Climb an infinite ladder?
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∀ \( n \in \mathbb{N} \)
Climb an infinite ladder?

∀ \( k \), \( P(k) = \Rightarrow P(k+1) \)

\( P(0) = \Rightarrow P(1) = \Rightarrow P(2) = \Rightarrow P(3) \ldots \)

(\( \forall n \in \mathbb{N} \)) \( P(n) \)

Your favorite example of forever... or the natural numbers...
Climb an infinite ladder?

\[ P(0) \]

\[ \forall k, P(k) \implies P(k + 1) \]
Climb an infinite ladder?

∀k, P(k) \implies P(k + 1)

P(0) \implies P(1) \implies P(2) \implies \ldots

\forall n \in \mathbb{N} \quad P(n)

Your favorite example of forever.
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P(0) \\
\forall k, P(k) \implies P(k+1) \\
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P(n) \implies P(n+1)
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\[
\forall k, P(k) \implies P(k+1)
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P(0) \implies P(1) \implies P(2) \implies P(3) \ldots
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Climb an infinite ladder?

\[
P(n + 3) \quad P(n + 2) \quad P(n + 1) \quad P(n) \quad \ldots \quad \ldots \quad P(3) \quad P(2) \quad P(1) \quad P(0)
\]

\[
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(\forall n \in \mathbb{N}) P(n)
Climb an infinite ladder?

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Your favorite example of forever..
Climb an infinite ladder?

∀k, P(k) → P(k + 1)

P(0) → P(1) → P(2) → P(3) ...

(∀n ∈ N) P(n)

Your favorite example of forever..or the natural numbers...
Horses of the same color...

**Theorem:** All horses have the same color.
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Base Case: $P(1)$ - trivially true.
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First $k$ have same color by $P(k)$. $1, 2, 3, \ldots, k, k + 1$
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- A horse in the middle in common! $1, 2, 3, \ldots, k, k + 1$
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- A horse in the middle in common! \(1, 2, 3, \ldots, k, k + 1\)
- All $k$ must have the same color. \(1, 2, 3, \ldots, k, k + 1\)
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How about $P(1) \implies P(2)$?
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**Theorem:** All horses have the same color.

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**New Base Case:** $P(2)$: there are two horses with same color.

Induction Hypothesis: $P(k)$ - Any $k$ horses have the same color.

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Fix base case.
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(There are two horses is $\neq$ For every pair of two horses!!!)
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Of course it doesn’t work.
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Of course it doesn’t work.

More subtle to catch errors in proofs of correct theorems!!
Sad Islanders...

Island with 100 possibly blue-eyed and green-eyed inhabitants.
Sad Islanders...

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Any islander who knows they have green eyes must “leave the island” that day.
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No islander knows their own eye color, but knows everyone else's.

Visitor: “I see someone has green eyes.”

Result: What happens?

(A) Nothing, no information was added.

(B) Information was added, maybe?

(C) They all leave the island.

(D) They all leave the island on day 100.

On day 100, they all leave.

Why?
Sad Islanders...

Island with 100 possibly blue-eyed and green-eyed inhabitants.

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Why?
They know induction.

Thm: If there are \( n \) villagers with green eyes they leave on day \( n \).
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Proof:
Base: $n = 1$. Person with green eyes leaves on day 1.
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Base: $n = 1$. Person with green eyes leaves on day 1.

Induction hypothesis:
If $n$ people with green eyes, they would leave on day $n$. 

I like the island.
SAD.

Wait! Visitor added no information.
They know induction.

Thm: If there are \( n \) villagers with green eyes they leave on day \( n \).

**Proof:**
Base: \( n = 1 \). Person with green eyes leaves on day 1.

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If \( n \) people with green eyes, they would leave on day \( n \).

Induction step:
On day \( n + 1 \), a green eyed person sees \( n \) people with green eyes.
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Induction step:
On day $n+1$, a green eyed person sees $n$ people with green eyes. But they didn’t leave.
They know induction.

Thm: If there are $n$ villagers with green eyes they leave on day $n$.

Proof:
Base: $n = 1$. Person with green eyes leaves on day 1.
Induction hypothesis:
If $n$ people with green eyes, they would leave on day $n$.
Induction step:
On day $n + 1$, a green eyed person sees $n$ people with green eyes.
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So there must be $n + 1$ people with green eyes.
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One of them, is me.
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Wait! Visitor added no information.
Common Knowledge.

