Graphs!
Graphs!
Definitions: model.
Graphs!
  Definitions: model.
  Fact!
Lecture 5: Graphs.

Graphs!
Definitions: model.
Fact!
Graphs!
  Definitions: model.
  Fact!
Planar graphs.
Graphs!
Definitions: model.
Fact!
Planar graphs.
Euler Again!!!!
Map Coloring.
Map Coloring.

A diagram showing a map with regions colored in different colors to demonstrate the concept of map coloring.

Theorem: Four colors enough.
Map Coloring.

Fewer Colors?
Yes! Three colors.

Fewer Colors?
Four colors required!

Theorem: Four colors enough.
Map Coloring.

Fewer Colors?

Yes! Three colors.

Fewer Colors?

Four colors required!

Theorem: Four colors enough.

Fewer Colors?
Map Coloring.

Yes! Three colors.
Map Coloring.

Yes! Three colors.

Four colors required!

Theorem: Four colors enough.
Map Coloring.

Yes! Three colors.

Four colors required!

Theorem: Four colors enough.
Map Coloring.

- Fewer Colors? Yes! Three colors.
- Fewer Colors? Four colors required!
- Theorem: Four colors enough.
Map Coloring.

Yes! Three colors.

Four colors required!

Theorem: Four colors enough.
Map Coloring.

Yes! Three colors.

Four colors required!
Theorem: Four colors enough.
Map Coloring.

Yes! Three colors.

Four colors required!

Theorem: Four colors enough.
Map Coloring.

Fewer Colors?

No! Four colors required!

Theorem: Four colors enough.
Map Coloring.

Yes! Three colors.

Four colors required!

Theorem: Four colors enough.
Map Coloring.

Four colors required!
Map Coloring.

Four colors required!

Theorem: Four colors enough.
Scheduling: coloring.

Exam Slot 1.
Exam Slot 2.
Exam Slot 3.
Scheduling: coloring.

Exam Slot 1.

Exam Slot 2.

Exam Slot 3.

61A

61B

61C

70

170

70
Scheduling: coloring.

Exam Slot 1.
Exam Slot 2.
Exam Slot 3.
Scheduling: coloring.

- Exam Slot 1.
- Exam Slot 2.
- Exam Slot 3.
Scheduling: coloring.

Exam Slot 1.
Exam Slot 2.
Exam Slot 3.
Scheduling: coloring.

Diagram showing the connections between 61A, 61B, 61C, 70, and 170.
Scheduling: coloring.
Scheduling: coloring.
Scheduling: coloring.
Scheduling: coloring.
Scheduling: coloring.
Scheduling: coloring.
Scheduling: coloring.
Scheduling: coloring.
Scheduling: coloring.

Exam Slot 1.
Exam Slot 2.
Exam Slot 3.
Graphs: formally.

Graph:

\[ G = (V, E) \]

- \( V \): set of vertices. \( \{A, B, C, D\} \)
- \( E \subseteq V \times V \): set of edges. \( \{(A, B), (A, B), (A, C), (A, C), (B, D), (A, D), (C, D)\} \)

For CS 70, usually simple graphs. No parallel edges. Multigraph above.
Graphs: formally.

Graph: $G = (V, E)$. 

$V$ - set of vertices. 
$\{A, B, C, D\}$ 

$E \subseteq V \times V$ - set of edges. 
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$. 

For CS 70, usually simple graphs. 
No parallel edges. 
Multigraph above.
Graphs: formally.

Graph: $G = (V, E)$.

$V$ - set of vertices.
Graphs: formally.

Graph: $G = (V, E)$.  
$V$ - set of vertices.  
{$A, B, C, D$}
Graphs: formally.

Graph: $G = (V, E)$.
- $V$ - set of vertices.
  - $\{A, B, C, D\}$
- $E \subseteq V \times V$ -
Graphs: formally.

Graph: $G = (V, E)$.

$V$ - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.
Graphs: formally.

Graph: $G = (V, E)$.

$V$ - set of vertices.

\{A, B, C, D\}

$E \subseteq V \times V$ - set of edges.

\{\{A, B\}\}
Graphs: formally.

Graph: $G = (V, E)$.
- $V$ - set of vertices.
  - $\{A, B, C, D\}$
- $E \subseteq V \times V$ - set of edges.
  - $\{\{A, B\}, \{A, B\}\}$
Graphs: formally.

Graph: \( G = (V, E) \).
- \( V \) - set of vertices.
  - \( \{A, B, C, D\} \)
- \( E \subseteq V \times V \) - set of edges.
  - \( \{\{A, B\}, \{A, B\}, \{A, C\}, \} \)
Graphs: formally.

Graph: \( G = (V, E) \).
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  - \( \{A, B, C, D\} \)
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  - \( \{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\} \).
Graphs: formally.

Graph: $G = (V, E)$.

$V$ - set of vertices.

