Today

Random Variables.

Quick Review: Probability. Some Rules.

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  - Inclusion/Exclusion: $Pr[A \cup B] = Pr[A] + Pr[B] - Pr[A \cap B]$.
  - Simple Total Probability: $Pr[B] = Pr[A \cap B] + Pr[\overline{A} \cap B]$.
  - Complement: $Pr[\overline{A}] = 1 - Pr[A]$.
  - Union Bound. Total Probability.
- **Conditional Probability:** $Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]}$
- Bayes' Rule: $Pr[A_m|B] = p_m q_m / (p_1 q_1 + \cdots + p_M q_M)$.

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1. Random Variables.
2. Expectation
3. Distributions.

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The number is a (known) function of the outcome.

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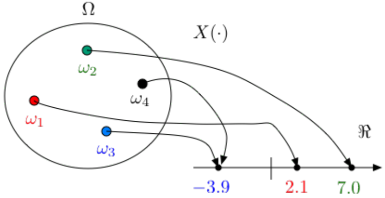

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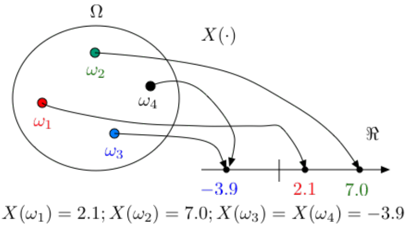

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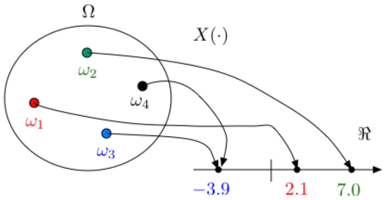

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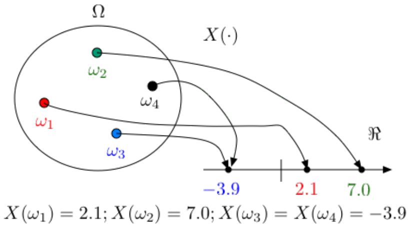

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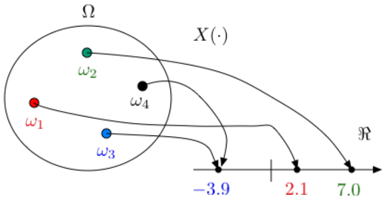

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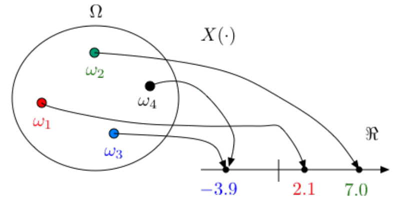

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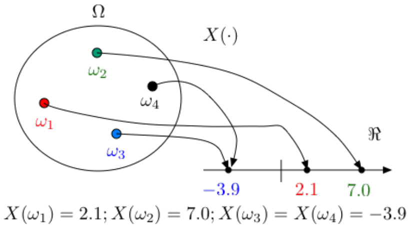

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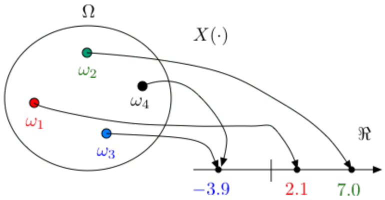

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Note:Random variable induces partition:
$A_y = \{\omega \in \Omega : X(\omega) = y\} = X^{-1}(y)$

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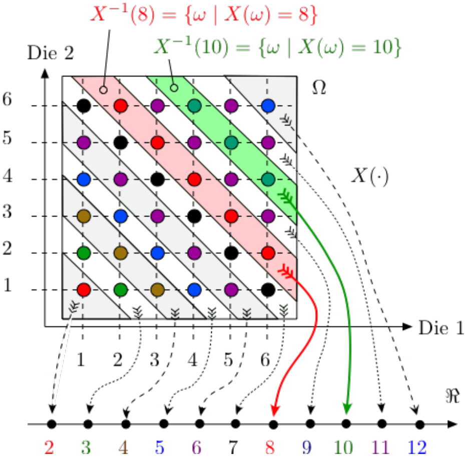

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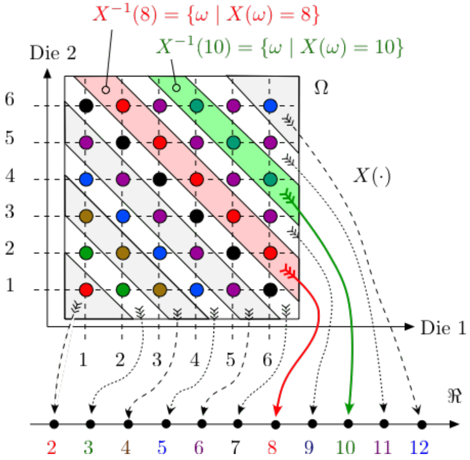

