Survey: How Helpful, How are You (doing)

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Counts: how helpful?

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	How?	Lecture	Homework	Course notes	Office hours	Piazza	Discussions	Slides
1	2.0	34.0	11.0	nan	5.0	12.0	nan	10.0
2	19.0	30.0	24.0	12.0	7.0	26.0	1.0	13.0
3	40.0	31.0	34.0	24.0	23.0	29.0	14.0	19.0
4	37.0	14.0	28.0	37.0	21.0	26.0	37.0	4.0
5	11.0	6.0	18.0	51.0	23.0	20.0	71.0	2.0

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Correlation Coefficients (sample)

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MMSE (sample based)

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1	1.5	2.2	1.6	nan	2.0	2.0	nan	1.9
2	2.0	2.2	2.1	1.7	3.0	2.6	1.0	2.8
3	2.4	2.8	2.7	2.1	2.3	2.4	2.2	3.2
4	3.1	3.0	3.0	2.8	2.3	2.6	2.5	2.5
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4	3.1	3.0	3.0	2.8	2.3	2.6	2.5	2.5
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CS70: Lecture 27

1. Review: Continuous Probability
2. Bayes' Rule with Continuous RVs
3. Normal Distribution
4. Central Limit Theorem
5. Confidence Intervals
6. Wrapup.

Continuous Probability

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5. Variance: $var[X] = E[(X - E[X])^2] = E[X^2] - E[X]^2$.

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5. Variance: $var[X] = E[(X - E[X])^2] = E[X^2] - E[X]^2$.
6. Variance of Sum of Independent RVs: If X_n are pairwise independent, $var[X_1 + \cdots + X_n] = var[X_1] + \cdots + var[X_n]$

Continuous RV and Bayes' Rule

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Example 1:

Continuous RV and Bayes' Rule

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W.p. $1/2$, X, Y are i.i.d. $\mathit{Expo}(1)$ and w.p. $1/2$, they are i.i.d. $\mathit{Expo}(3)$.

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Calculate $E[Y|X = x]$.

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Let B be the event that $X \in [x, x + \delta]$ where $0 < \delta \ll 1$.

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Then,

$$Pr[A|B] = \frac{(1/2)Pr[B|A]}{(1/2)Pr[B|A] + (1/2)Pr[B|\bar{A}]}$$

Continuous RV and Bayes' Rule

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&= \frac{\exp\{-x\}}{\exp\{-x\} + 3\exp\{-3x\}} = \frac{e^{2x}}{3+e^{2x}}.
\end{aligned}
$$

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Now,

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Now,

$$\begin{aligned} E[Y|X=x] &= E[Y|A]Pr[A|X=x] + E[Y|\bar{A}]Pr[\bar{A}|X=x] \\ &= 1 \times Pr[A|X=x] + (1/3)Pr[\bar{A}|X=x] \end{aligned}$$

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We used $Pr[Z \in [x, x+\delta]] \approx f_Z(x)\delta$ and given A one has $f_X(x) = \exp\{-x\}$ whereas given $\bar{A}$ one has $f_X(x) = 3\exp\{-3x\}$.

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W.p. $1/2$, Bob is a good dart player and shoots uniformly in a circle with radius 1. Otherwise, Bob is a very good dart player and shoots uniformly in a circle with radius $1/2$.

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W.p. $1/2$, Bob is a good dart player and shoots uniformly in a circle with radius 1. Otherwise, Bob is a very good dart player and shoots uniformly in a circle with radius $1/2$.

The first dart of Bob is at distance 0.3 from the center of the target.

