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Integration by Parts: $\int udv = uv - \int vdu$.

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5. Target: $f_X(x) = 2x \cdot 1\{0 \leq x \leq 1\}; F_X(x) = x^2$ for $0 \leq x \leq 1$.
6. Joint pdf: $Pr[X \in (x, x+\delta), Y = (y, y+\delta)) = f_{X,Y}(x,y)\delta^2$.
   6.1 Conditional Distribution: $f_{X|Y}(x,y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$.
   6.2 Independence: $f_{X|Y}(x,y) = f_X(x)$

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What is true?
X has CDF $F(x)$ and PDF $f(x)$.

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(A), (B), (D) think events, (E) think event and density.

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(C) confuses probability density of outcome with value of outcome.

Discrete/Continuous

Discrete: Probability of outcome $\rightarrow$ random variables, events.

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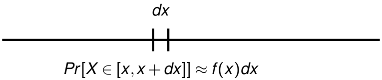

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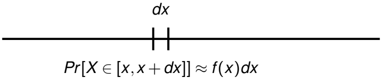

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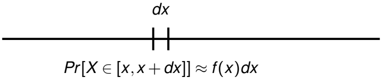

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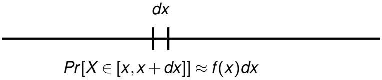

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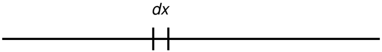

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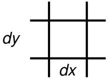

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Sample Space:Ω, $Pr[\omega]$.

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Random variables: $X(\omega)$.
Distribution: $Pr[X = x]$
$\sum_x Pr[X = x] = 1$.

Random Variable: X
Event: $A = [a, b]$, $Pr[X \in A]$,
CDF: $F(x) = Pr[X \leq x]$.
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Continuous as Discrete.
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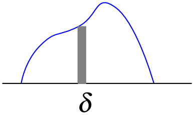

Probability Rules are all good.

Conditional Probability.

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Events: A, B

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Discrete: “Heads”, “Tails”, $X = 1$, $Y = 5$.

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Conditional Probability.

Events: A, B

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$Pr[$“Second Heads”$] = Pr[HH] + Pr[HT]$
B is First coin heads.

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Conditional Probability.

Events: A, B

Discrete: "Heads", "Tails", $X = 1,\ Y = 5$.

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$Pr[X \in [.45, .55]] = Pr[X \in [.45, .50]] + Pr[X \in (.50, .55]]$

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Events: A, B

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B is $X \in [0, .5]$

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Events: A, B

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Total Probability Rule: $Pr[A] = Pr[A \cap B] + Pr[A \cap \overline{B}]$
$Pr[\text{“Second Heads”}] = Pr[HH] + Pr[HT]$
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Probability Rules are all good.

Conditional Probability.

Events: A, B

Discrete: "Heads", "Tails", $X = 1,\ Y = 5$.

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Corollary: For independent random variables, $f_{X|Y}(x,y) = f_X(x)$.

Expo(λ)

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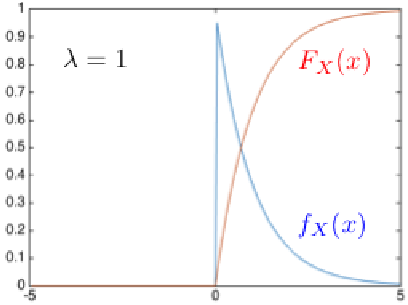

