bars: |⭐|
Stars and Bars.

How many ways to add up \( n \) natural numbers to equal \( k \)?

Or: \( k \) choices from set of \( n \) possibilities with replacement.

Sample with replacement where order just doesn’t matter.

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: \(*\cdots\cdots\cdots\cdots\cdots\cdots\\) .

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars: \(*\,|\,*\,|\,**\) .

Alice: 0, Bob: 1, Eve: 4.
Stars and Bars: \(|\,*\|\,**\cdots\cdots\cdots\cdots\cdots\cdots\) .
Stars and Bars.

How many ways to add up $n$ natural numbers to equal $k$?

Or: $k$ choices from set of $n$ possibilities with replacement.

**Sample with replacement where order just doesn’t matter.**

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: ⭐⭐⭐⭐⭐.

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars: ⭐⭐|⭐|⭐⭐.

Alice: 0, Bob: 1, Eve: 4.
Stars and Bars: |⭐|⭐⭐⭐⭐.

Each split $\implies$ a sequence of stars and bars.
Stars and Bars.

How many ways to add up $n$ natural numbers to equal $k$?
Or: $k$ choices from set of $n$ possibilities with replacement.
**Sample with replacement where order just doesn’t matter.**

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: ★★★★★.

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars: ★★ | ★ | ★★.

Alice: 0, Bob: 1, Eve: 4.
Stars and Bars: | ★ | ★★★★.

Each split $\implies$ a sequence of stars and bars.
Each sequence of stars and bars $\implies$ a split.
Stars and Bars.

How many ways to add up $n$ natural numbers to equal $k$?

Or: $k$ choices from set of $n$ possibilities with replacement.

**Sample with replacement where order just doesn’t matter.**

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: ⭐⭐⭐⭐⭐.

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars: ⭐⭐|⭐|⭐⭐.

Alice: 0, Bob: 1, Eve: 4.
Stars and Bars: |⭐|⭐⭐⭐⭐.

Each split $\implies$ a sequence of stars and bars.
Each sequence of stars and bars $\implies$ a split.
Stars and Bars.

How many ways to add up $n$ natural numbers to equal $k$?

Or: $k$ choices from set of $n$ possibilities with replacement.

**Sample with replacement where order just doesn’t matter.**

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: ★★★★★.

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars: ★★|★|★★.

Alice: 0, Bob: 1, Eve: 4.
Stars and Bars: |★|★★★★.

Each split $\implies$ a sequence of stars and bars.
Each sequence of stars and bars $\implies$ a split.

**Counting Rule:** if there is a one-to-one mapping between two sets they have the same size!
Stars and Bars.

How many different 5 star and 2 bar diagrams?
Stars and Bars.

How many different 5 star and 2 bar diagrams?
7 positions in which to place the 2 bars.
Stars and Bars.

How many different 5 star and 2 bar diagrams?
7 positions in which to place the 2 bars.
\( \binom{7}{2} \) ways to do so and
Stars and Bars.

How many different 5 star and 2 bar diagrams?
7 positions in which to place the 2 bars.
\(\binom{7}{2}\) ways to do so and \(\binom{7}{2}\) ways to split 5$ among 3 people.
Stars and Bars.

How many different 5 star and 2 bar diagrams?
7 positions in which to place the 2 bars.
\[ \binom{7}{2} \] ways to do so and \[ \binom{7}{2} \] ways to split 5$ among 3 people.
Ways to add up \( n \) natural numbers to sum to \( k \)?
How many different 5 star and 2 bar diagrams?
7 positions in which to place the 2 bars.
\( \binom{7}{2} \) ways to do so and \( \binom{7}{2} \) ways to split 5$ among 3 people.
Ways to add up \( n \) natural numbers to sum to \( k \) or

“ \( k \) from \( n \) with replacement where order doesn’t matter.”
How many different 5 star and 2 bar diagrams?
7 positions in which to place the 2 bars.
(\(\binom{7}{2}\)) ways to do so and (\(\binom{7}{2}\)) ways to split 5$ among 3 people.
Ways to add up \(n\) natural numbers to sum to \(k\)? or

“\(k\) from \(n\) with replacement where order doesn’t matter.”