Using knowledge about what other people’s knowledge (your eye color) is.

On day 1, everyone knows everyone sees more than zero.

On day 2, everyone knows everyone sees more than one.

. . .

On day 99, everyone knows no one sees 98 since everyone knows everyone else does not see 97...

On day 100, . . .uh oh!

Another example:
Emperor’s new clothes!
No one knows other people see that he has no clothes.
Until kid points it out.

Political arguments?
Everyone knows the arguments, everyone knows everyone knows the arguments....

The islanders didn’t talk.
Induction is quieter.
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  Everyone knows the arguments,
  everyone knows everyone knows the arguments.....
The islanders didn’t talk.
Common Knowledge.

Using knowledge about what other people’s knowledge (your eye color) is.

On day 1, everyone knows everyone sees more than zero.
On day 2, everyone knows everyone sees more than one.

... 

On day 99, everyone knows no one sees 98 since everyone knows everyone else does not see 97...

On day 100, ...uh oh!

Another example:
Emperor’s new clothes!
  No one knows other people see that he has no clothes.
  Until kid points it out.

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Strong Induction and Recursion.

Thm: For every natural number $n \geq 12$, $n = 4x + 5y$. 

Instead of proof, let's write some code!

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def find-x-y(n):
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Prove: Given $n$, returns $(x, y)$ where $n = 4x + 5y$, for $n \geq 12$.

Base cases: $P(12), P(13), P(14), P(15)$.

Yes.

Strong Induction step: Recursive call is correct: $P(n-4) \Rightarrow P(n)$.

$n-4 = 4(x'+1) + 5y'$

Slight differences: showed for all $n \geq 16$ that $\land i=1^n P(i) \Rightarrow P(n)$.
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Prove: Given \( n \), returns \((x, y)\) where \( n = 4x + 5y \), for \( n \geq 12 \).

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Yes.

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\( n - 4 = 4(x') + 5y' \) \( \Rightarrow \) \( n = 4(x' + 1) + 5y' \).

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Tidying up induction.

The induction principle works on the natural numbers.
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Can you do induction over other things? Yes.
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Any set where any subset of the set has a smallest element.
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Any set where any subset of the set has a smallest element.

In some sense, the natural numbers.
Stable Matching Problem

- $n$ candidates and $n$ jobs.
- Each job has a ranked preference list of candidates.
- Each candidate has a ranked preference list of jobs.
- How should they be matched?
- Maximize total satisfaction.
- Maximize number of first choices.
- Maximize worse off.
- Minimize difference between preference ranks.
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The best laid plans..

Consider the pairs..

- (Anthony) Davis and Pelicans
- (Lonzo) Ball and Lakers
The best laid plans..

Consider the pairs..

► (Anthony) Davis and Pelicans
► (Lonzo) Ball and Lakers

Davis prefers the Lakers.
The best laid plans..

Consider the pairs..

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Davis prefers the Lakers.
Lakers prefer Davis.
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Consider the pairs..

- (Anthony) Davis and Pelicans
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Lakers prefer Davis.
Uh..oh.
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Consider the pairs..

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Davis prefers the Lakers.
Lakers prefer Davis.
Uh..oh. Sad Lonzo and Pelicans.
So..

Produce a matching where there are no crazy moves!
So..

Produce a matching where there are no crazy moves!

**Definition:** A **matching** is disjoint set of $n$ job-candidate pairs.
So..

Produce a matching where there are no crazy moves!

**Definition:** A matching is disjoint set of $n$ job-candidate pairs.

Example: A matching $S = \{(\text{Lakers}, \text{Ball}); (\text{Pelicans}, \text{Davis})\}$. 
Produce a matching where there are no crazy moves!

**Definition:** A *matching* is disjoint set of $n$ job-candidate pairs.

Example: A matching $S = \{(\text{Lakers}, \text{Ball}); (\text{Pelicans}, \text{Davis})\}$.

**Definition:** A *rogue couple* $b, g^*$ for a pairing $S$: $b$ and $g^*$ prefer each other to their partners in $S$. 
Produce a matching where there are no crazy moves!

**Definition:** A matching is disjoint set of $n$ job-candidate pairs.

Example: A matching $S = \{ (Lakers, Ball); (Pelicans, Davis) \}$.