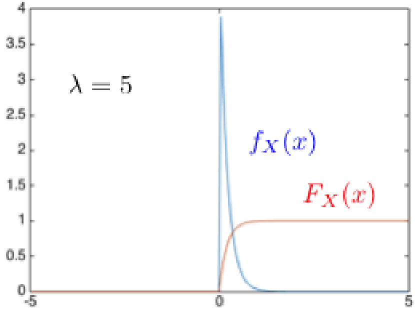

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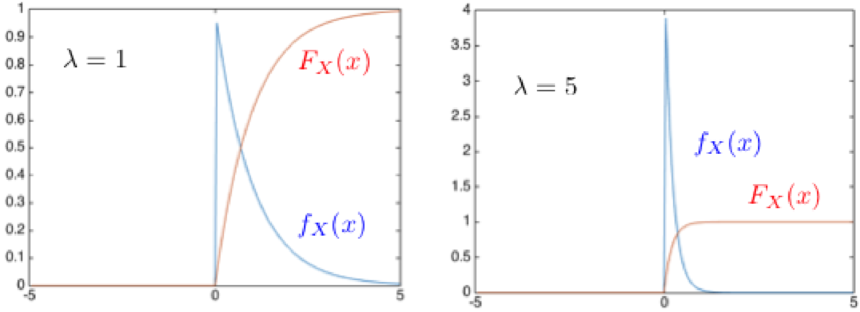

Note that $Pr[X > t] = e^{-\lambda t}$ for $t > 0$.

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Replace b by $b-a$, use $X = U[0,1]$, then $Y = a + (b-a)X$ is $U[a,b]$.

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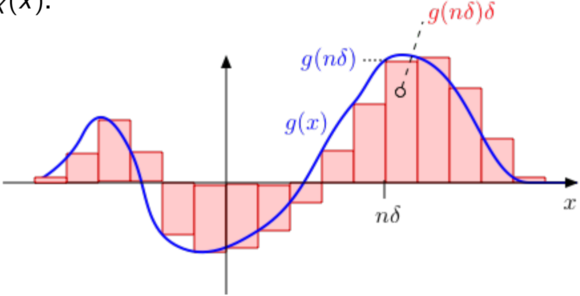

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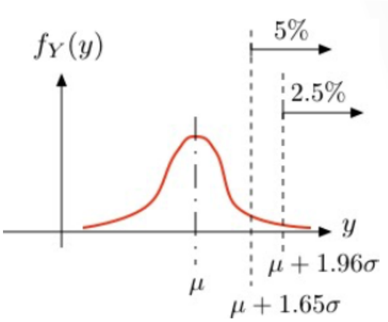

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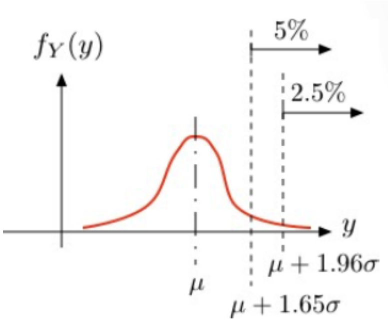

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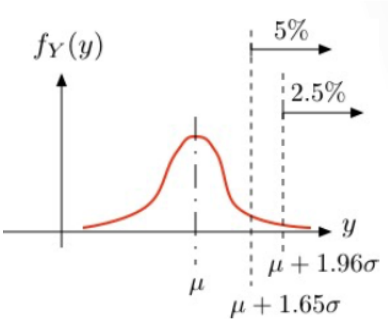

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Let $X_1, X_2, \ldots$ be i.i.d. with $E[X_1] = \mu$ and $var(X_1) = \sigma^2$. Define

$$S_n := \frac{A_n - \mu}{\sigma/\sqrt{n}} = \frac{X_1 + \cdots + X_n - n\mu}{\sigma\sqrt{n}}.$$

Then,

$$S_n \to \mathcal{N}(0,1), \text{as } n \to \infty.$$

That is,

$$Pr[S_n \leq \alpha] \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha} e^{-x^2/2} dx.$$

Proof: See EE126.

Note:

$$E(S_n) = \frac{1}{\sigma/\sqrt{n}}(E(A_n) - \mu) = 0$$

$$Var(S_n) = \frac{1}{\sigma^2/n} Var(A_n) = 1.$$

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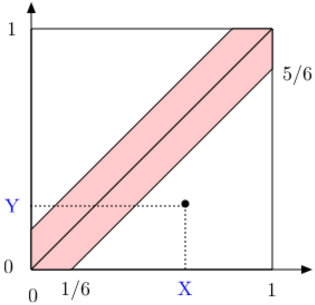