In general, \(k\) stars \(n-1\) bars.

\[\begin{align*}
\star \star \mid \star \mid \cdots \mid \star \star.
\end{align*}\]
How many different 5 star and 2 bar diagrams?  
7 positions in which to place the 2 bars.  
\binom{7}{2} ways to do so and \binom{7}{2} ways to split 5$ among 3 people. 
Ways to add up \( n \) natural numbers to sum to \( k \) or

“\( k \) from \( n \) with replacement where order doesn’t matter.”

In general, \( k \) stars \( n - 1 \) bars.

\[
\begin{align*}
&\text{★★} \mid \text{★} \mid \cdots \mid \text{★★}.
\end{align*}
\]

\( n + k - 1 \) positions from which to choose \( n - 1 \) bar positions.
Stars and Bars.

How many different 5 star and 2 bar diagrams?
7 positions in which to place the 2 bars.
\( \binom{7}{2} \) ways to do so and \( \binom{7}{2} \) ways to split 5$ among 3 people.
Ways to add up \( n \) natural numbers to sum to \( k \)? or

“ \( k \) from \( n \) with replacement where order doesn’t matter.”

In general, \( k \) stars \( n - 1 \) bars.

\[
\begin{array}{cccccc}
  \ast \ast \mid \ast \mid \cdots \mid \ast \ast \\
\end{array}
\]

\( n + k - 1 \) positions from which to choose \( n - 1 \) bar positions.

\[
\binom{n+k-1}{n-1}
\]
Mark what’s correct:
(A) ways to split 5 dollars among 3: \( \binom{7}{2} \)
(B) ways to split \( n \) dollars among \( k \): \( \binom{n+k-1}{k-1} \)
(C) ways to split 3 dollars among 5: \( \binom{7}{5} \)
(D) ways to split 5 dollars among 3: \( \binom{7}{5} \)
Mark what’s correct:

(A) ways to split 5 dollars among 3: \( \binom{7}{2} \)

(B) ways to split \( n \) dollars among \( k \): \( \binom{n+k-1}{k-1} \)

(C) ways to split 3 dollars among 5: \( \binom{7}{5} \)

(D) ways to split 5 dollars among 3: \( \binom{7}{5} \)

(A), (B), (D) are correct.
A technique to prove identities by counting arguments!
Combinatorial Proofs - 1

A technique to prove identities by counting arguments!

**Theorem:** \( \binom{n}{k} = \binom{n}{n-k} \)
A technique to prove identities by counting arguments!

**Theorem:** \( \binom{n}{k} = \binom{n}{n-k} \)

**Proof:** How many subsets of size \( k \)?
A technique to prove identities by counting arguments!

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A technique to prove identities by counting arguments!

**Theorem:** \( \binom{n}{k} = \binom{n}{n-k} \)

**Proof:** How many subsets of size \( k \)? \( \binom{n}{k} \)

How many subsets of size \( k \)?

Choose a subset of size \( n-k \).
A technique to prove identities by counting arguments!

**Theorem:** \( \binom{n}{k} = \binom{n}{n-k} \)

**Proof:** How many subsets of size \( k \)？ \( \binom{n}{k} \)

How many subsets of size \( k \)？
Choose a subset of size \( n-k \)
and what’s left out
A technique to prove identities by counting arguments!

**Theorem:** \( \binom{n}{k} = \binom{n}{n-k} \)

**Proof:** How many subsets of size \( k \)? \( \binom{n}{k} \)

How many subsets of size \( k \)?
Choose a subset of size \( n-k \)
and what’s left out is a subset of size \( k \).
A technique to prove identities by counting arguments!