**Definition:** A rogue couple $b, g^*$ for a pairing $S$: $b$ and $g^*$ prefer each other to their partners in $S$.

Example: Davis and Lakers are a rogue couple in $S$. 
A stable matching??

Given a set of preferences.
A stable matching??

Given a set of preferences.
Is there a stable matching?
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Consider a single type version: stable roommates.

<table>
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<tr>
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Graph representation:

- A connected to C
- B connected to D
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| A | B | C | D |
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\[ \text{A} \rightarrow \text{C} \quad \text{B} \rightarrow \text{D} \]

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A \rightarrow B, C \rightarrow D
A stable matching??

Given a set of preferences.

Is there a stable matching?

How does one find it?

Consider a single type version: stable roommates.

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A --- B

C --- D
A stable matching??

Given a set of preferences.
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![Diagram of matching pairs]
A stable matching??

Given a set of preferences.
Is there a stable matching?
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![Diagram](image-url)
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Is there a stable matching?
How does one find it?

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A \rightarrow B
B \rightarrow C
C \rightarrow D
D \rightarrow A
Example.

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Example.

| Jobs | | Candidates |
|------|-----------------|
| A    | 1 2 3           | 1   | C A B |
| B    | X 2 3           | 2   | A B C |
| C    | 2 1 3           | 3   | A C B |

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12 / 28
Example.

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The Propose and Reject Algorithm.

Each Day:
1. Each job proposes to its favorite candidate on its list.
2. Each candidate rejects all but their favorite proposer (whom they put on a string.)
3. Rejected job crosses rejecting candidate off its list.

Stop when each job gets exactly one proposal.

Does this terminate?

...produce a matching?

...a stable matching?

Do jobs or candidates do "better"?
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Do jobs or candidates do “better”?
Termination.
Termination.

Every non-terminated day a job crossed an item off the list.
Termination.

Every non-terminated day a job crossed an item off the list.

Total size of lists?
Termination.

Every non-terminated day a job crossed an item off the list.
Total size of lists? $n$ jobs, $n$ length list.
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Terminates in $\leq n^2$ steps!
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If on day $t$, a candidate $g$ has a job $b$ on a string, any job, $b'$, on candidate $g$'s string for any day $t'$ > $t$ is at least as good as $b$.

Example: Candidate "Alice" has job "Amalmagated Concrete" on string on day 5. She has job "Amalmagated Asphalt" on string on day 7. Does Alice prefer "Amalmagated Asphalt" or "Amalmagated Concrete"?

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Day 10: Can Alice have "Amalmagated Asphalt" on her string? Yes. Alice prefers day 10 job as much as day 7 job. Here, $b = b'$.

Why is lemma true?

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Proof:
$P(k)$ – "job on $g$’s string is at least as good as $b$ on day $t+k"$
$P(0)$ – true. Candidate has $b$ on string.
Assume $P(k)$. Let $b'$ be job on string on day $t+k$.
On day $t+k+1$, job $b'$ comes back. Candidate $g$ can choose $b'$, or do better with another job, $b''$. That is, $b' \geq b$ by induction hypothesis.
And $b''$ is better than $b'$ by algorithm.
$= \Rightarrow$ Candidate does at least as well as with $b$.

$P(k) = \Rightarrow P(k+1)$.
And by principle of induction, lemma holds for every day after $t$. 

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Question: It just gets better for candidates, because?

(A) Induction on days.

(B) When the economy is good.

(C) The candidate can always keep the job on the string.

Sure on (B), but that Econ not CS.

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**Lemma:** Every job is matched at end.
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**Proof:**

- If not, a job $b$ must have been rejected $n$ times.
- Every candidate has been proposed to by $b$, and Improvement lemma implies each candidate has a job on a string.
- And each job is on at most one string.
- $n$ candidates and $n$ jobs.
- Same number of each.
- $b$ must be on some candidate's string!
- Contradiction.
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$\implies$ each candidate has a job on a string.
and each job is on at most one string.
$n$ candidates and $n$ jobs. Same number of each.
Matching when done.

**Lemma:** Every job is matched at end.

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Contradiction.
Question: The argument for termination uses.

(A) Implies: no unmatched job at end.

(B) Improvement Lemma: every candidate matched.

(C) Algorithm: unmatched job would ask everyone.

(D) Implies: every one gets their favorite job.
Matching is Stable.