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(a) We use Bayes' Rule:

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Note: If uniform in radius r, then $Pr[X \leq x] = (\pi x^2)/(\pi r^2)$, so that $f_X(x) = 2x/(r^2)$.
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$$\begin{aligned} Pr[VG|0.3] &= \frac{Pr[VG]Pr[\approx 0.3|VG]}{Pr[VG]Pr[\approx 0.3|VG] + Pr[G]Pr[\approx 0.3|G]} \\ &= \frac{0.5 \times 2(0.3)\varepsilon/(0.5^2)}{0.5 \times 2(0.3)\varepsilon/(0.5^2) + 0.5 \times 2\varepsilon(0.3)} = 0.8. \end{aligned}$$

(b) $E[X] = 0.8 \times 0.5 \times \frac{2}{3} + 0.2 \times \frac{2}{3}$

Continuous RV and Bayes' Rule

Example 2:

W.p. $1/2$, Bob is a good dart player and shoots uniformly in a circle with radius 1. Otherwise, Bob is a very good dart player and shoots uniformly in a circle with radius $1/2$.

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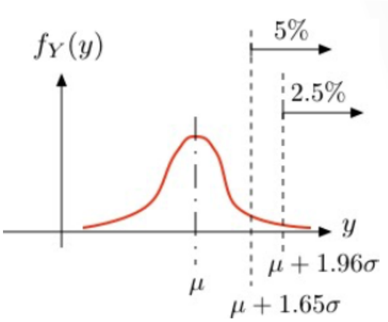

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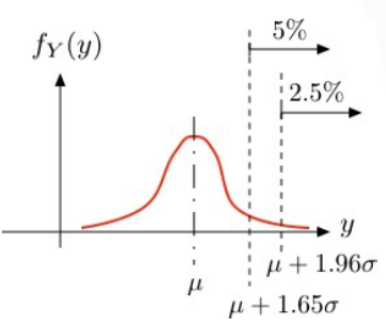

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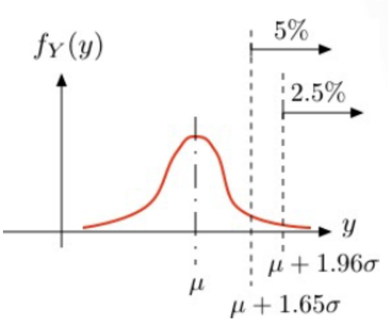

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Let $X_1, X_2, \ldots$ be i.i.d. with mean μ and variance σ^2. Let

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Thus, the CLT provides a smaller confidence interval.

Summary

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CS70: Wrapping Up.

Random Thoughts

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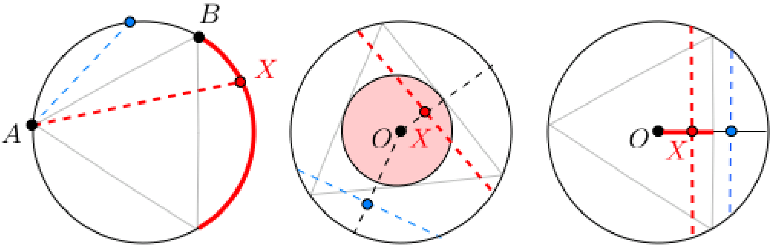
- Beware of statistics reported in the media!

Choosing at Random: Bertrand’s Paradox:poll.



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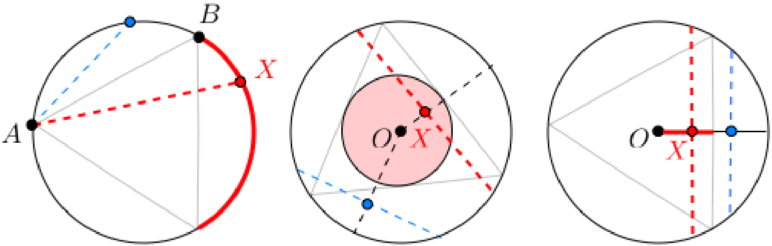

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Which is largest probability? (A) (B) (C)