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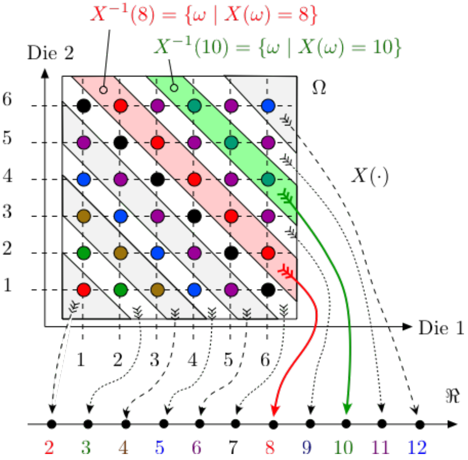

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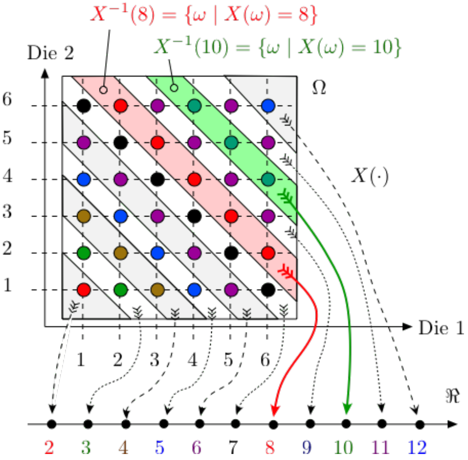

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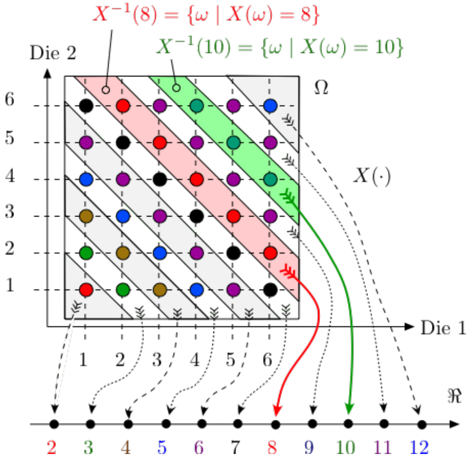

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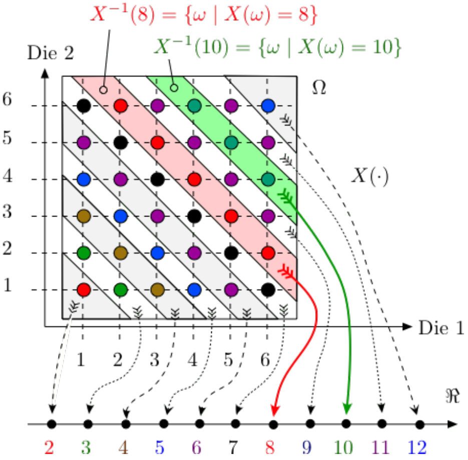

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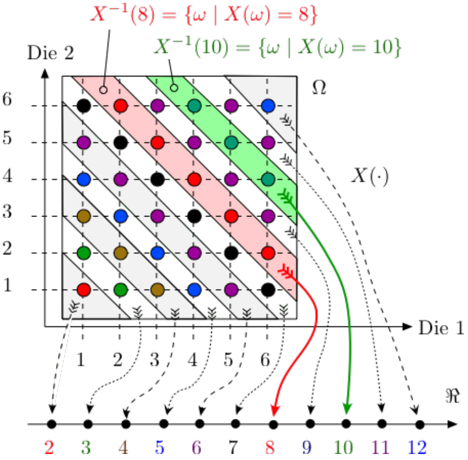

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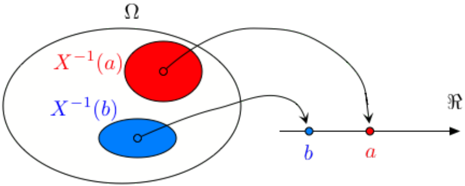

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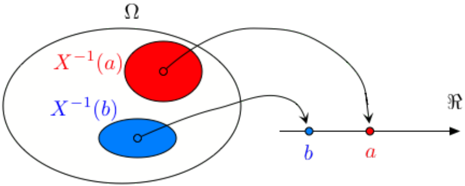
Definition: The **distribution** of a random variable X, is $\{(a, Pr[X = a]) : a \in \mathscr{A}\}$, where $\mathscr{A}$ is the range of X.