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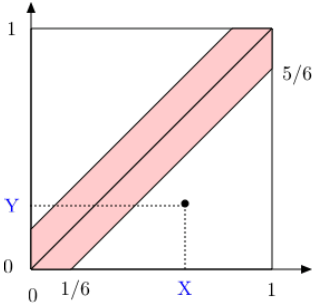

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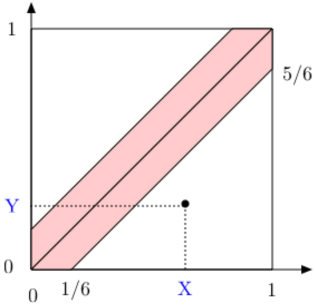

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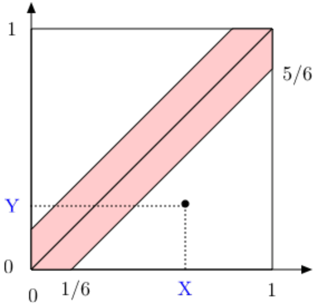

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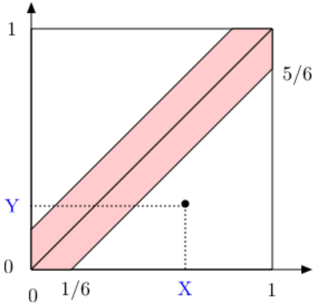

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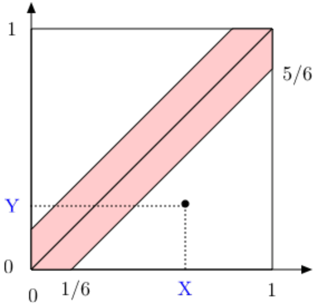

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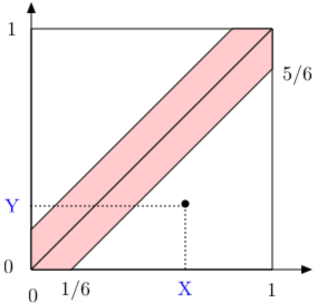

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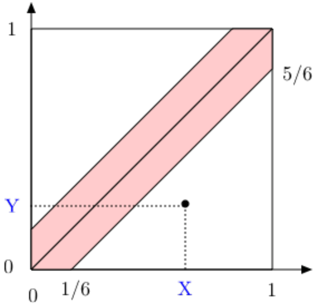

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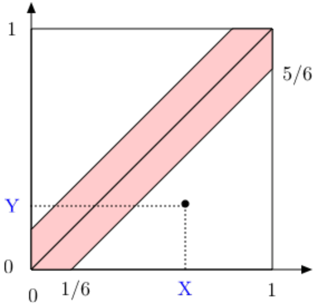

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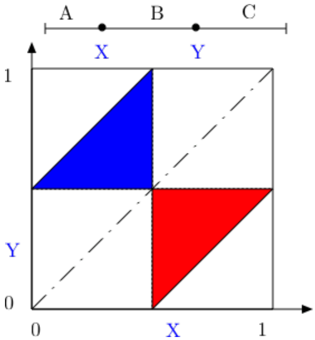

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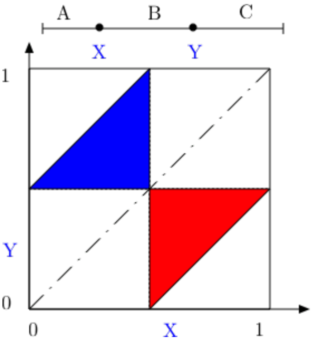

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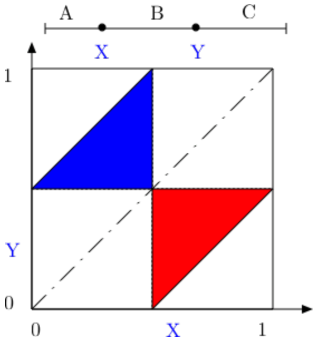

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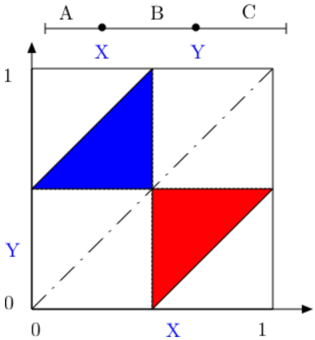