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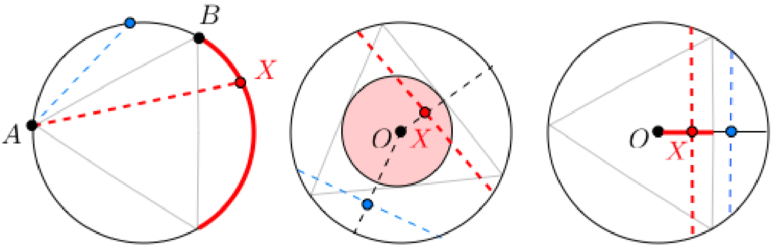

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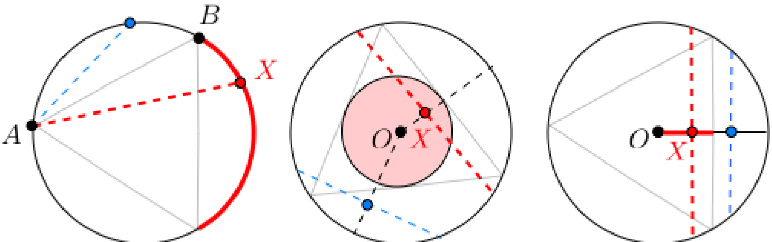

The figures corresponds to three ways of choosing a chord "at random."

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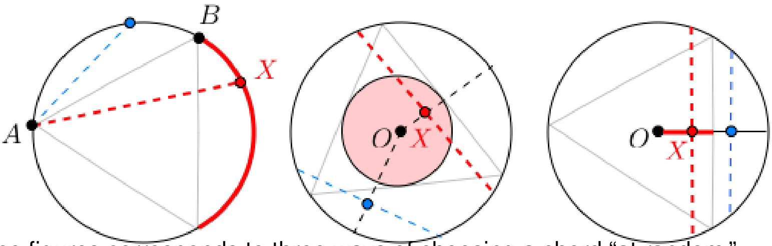

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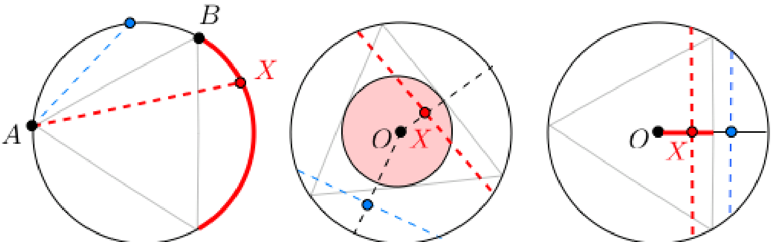

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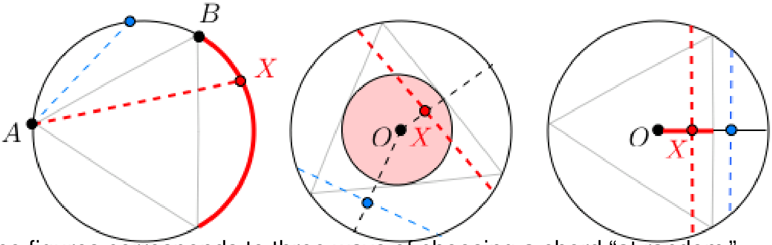

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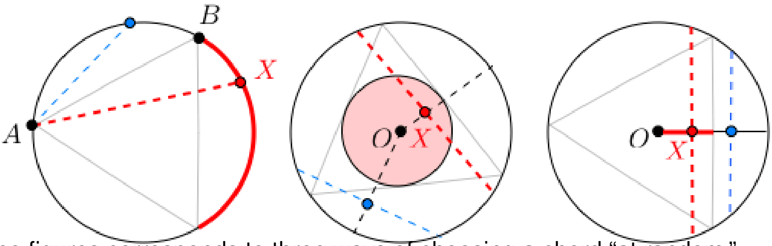