**Theorem:** \( \binom{n}{k} = \binom{n}{n-k} \)

**Proof:** How many subsets of size \( k \)? \( \binom{n}{k} \)

How many subsets of size \( k \)?
Choose a subset of size \( n - k \)
and what’s left out is a subset of size \( k \).
Choosing a subset of size \( k \) is same
A technique to prove identities by counting arguments!

**Theorem:** \( \binom{n}{k} = \binom{n}{n-k} \)

**Proof:** How many subsets of size \( k \)? \( \binom{n}{k} \)

How many subsets of size \( k \)?
- Choose a subset of size \( n - k \)
  - and what’s left out is a subset of size \( k \).
- Choosing a subset of size \( k \) is same as choosing \( n - k \) elements to not take.
A technique to prove identities by counting arguments!

**Theorem:** \( \binom{n}{k} = \binom{n}{n-k} \)

**Proof:** How many subsets of size \( k \)? \( \binom{n}{k} \)

How many subsets of size \( k \)?
Choose a subset of size \( n-k \)
   and what's left out is a subset of size \( k \).
Choosing a subset of size \( k \) is same
   as choosing \( n-k \) elements to not take.
\( \implies \binom{n}{n-k} \) subsets of size \( k \).
A technique to prove identities by counting arguments!

**Theorem:** \( \binom{n}{k} = \binom{n}{n-k} \)

**Proof:** How many subsets of size \( k \)？ \( \binom{n}{k} \)

How many subsets of size \( k \)？
Choose a subset of size \( n-k \)
and what’s left out is a subset of size \( k \).
Choosing a subset of size \( k \) is same
as choosing \( n-k \) elements to not take.
\( \Longrightarrow \binom{n}{n-k} \) subsets of size \( k \).
Theorem: \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

Proof: How many size \( k \) subsets of \( n+1 \)?
Theorem: \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

Proof: How many size \( k \) subsets of \( n+1 \)? \( \binom{n+1}{k} \).

Without and with first element \( \rightarrow \) disjoint.

So,

\[ \binom{n}{k} - 1 + \binom{n}{k} = \binom{n+1}{k}. \]
Theorem: \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

Proof: How many size \( k \) subsets of \( n+1 \)? \( \binom{n+1}{k} \).

How many size \( k \) subsets of \( n+1 \)?
Combinatorial Proofs - 2

**Theorem:** \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

**Proof:** How many size \( k \) subsets of \( n+1 \)? \( \binom{n+1}{k} \).

How many size \( k \) subsets of \( n+1 \)?
How many contain the first element?
Theorem: \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

Proof: How many size \( k \) subsets of \( n+1 \)? \( \binom{n+1}{k} \).

How many size \( k \) subsets of \( n+1 \)?
How many contain the first element?
Chose first element,
Combinatorial Proofs - 2

**Theorem:** \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

**Proof:** How many size \( k \) subsets of \( n+1 \)? \( \binom{n+1}{k} \).

How many size \( k \) subsets of \( n+1 \)?
How many contain the first element?
Chose first element, need \( k-1 \) more from remaining \( n \) elements.
Theorem: \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

Proof: How many size \( k \) subsets of \( n+1 \)? \( \binom{n+1}{k} \).

How many size \( k \) subsets of \( n+1 \)?

How many contain the first element?

Chose first element, need \( k - 1 \) more from remaining \( n \) elements.

\[ \Rightarrow \binom{n}{k-1} \]
**Theorem:** \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

**Proof:** How many size \( k \) subsets of \( n+1 \)? \( \binom{n+1}{k} \).

How many size \( k \) subsets of \( n+1 \)?