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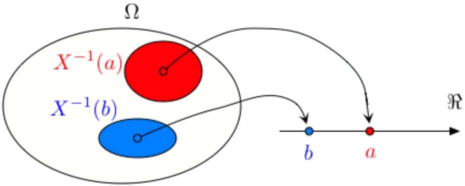

$$Pr[X = a] := Pr[X^{-1}(a)] \text{ where } X^{-1}(a) := \{\omega \mid X(\omega) = a\}.$$

Handing back assignments

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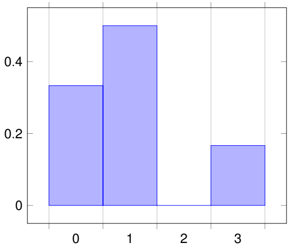

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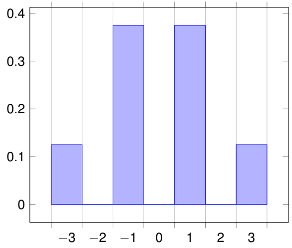

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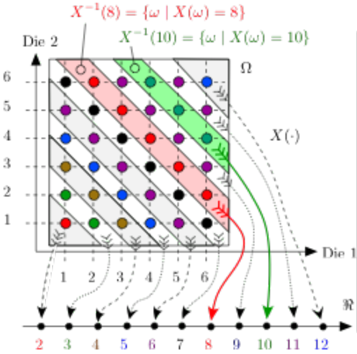

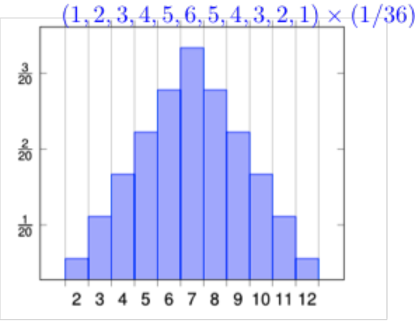

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The subjectivist(bayesian) interpretation of $E[X]$ is less obvious.

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Distributive property of multiplication over addition.

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This holds for a uniform probability space.

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Let’s cover some.

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Flip n coins with heads probability p.

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What is the probability of ω if ω has i heads?
Probability of heads in any position is p.

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Flip n coins with heads probability p.

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Probability of "$X = i$" is sum of $Pr[\omega]$, $\omega \in$ "$X = i$".

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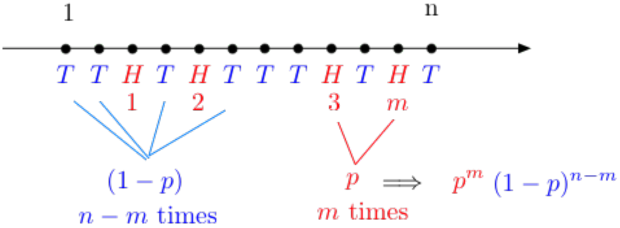

$\binom{n}{m}$ outcomes with m Hs and $n - m$ Ts

$$\Longrightarrow \boxed{Pr[X = m] = \binom{n}{m} p^m (1-p)^{n-m}}$$

Error channel and...

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Also distribution in polling, experiments, etc.

Expectation of Binomial Distibution

Parameter p and n. What is expectation?

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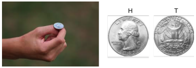

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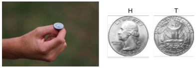

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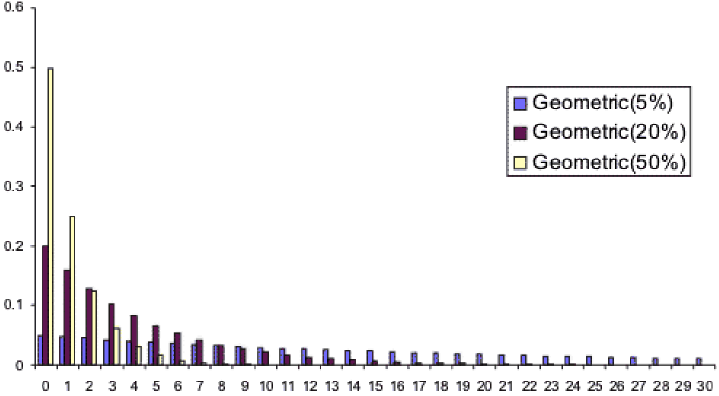

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Example: $Pr[2\lambda \text{ arrivals }]$?

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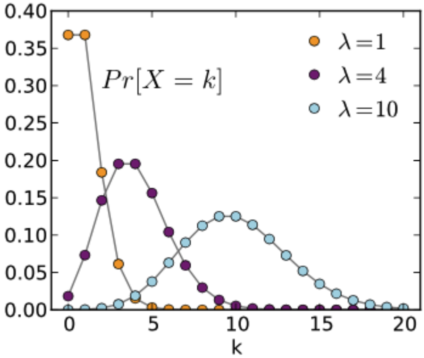

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