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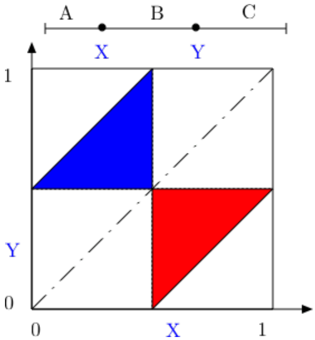

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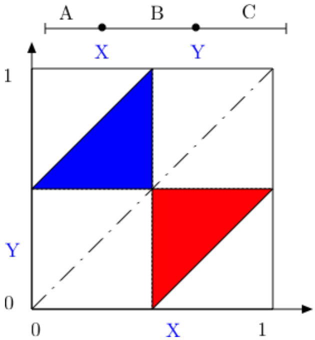

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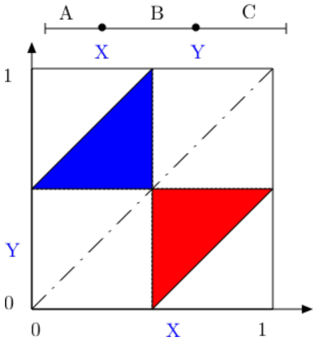

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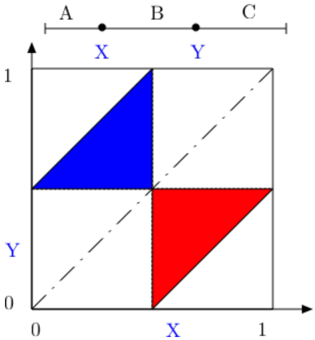

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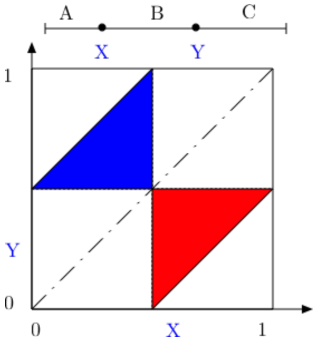

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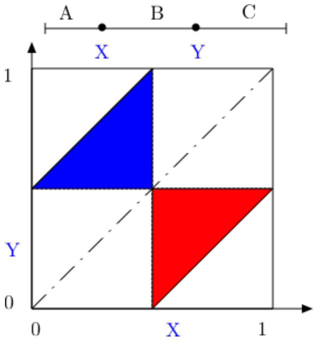

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Thus, $Pr[\text{make triangle}] = 1/4$.

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Since, $\int_0^\infty x\lambda e^{-\lambda x} dx = \lambda[-\frac{xe^{-\lambda x}}{\lambda} - \frac{e^{-\lambda x}}{\lambda^2}]_0^\infty = \frac{1}{\lambda}$.

$$E[Z] = \int_0^\infty z f_Z(z) dz =$$

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$$f_Z(z) = \lambda e^{-\lambda z} + \mu e^{-\mu z} - (\lambda + \mu)e^{-(\lambda+\mu)z}, \forall z > 0.$$

Since, $\int_0^\infty x\lambda e^{-\lambda x}dx = \lambda[-\frac{xe^{-\lambda x}}{\lambda} - \frac{e^{-\lambda x}}{\lambda^2}]_0^\infty = \frac{1}{\lambda}$.

$$E[Z] = \int_0^\infty z f_Z(z)dz = \frac{1}{\lambda} + \frac{1}{\mu} - \frac{1}{\lambda + \mu}.$$

Maximum of Two Exponentials

Let $X = Expo(\lambda)$ and $Y = Expo(\mu)$ be independent.
Define $Z = \max\{X, Y\}$.

Calculate $E[Z]$.

We compute f_Z, then integrate.

One has

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(A) Z is exponential.
(B) Parameter is n.
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(C) is an argument for (A), (B) and (D).

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Hence,

$$E[Z] = A_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n} = H(n).$$

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For instance, if $n = 16$, then $SNR(dB) \approx 112dB$.

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Continuous Probability

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- Sums become integrals, ...
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