**Lemma:** There is no rogue couple for the matching formed by traditional marriage algorithm.
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\[
\begin{array}{c}
  b^* \quad \text{----} \quad g^* \\
  b \quad \text{-----} \quad g \\
\end{array}
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\begin{align*}
  b^* & \sim g^* \\
  b & \sim g \\
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\(b\) prefers \(g^*\) to \(g\).
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    b^* & \quad \quad g^* \quad \quad b \quad \text{prefers } g^* \text{ to } g. \\
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    b^* & \quad \quad \quad g^* \\
    b & \quad \quad \quad g
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\(b\) prefers \(g^*\) to \(g\).
\(g^*\) prefers \(b\) to \(b^*\).

Job \(b\) proposes to \(g^*\) before proposing to \(g\).
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    b^* & \quad \vdash \quad g^* & \quad b \text{ prefers } g^* \text{ to } g. \\
    b & \quad \dashv \quad g & \quad g^* \text{ prefers } b \text{ to } b^*.
\end{align*}
\]

Job \(b\) proposes to \(g^*\) before proposing to \(g\).
So \(g^*\) rejected \(b\) (since he moved on)
Matching is Stable.

**Lemma:** There is no rogue couple for the matching formed by traditional marriage algorithm.

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Assume there is a rogue couple; \((b, g^*)\)

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\begin{array}{c}
  b^* \quad g^* \\
  b \quad g
  \end{array}
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Assume there is a rogue couple; \((b, g^*)\)

1. \(b^* \quad \underbrace{\text{\underbrace{\quad g^*}}} \quad b \text{ prefers } g^* \text{ to } g.\)
2. \(b \quad \underbrace{\text{\underbrace{\quad g}} \quad g^* \text{ prefers } b \text{ to } b^*.}\)

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Contradiction!
Question: The SMA produces a stable pairing is a proof by?

(A) Contradiction.
(B) Uses the improvement lemma.
(C) Induction.
(D) Direct.
Good for jobs? candidates?

Is the Job-Proposes better for jobs?
Good for jobs? candidates?

Is the Job-Proposes better for jobs? for candidates?

Definition: A matching is \( x \)-optimal if \( x \)'s partner is its best partner in any stable pairing.

Definition: A matching is \( x \)-pessimal if \( x \)'s partner is its worst partner in any stable pairing.

Definition: A matching is job optimal if it is \( x \)-optimal for all jobs \( x \).

...and so on for job pessimal, candidate optimal, candidate pessimal.

Claim: The optimal partner for a job must be first in its preference list.

True? False!

Subtlety here: Best partner in any stable matching. As well as you can be in a globally stable solution!

Question: Is there a job or candidate optimal matching? Is it possible: \( b \)-optimal pairing different from the \( b \)'-optimal matching! Yes? No?
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Yes? No?
Understanding Optimality: by example.

A: 1,2 1: A,B
B: 1,2 2: B,A

Consider pairing: (A, 1), (B, 2).

Stable? Yes.
Optimal for B? Notice: only one stable pairing. So this is the best B can do in a stable pairing. So optimal for B.
Also optimal for A, 1 and 2.
Also pessimal for A, B, 1 and 2.

Pairing S: (A, 1), (B, 2).
Stable? Yes.

Pairing T: (A, 2), (B, 1).
Also Stable.
Which is optimal for A? S
Which is optimal for B? S
Which is optimal for 1? T
Which is optimal for 2? T
Pessimality?
Understanding Optimality: by example.

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Pessimality? 23 / 28
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Stable?
Understanding Optimality: by example.

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Which is optimal for A?
Understanding Optimality: by example.

A: 1,2  
   1: A,B
B: 1,2  
   2: B,A

Consider pairing: (A, 1), (B, 2).

Stable? Yes.

Optimal for B?
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So optimal for B.

Also optimal for A, 1 and 2. Also pessimal for A,B,1 and 2.

A: 1,2  
   1: B,A
B: 2,1  
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Which is optimal for \(A\)? \(S\)  
Which is optimal for \(B\)?
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Which is optimal for A? S Why?
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Understanding Optimality: by example.

A: 1,2  
   1: A,B
B: 1,2  
   2: B,A

Consider pairing: \((A, 1), (B, 2)\).

Stable? Yes.

Optimal for \(B\)?
Notice: only one stable pairing.
So this is the best \(B\) can do in a stable pairing.
So optimal for \(B\).