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Combinatorial Proofs - 2

**Theorem:** \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

**Proof:** How many size \( k \) subsets of \( n+1 \)? \( \binom{n+1}{k} \).

How many size \( k \) subsets of \( n+1 \)?
How many contain the first element?
  Chose first element, need \( k - 1 \) more from remaining \( n \) elements.
  \[ \implies \binom{n}{k-1} \]

How many don’t contain the first element?
Theorem: \((\binom{n+1}{k}) = \binom{n}{k} + \binom{n}{k-1}\).

Proof: How many size \(k\) subsets of \(n+1\)? \((\binom{n+1}{k})\).

How many size \(k\) subsets of \(n+1\)?
How many contain the first element?
  Chose first element, need \(k-1\) more from remaining \(n\) elements.
  \(\Longrightarrow \binom{n}{k-1}\)

How many don’t contain the first element?
Need to choose \(k\) elements from remaining \(n\) elts.
Theorem: $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

Proof: How many size $k$ subsets of $n+1$? $\binom{n+1}{k}$.

How many size $k$ subsets of $n+1$?
How many contain the first element?
Chose first element, need $k-1$ more from remaining $n$ elements.
\[\implies \binom{n}{k-1}\]

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Need to choose $k$ elements from remaining $n$ elts.
\[\implies \binom{n}{k}\]
Theorem: \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

Proof: How many size \( k \) subsets of \( n+1 \)? \( \binom{n+1}{k} \).

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   Chose first element, need \( k-1 \) more from remaining \( n \) elements.
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Sum Rule: size of union of disjoint sets of objects.
**Theorem:** \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

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How many don’t contain the first element?

Need to choose \( k \) elements from remaining \( n \) elts.

\[ \implies \binom{n}{k} \]

**Sum Rule:** size of union of disjoint sets of objects.

Without and with first element \( \rightarrow \) disjoint.
Theorem: \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

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  \[ \implies \binom{n}{k} \]

Sum Rule: size of union of disjoint sets of objects.
  Without and with first element \( \rightarrow \) disjoint.

So, \( \binom{n}{k-1} + \binom{n}{k} \)
Theorem: \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

Proof: How many size \( k \) subsets of \( n + 1 \)? \( \binom{n+1}{k} \).

How many size \( k \) subsets of \( n + 1 \)?
How many contain the first element?
  Chose first element, need \( k - 1 \) more from remaining \( n \) elements.
  \[ \Rightarrow \binom{n}{k-1} \]

How many don’t contain the first element?
  Need to choose \( k \) elements from remaining \( n \) elts.
  \[ \Rightarrow \binom{n}{k} \]

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So, \( \binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k} \). \( \square \)
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**Theorem:** $2^n = \binom{n}{n} + \binom{n}{n-1} + \cdots + \binom{n}{0}$
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- element $i$ **is in** or **is not** in the subset: 2 poss.
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Sum over $i$ to get total number of subsets.
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Simple Inclusion/Exclusion

**Sum Rule:** For disjoint sets $S$ and $T$, $|S \cup T| = |S| + |T|$

Used to reason about all subsets
by adding number of subsets of size 1, 2, 3, \ldots
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![Venn Diagram]
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In $T. \implies |T|$

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General version of the above rule (for $n$ sets) in the notes.
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$S =$ numbers with 7 as first digit. $|S| = 10^9$
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$S \cap T = $ numbers with 7 as first and second digit. $|S \cap T| = 10^8$.

Answer: $|S| + |T| - |S \cap T| = 10^9 + 10^9 - 10^8$. 
Summary.

First rule: \( n_1 \times n_2 \cdots \times n_3 \).
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$k$ Samples with replacement from $n$ items: $n^k$. 
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Sample without replacement: \( \frac{n!}{(n-k)!} \).
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**First rule:** $n_1 \times n_2 \cdots \times n_3$.

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**Second rule:** when order doesn’t matter divide (when possible)
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Second rule: when order doesn’t matter divide (when possible)
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“\( n \) choose \( k \)”
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- Stable Matching
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- Polynomials
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- Computability
- Counting...

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Good Studying and Good Luck!!!