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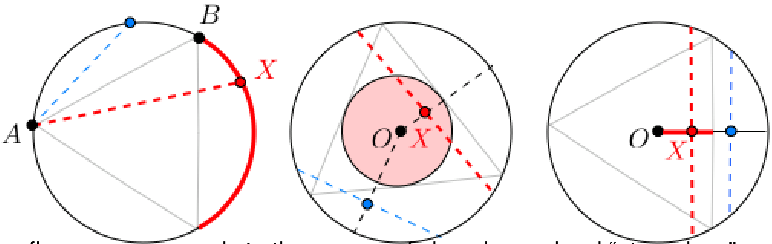

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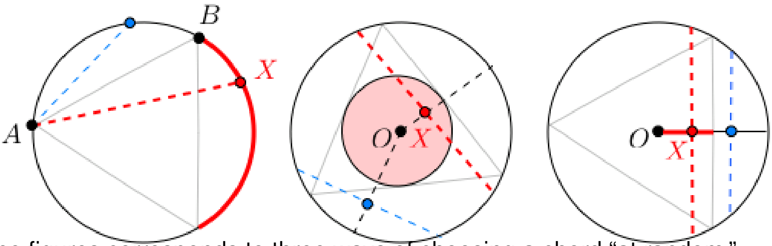

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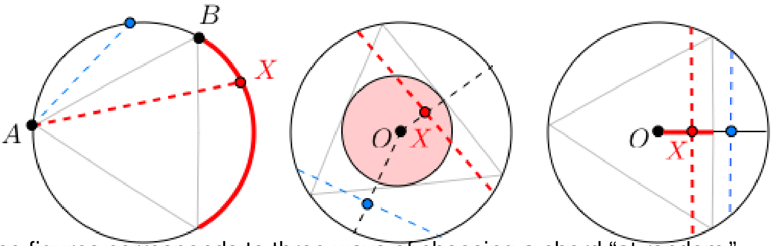

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  E.g., remember facts that confirm beliefs and forget others.

Confirmation Bias: An experiment

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There are two bags.

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As one draws balls one at time, one asks people to declare whether they think one draws from the first or second bag.

Surprisingly, people tend to be reinforced in their original belief, even when the evidence accumulates against it.

Report Data not Opinion!

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Each person pulls ball, reports opinion on which bag:

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Being Rational: ‘Thinking, Fast and Slow’

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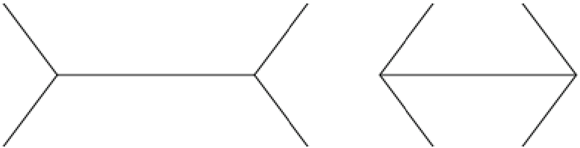

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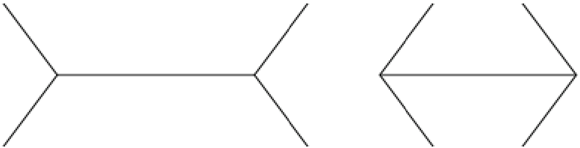

It is difficult to think clearly!

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...and (most important) your integrity.

What to Remember?

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Professor, what should I remember about probability from this course?

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You have learned a lot in this course!

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Proofs,

Parting Thoughts

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Proofs, Graphs,

Parting Thoughts

You have learned a lot in this course!

Proofs, Graphs, Mod(p),

Parting Thoughts

You have learned a lot in this course!

Proofs, Graphs, Mod(p), RSA,

Parting Thoughts

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Proofs, Graphs, Mod(p), RSA, Reed-Solomon,

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Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability,

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Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... ,

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Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... ,
how to handle stress,

Parting Thoughts

You have learned a lot in this course!

Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... ,
how to handle stress, how to sleep less,

Parting Thoughts

You have learned a lot in this course!

Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... ,

how to handle stress, how to sleep less, how to keep smiling, ...

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You have learned a lot in this course!

Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... ,
how to handle stress, how to sleep less, how to keep smiling, ...
Difficult course?

Parting Thoughts

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Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... ,
how to handle stress, how to sleep less, how to keep smiling, ...
Difficult course? Perhaps.

Parting Thoughts

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Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... ,
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Difficult course? Perhaps. Mind expanding!