Also optimal for \(A\), 1 and 2. Also pessimal for \(A,B,1\) and 2.

A: 1,2  
   1: B,A
B: 2,1  
   2: A,B

Pairing \(S\): \((A, 1), (B, 2)\). Stable? Yes.

Pairing \(T\): \((A, 2), (B, 1)\). Also Stable.

Which is optimal for \(A\)? \(S\)  
Which is optimal for \(B\)? \(S\)
Which is optimal for 1?
Understanding Optimality: by example.

A: 1,2  
1: A,B
B: 1,2  
2: B,A

Consider pairing: (A,1), (B,2).

Stable? Yes.

Optimal for B? 
Notice: only one stable pairing.
So this is the best B can do in a stable pairing.
So optimal for B.

Also optimal for A, 1 and 2. Also pessimal for A,B,1 and 2.

A: 1,2  
1: B,A
B: 2,1  
2: A,B

Pairing S: (A,1), (B,2). Stable? Yes.

Pairing T: (A,2), (B,1). Also Stable.

Which is optimal for A? S        Which is optimal for B? S
Which is optimal for 1? T
Understanding Optimality: by example.

A: 1,2  1: A,B
B: 1,2  2: B,A

Consider pairing: \((A, 1), (B, 2)\).

Stable? Yes.

Optimal for \(B\)?
Notice: only one stable pairing.
So this is the best \(B\) can do in a stable pairing.
So optimal for \(B\).

Also optimal for \(A\), 1 and 2. Also pessimal for \(A,B,1\) and 2.

A: 1,2  1: B,A
B: 2,1  2: A,B

Pairing \(S\): \((A, 1), (B, 2)\). Stable? Yes.

Pairing \(T\): \((A, 2), (B, 1)\). Also Stable.

Which is optimal for \(A\)? \(S\)       Which is optimal for \(B\)? \(S\)
Which is optimal for 1? \(T\)       Which is optimal for 2?
Understanding Optimality: by example.

A: 1,2
B: 1,2

1: A,B
2: B,A

Consider pairing: (A, 1), (B, 2).

Stable? Yes.

Optimal for B?
Notice: only one stable pairing.
So this is the best B can do in a stable pairing.
So optimal for B.

Also optimal for A, 1 and 2. Also pessimal for A,B, 1 and 2.

A: 1,2
B: 2,1

1: B,A
2: A,B

Pairing S: (A, 1), (B, 2). Stable? Yes.

Pairing T: (A, 2), (B, 1). Also Stable.

Which is optimal for A? S
Which is optimal for B? S
Which is optimal for 1? T
Which is optimal for 2? T
Understanding Optimality: by example.

Consider pairing: \((A, 1), (B, 2)\).

Stable? Yes.

Optimal for \(B\)?
Notice: only one stable pairing.
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Pairing \(S\): \((A, 1), (B, 2)\). Stable? Yes.
Pairing \(T\): \((A, 2), (B, 1)\). Also Stable.

Which is optimal for \(A\)? \(S\)  Which is optimal for \(B\)? \(S\)
Which is optimal for 1? \(T\)  Which is optimal for 2? \(T\)
Pessimality?
Job Propose and Candidate Reject is optimal!

For jobs?

Theorem:
Job Propose and Reject produces a job-optimal pairing.

Proof:
Assume not: there is a job $b$ does not get optimal candidate, $g$.

There is a stable pairing $S$ where $b$ and $g$ are paired.

Let $t$ be first day job $b$ gets rejected by its optimal candidate $g$ who it is paired with in stable pairing $S$.

$b^*$ knocks $b$ off of $g$'s string on day $t = \Rightarrow g$ prefers $b^*$ to $b$.

By choice of $t$, $b^*$ likes $g$ at least as much as optimal candidate.

$\Rightarrow b^*$ prefers $g$ to its partner $g^*$ in $S$.

Rogue couple for $S$.

So $S$ is not a stable pairing.

Contradiction.

Notes:
$S$ - stable.
$(b^*, g^*) \in S$.

But $(b^*, g^*)$ is rogue couple!

Used Well-Ordering principle...

Induction.
Job Propose and Candidate Reject is optimal!

For jobs? For candidates?

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Let \( t \) be first day job \( b \) gets rejected by its optimal candidate \( g \) who it is paired with in stable pairing \( S \).

\( b^\ast \) knocks \( b \) off of \( g \)'s string on day \( t \).

\[ \Rightarrow g \] prefers \( b^\ast \) to \( b \).

By choice of \( t \), \( b^\ast \) likes \( g \) at least as much as optimal candidate.

\[ \Rightarrow b^\ast \] prefers \( g \) to its partner \( g^\ast \) in \( S \).

Rogue couple for \( S \).

So \( S \) is not a stable pairing.

Contradiction.

Notes:

\( S \) - stable.

\((b^\ast, g^\ast) \in S\).

But \((b^\ast, g)\) is rogue couple!

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Job Propose and Candidate Reject is optimal!

For jobs? For candidates?

**Theorem:** Job Propose and Reject produces a job-optimal pairing.
Job Propose and Candidate Reject is optimal!

For jobs? For candidates?

**Theorem:** Job Propose and Reject produces a job-optimal pairing.

**Proof:**
Job Propose and Candidate Reject is optimal!

For jobs? For candidates?

**Theorem**: Job Propose and Reject produces a job-optimal pairing.

**Proof**: Assume not:

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By choice of $t$, $b^*$ likes $g$ at least as much as optimal candidate $= \Rightarrow b^*$ prefers $g$ to its partner $g^*$ in $S$.

**Rogue couple for $S$.**

So $S$ is not a stable pairing.

**Notes:**

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Notes:
$S$ - stable.
$(b^*, g)$ $\in$ $S$.
But $(b^*, g)$ is rogue couple!

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\( b^* \) - knocks \( b \) off of \( g \)’s string on day \( t \) \( \implies \) \( g \) prefers \( b^* \) to \( b \)
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$b^*$ - knocks $b$ off of $g$’s string on day $t \implies g$ prefers $b^*$ to $b$

By choice of $t$, $b^*$ likes $g$ at least as much as optimal candidate.
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$b^*$ - knocks $b$ off of $g$’s string on day $t$ $\implies$ $g$ prefers $b^*$ to $b$

By choice of $t$, $b^*$ likes $g$ at least as much as optimal candidate.

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By choice of $t$, $b^*$ likes $g$ at least as much as optimal candidate.

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Rogue couple for $S$. 

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By choice of \( t \), \( b^* \) likes \( g \) at least as much as optimal candidate.

\( \implies b^* \) prefers \( g \) to its partner \( g^* \) in \( S \).

Rogue couple for \( S \).
So \( S \) is not a stable pairing.
Job Propose and Candidate Reject is optimal!

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By choice of $t$, $b^*$ likes $g$ at least as much as optimal candidate.

$\implies b^*$ prefers $g$ to its partner $g^*$ in $S$.

Rogue couple for $S$.
So $S$ is not a stable pairing. Contradiction.
Job Propose and Candidate Reject is optimal!

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By choice of $t$, $b^*$ likes $g$ at least as much as optimal candidate.  

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Notes:
Job Propose and Candidate Reject is optimal!

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Notes: $S$ - stable.
Job Propose and Candidate Reject is optimal!

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By choice of $t$, $b^*$ likes $g$ at least as much as optimal candidate.

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Rogue couple for $S$.
So $S$ is not a stable pairing. Contradiction. $\square$

Notes: $S$ - stable. $(b^*, g^*) \in S$. 
Job Propose and Candidate Reject is optimal!

For jobs? For candidates?

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Assume not: there is a job $b$ does not get optimal candidate, $g$.

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By choice of $t$, $b^*$ likes $g$ at least as much as optimal candidate.

$\implies b^*$ prefers $g$ to its partner $g^*$ in $S$.

Rogue couple for $S$.
So $S$ is not a stable pairing. Contradiction.

Notes: $S$ - stable. $(b^*, g^*) \in S$. But $(b^*, g)$
Job Propose and Candidate Reject is optimal!

For jobs? For candidates?

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By choice of $t$, $b^*$ likes $g$ at least as much as optimal candidate.

$\implies b^*$ prefers $g$ to its partner $g^*$ in $S$.

Rogue couple for $S$. So $S$ is not a stable pairing. Contradiction.

Notes: $S$ - stable. $(b^*, g^*) \in S$. But $(b^*, g)$ is rogue couple!
Job Propose and Candidate Reject is optimal!

For jobs? For candidates?