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Difficult course? Perhaps. Mind expanding! I believe!!
Useful?

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Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... ,
how to handle stress, how to sleep less, how to keep smiling, ...
Difficult course? Perhaps. Mind expanding! I believe!!
Useful? You bet!

Derivative of sine?

$\sin(x)$.

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What is x? An angle in radians.

Derivative of sine?

$\sin(x)$.

What is x? An angle in radians.

Let's call it θ and do derivative of $\sin\theta$.

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θ - Length of arc of unit circle

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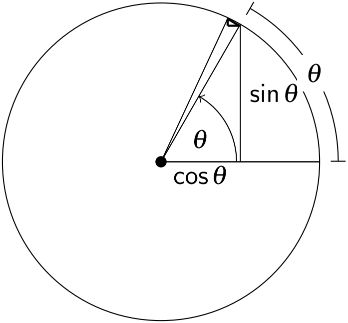

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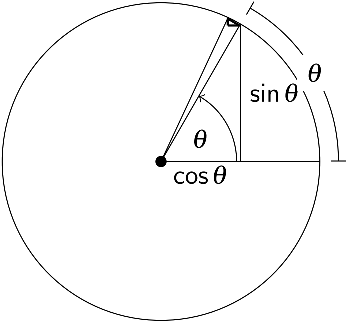

Rise.

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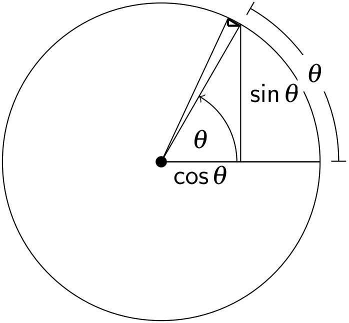
θ - Length of arc of unit circle



Rise. Similar triangle!!!

Derivative of sine?

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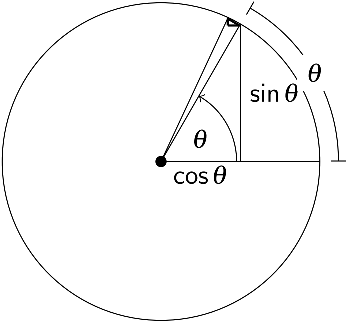

Rise. Similar triangle!!!

"Run" is change in radians

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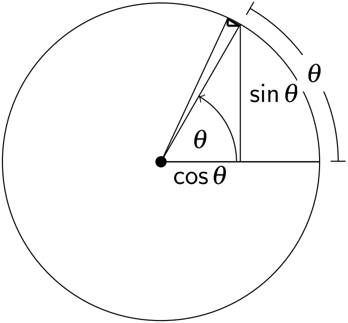

Rise. Similar triangle!!!

"Run" is change in radians which is $\approx$length of hypotenuse.

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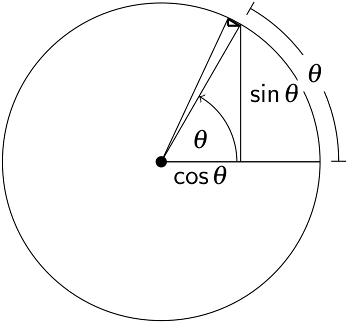

Rise. Similar triangle!!!

"Run" is change in radians which is $\approx$length of hypotenuse.

"Rise" is cosine times length of hypotenuse.

Derivative of sine?

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What is x? An angle in radians.

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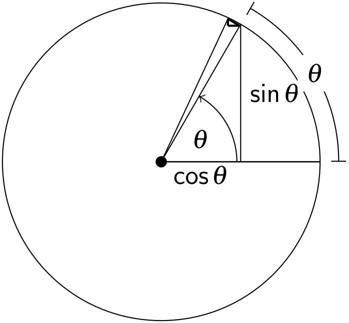

Rise. Similar triangle!!!

"Run" is change in radians which is $\approx$length of hypotenuse.