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Assume not: there is a job $b$ does not get optimal candidate, $g$.

There is a stable pairing $S$ where $b$ and $g$ are paired.

Let $t$ be first day job $b$ gets rejected by its optimal candidate $g$ who it is paired with in stable pairing $S$.

$b^* \ - \ \text{knocks} \ \ b \ \text{off of} \ \ g\text{’s string on day } t \implies g \ \text{prefers} \ b^* \ \text{to} \ b$

By choice of $t$, $b^*$ likes $g$ at least as much as optimal candidate.

$\implies b^* \ \text{prefers} \ g \ \text{to its partner} \ g^* \ \text{in} \ S$.

Rogue couple for $S$.
So $S$ is not a stable pairing. Contradiction.

Notes: $S$ - stable. $(b^*, g^*) \in S$. But $(b^*, g)$ is rogue couple!

Used Well-Ordering principle...
Job Propose and Candidate Reject is optimal!

For jobs? For candidates?

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\( \implies \) \( b^* \) prefers \( g \) to its partner \( g^* \) in \( S \).

Rogue couple for \( S \).
So \( S \) is not a stable pairing. Contradiction.

Notes: \( S \) - stable. \( (b^*, g^*) \in S \). But \( (b^*, g) \) is rogue couple!
Used Well-Ordering principle...Induction.
How about for candidates?

Theorem:
Job Propose and Reject produces candidate-pessimal pairing.

\( T \) – pairing produced by JPR.
\( S \) – worse stable pairing for candidate \( g \).

In \( T \), \((g, b)\) is pair.
In \( S \), \((g, b^*)\) is pair.
\( g \) prefers \( b \) to \( b^* \).
\( T \) is job optimal, so \( b \) prefers \( g \) to its partner in \( S \).

\((g, b)\) is Rogue couple for \( S \).
\( S \) is not stable.

Contradiction.

Notes:
Not really induction.
Structural statement: Job optimality \( \implies \) Candidate pessimality.
How about for candidates?

**Theorem:** Job Propose and Reject produces candidate-pessimal pairing.

$(g, b)$ is pair.

$(g, b^*)$ is pair.

$g$ prefers $b$ to $b^*$.

$T$ is job optimal, so $b$ prefers $g$ to its partner in $S$.

$(g, b)$ is Rogue couple for $S$.

$S$ is not stable.

Contradiction.

Notes: Not really induction.

Structural statement: Job optimality $\Rightarrow$ Candidate pessimality.
How about for candidates?

**Theorem:** Job Propose and Reject produces candidate-pessimal pairing.

$T$ – pairing produced by JPR.
How about for candidates?

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How about for candidates?

**Theorem:** Job Propose and Reject produces candidate-pessimal pairing.

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$g$ prefers $b$ to $b^*$.

$T$ is job optimal, so $b$ prefers $g$ to its partner in $S$. 

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$(g, b)$ is Rogue couple for $S$. 

Notes: Not really induction. Structural statement: Job optimality \implies Candidate pessimality.
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Contradiction.

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**Contradiction.**

Notes: Not really induction.
   Structural statement: Job optimality
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**Contradiction.**

Notes: Not really induction.

  Structural statement: Job optimality $\implies$ Candidate pessimality.
Quick Questions.

How does one make it better for candidates?
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Propose and Reject - stable matching algorithm. One side proposes.
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Jobs Propose $\implies$ job optimal.
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Candidates propose. $\implies$ optimal for candidates.
Residency Matching..
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The method was used to match residents to hospitals.
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Hospital optimal....
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The method was used to match residents to hospitals. Hospital optimal.... ..until 1990’s...
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Another variation: couples.
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Takeaways.

Analysis of cool algorithm with interesting goal: stability.
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“Economic”: different utilities.
Takeaways.

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“Economic”: different utilities.
Definition of optimality: best utility in stable world.
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Action gives better results for individuals but gives instability.
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Induction over steps of algorithm.
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Proofs carefully use definition:
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Induction over steps of algorithm.

Proofs carefully use definition:

Stability:
- Improvement Lemma: every day the job gets to choose.

Optimality proof:
- Job Optimality:
  - contradiction of the existence of a better *stable* pairing.
- Candidate Pessimality:
  - contradiction plus cuz job optimality implies better pairing.
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Life Lesson: ask,
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Life Lesson: ask, you will do better even if rejection is hard.