"Rise" is cosine times length of hypotenuse.

Ratio of rise/run is cosine of angle!

Arguments, reasoning.

What you know: slope, limit.

Arguments, reasoning.

What you know: slope, limit.
Plus: definition.

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.

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  Minimization, optimization, ...

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, ...

Knowing how to program

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, …..

Knowing how to program plus some syntax (google) gives the ability to program.

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, ...

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, ...

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, …..

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition gives calculus.

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, ...

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition gives calculus.

Discrete Math: basics are counting, how many, when are two sets the same size?

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, ...

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition gives calculus.

Discrete Math: basics are counting, how many, when are two sets the same size?

Probability:

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, ...

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition gives calculus.

Discrete Math: basics are counting, how many, when are two sets the same size?

Probability: division.

Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
  Minimization, optimization, …..

Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition gives calculus.

Discrete Math: basics are counting, how many, when are two sets the same size?

Probability: division.

...plus reasoning.

What's Next?

Professor,

What’s Next?

Professor, I loved this course so much!

What's Next?

Professor, I loved this course so much!
I want to learn more about discrete math and probability!

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- CS170: Efficient Algorithms and Intractable Problems a.k.a. Introduction to CS Theory:

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- CS170: Efficient Algorithms and Intractable Problems a.k.a. Introduction to CS Theory: Graphs, Dynamic Programming, Complexity.

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- CS170: Efficient Algorithms and Intractable Problems a.k.a.
  Introduction to CS Theory: Graphs, Dynamic Programming, Complexity.
- EE126: Probability in EECS: An Application-Driven Course:

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- CS170: Efficient Algorithms and Intractable Problems a.k.a. Introduction to CS Theory: Graphs, Dynamic Programming, Complexity.
- EE126: Probability in EECS: An Application-Driven Course: PageRank, Digital Links, Tracking, Speech Recognition, Planning, etc.

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- CS170: Efficient Algorithms and Intractable Problems a.k.a. Introduction to CS Theory: Graphs, Dynamic Programming, Complexity.
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- CS189: Introduction to Machine Learning:

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- CS188: Artificial Intelligence: Hidden Markov Chains, Bayes Networks, Neural Networks.
- CS189: Introduction to Machine Learning: Regression, Neural Networks, Learning, etc.

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- CS188: Artificial Intelligence: Hidden Markov Chains, Bayes Networks, Neural Networks.
- CS189: Introduction to Machine Learning: Regression, Neural Networks, Learning, etc. Programming experiments with real-world applications.

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- EE121: Digital Communication:

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- CS188: Artificial Intelligence: Hidden Markov Chains, Bayes Networks, Neural Networks.
- CS189: Introduction to Machine Learning: Regression, Neural Networks, Learning, etc. Programming experiments with real-world applications.
- EE121: Digital Communication: Coding for communication and storage.

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- CS188: Artificial Intelligence: Hidden Markov Chains, Bayes Networks, Neural Networks.
- CS189: Introduction to Machine Learning: Regression, Neural Networks, Learning, etc. Programming experiments with real-world applications.
- EE121: Digital Communication: Coding for communication and storage.
- EE223: Stochastic Control.

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- EE121: Digital Communication: Coding for communication and storage.
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- EE229A: Information Theory; EE229B: Coding Theory.

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Final Thoughts

Final Thoughts

More precisely:

Final Thoughts

More precisely: Some thoughts about the final ....

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides;

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes;

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems;

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW
- Approximate Coverage: Probability 2/3, Discrete Math: 1/3.

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW
- Approximate Coverage: Probability 2/3, Discrete Math: 1/3.
- Every question topic covered in at least two places. Most will be covered in all places.

Finally....

Finally....

Thanks for taking the course!

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Thanks to the CS70 Staff:

Finally....

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Thanks to the CS70 Staff:

- The Terrific Tutors
- The Rigorous Readers
- The Thrilling TAs
- The Amazing Assistants

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Good studying!!!