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$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.
Graphs: formally.

Graph: $G = (V, E)$.

$V$ - set of vertices.
\{A, B, C, D\}

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\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}.

For CS 70, usually simple graphs.
No parallel edges.
Graphs: formally.

Graph: \( G = (V, E) \).

- \( V \) - set of vertices.
  - \( \{A, B, C, D\} \)
- \( E \subseteq V \times V \) - set of edges.
  - \( \{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\} \).

For CS 70, usually simple graphs.

- No parallel edges.

Multigraph above.
Directed Graphs

\[ G = (V, E). \]
Directed Graphs

\[ G = (V, E). \]

\[ V - \text{set of vertices.} \]
Directed Graphs

\[ G = (V, E). \]
\[ V - \text{set of vertices.} \]
\[ \{1, 2, 3, 4\} \]
Directed Graphs

\[ G = (V, E). \]

- \( V \) - set of vertices.
  \[ \{1, 2, 3, 4\} \]
- \( E \) - ordered pairs of vertices.
Directed Graphs

$G = (V, E)$.
$V$ - set of vertices.
$\{1, 2, 3, 4\}$
$E$ ordered pairs of vertices.
$\{(1, 2), \}$
Directed Graphs

\[ G = (V, E). \]
\[ V - \text{set of vertices.} \]
\[ \{1, 2, 3, 4\} \]
\[ E - \text{ordered pairs of vertices.} \]
\[ \{(1, 2), (1, 3), \} \]
Directed Graphs

\[ G = (V, E). \]

\( V \) - set of vertices.
\( \{1, 2, 3, 4\} \)

\( E \) ordered pairs of vertices.
\( \{(1, 2), (1, 3), (1, 4), \} \)

One way streets.
Tournament:
1 beats 2, ...
Precedence:
1 is before 2,..
Social Network:
Directed?
Undirected?
Friends.
Undirected.
Likes.
Directed.
Directed Graphs

\[ G = (V, E) \]

- **V**: set of vertices.
  \[ \{1, 2, 3, 4\} \]

- **E**: ordered pairs of vertices.
  \[ \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\} \]
Directed Graphs

$G = (V, E)$.

$V$ - set of vertices.
\{1, 2, 3, 4\}

$E$ ordered pairs of vertices.
\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}

One way streets.
Directed Graphs

\[ G = (V, E). \]

\( V \) - set of vertices.
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\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}

One way streets.
Tournament:
Directed Graphs

\[ G = (V, E). \]

- **V** - set of vertices.
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- **E** - ordered pairs of vertices.
  \[ \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\} \]

One way streets.
Tournament: 1 beats 2,
Directed Graphs

\[ G = (V, E) \]

- **V** - set of vertices.
  \[ \{1, 2, 3, 4\} \]
- **E** - ordered pairs of vertices.
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One way streets.
Tournament: 1 beats 2, ...
Precedence:
Directed Graphs

$G = (V, E)$.

$V$ - set of vertices.

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$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,
Directed Graphs

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One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network:
Directed Graphs

\[ G = (V, E). \]

- \( V \) - set of vertices.
  - \( \{1, 2, 3, 4\} \)
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  - \( \{(1,2), (1,3), (1,4), (2,4), (3,4)\} \)

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..
Social Network: Directed?
Directed Graphs

\[ G = (V, E). \]
\[ V - \text{set of vertices.} \]
\[ \{1, 2, 3, 4\} \]
\[ E - \text{ordered pairs of vertices.} \]
\[ \{(1,2), (1,3), (1,4), (2,4), (3,4)\} \]

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Directed Graphs

\[ G = (V, E). \]
\[ V \text{ - set of vertices. } \{1, 2, 3, 4\} \]
\[ E \text{ ordered pairs of vertices. } \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\} \]

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends.
Directed Graphs

$G = (V, E)$.

$V$ - set of vertices.
{1, 2, 3, 4}

$E$ ordered pairs of vertices.
{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)}

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Directed Graphs

\[ G = (V, E) \]

- **V** - set of vertices.
  \[ \{1, 2, 3, 4\} \]
- **E** ordered pairs of vertices.
  \[ \{(1,2), (1,3), (1,4), (2,4), (3,4)\} \]

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
- Friends. Undirected.
- Likes.
Directed Graphs

\[ G = (V, E) \]

- **V** - set of vertices. \{1, 2, 3, 4\}
- **E** - ordered pairs of vertices. \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
    Friends. Undirected.
    Likes. Directed.
Directed Graphs

$G = (V, E)$.

$V$ - set of vertices.
\{1, 2, 3, 4\}

$E$ ordered pairs of vertices.
\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
  Friends. Undirected.
  Likes. Directed.
Graph Concepts and Definitions.

Graph: $G = (V, E)$

Neighbors of 10?

$\{1, 5, 7, 8\}$.

$u$ is a neighbor of $v$ if \{u, v\} $\in$ $E$.

Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

Degree of vertex 1?

2

Degree of vertex $u$ is the number of incident edges. Equals the number of neighbors in a simple graph.

Directed graph?

In-degree of 10?

1

Out-degree of 10?

3
Graph Concepts and Definitions.
Graph: $G = (V, E)$
- neighbors, adjacent, degree, incident, in-degree, out-degree
Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10?
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1,
Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5,
Graph Concepts and Definitions.

Graph: $G = (V, E)$

- neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7,
Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.
Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

$u$ is neighbor of $v$ if $\{u, v\} \in E$. 
Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

$u$ is neighbor of $v$ if $\{u, v\} \in E$.

Edge $\{10, 5\}$ is incident to

Edge $\{u, v\}$ is incident to $u$ and $v$. 

Degree of vertex 1?

Degree of vertex $u$ is number of incident edges. 

Equals number of neighbors in simple graph.

Directed graph?

In-degree of 10?

Out-degree of 10?
Graph Concepts and Definitions.

Graph: $G = (V, E)$

- neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

- $u$ is neighbor of $v$ if $\{u, v\} \in E$.

Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

- Edge $\{u, v\}$ is incident to $u$ and $v$.

Degree of vertex 1?
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

\( u \) is neighbor of \( v \) if \( \{u, v\} \in E \).

Edge \( \{10, 5\} \) is incident to vertex 10 and vertex 5.

Edge \( \{u, v\} \) is incident to \( u \) and \( v \).

Degree of vertex 1? 2
Graph Concepts and Definitions.

Graph: $G = (V, E)$

- neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

- $u$ is neighbor of $v$ if $\{u, v\} \in E$.

Edge \{10, 5\} is incident to vertex 10 and vertex 5.

- Edge $\{u, v\}$ is incident to $u$ and $v$.

Degree of vertex 1? 2

- Degree of vertex $u$ is number of incident edges.
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

\( u \) is neighbor of \( v \) if \( \{u, v\} \subseteq E \).

Edge \( \{10, 5\} \) is incident to vertex 10 and vertex 5.

Edge \( \{u, v\} \) is incident to \( u \) and \( v \).

Degree of vertex 1? 2

Degree of vertex \( u \) is number of incident edges.

Equals number of neighbors in simple graph.
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)
- neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.
- \( u \) is neighbor of \( v \) if \( \{u, v\} \in E \).
Edge \( \{10, 5\} \) is incident to vertex 10 and vertex 5.
- Edge \( \{u, v\} \) is incident to \( u \) and \( v \).
Degree of vertex 1? 2
- Degree of vertex \( u \) is number of incident edges.
  Equals number of neighbors in simple graph.
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

*\( u \) is neighbor of \( v \) if \( \{u, v\} \in E \).*

Edge \( \{10, 5\} \) is incident to vertex 10 and vertex 5.

Edge \( \{u, v\} \) is incident to \( u \) and \( v \).

Degree of vertex 1? 2

*Degree of vertex \( u \) is number of incident edges.*

Equals number of neighbors in simple graph.

Directed graph?
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

- neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1,5,7, 8.
- \( u \) is neighbor of \( v \) if \( \{u, v\} \in E \).

Edge \( \{10, 5\} \) is incident to vertex 10 and vertex 5.
- Edge \( \{u, v\} \) is incident to \( u \) and \( v \).

Degree of vertex 1? 2
- Degree of vertex \( u \) is number of incident edges.
- Equals number of neighbors in simple graph.

Directed graph?
- In-degree of 10?
Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

$u$ is neighbor of $v$ if $\{u, v\} \in E$.

Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

Edge $\{u, v\}$ is incident to $u$ and $v$.

Degree of vertex 1? 2

Degree of vertex $u$ is number of incident edges.

Equals number of neighbors in simple graph.

Directed graph?

In-degree of 10? 1
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

- neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

- \( u \) is neighbor of \( v \) if \( \{u, v\} \in E \).

Edge \( \{10, 5\} \) is incident to vertex 10 and vertex 5.

- Edge \( \{u, v\} \) is incident to \( u \) and \( v \).

Degree of vertex 1? 2

- Degree of vertex \( u \) is number of incident edges.
  
  Equals number of neighbors in simple graph.

Directed graph?

- In-degree of 10? 1
- Out-degree of 10?
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1,5,7, 8.

\( u \) is neighbor of \( v \) if \( \{u, v\} \in E \).

Edge \( \{10, 5\} \) is incident to vertex 10 and vertex 5.

Edge \( \{u, v\} \) is incident to \( u \) and \( v \).

Degree of vertex 1? 2

Degree of vertex \( u \) is number of incident edges.

Equals number of neighbors in simple graph.

Directed graph?

In-degree of 10? 1  Out-degree of 10? 3
Graph Concepts and Definitions.

Graph: $G = (V, E)$

- neighbors
- adjacent
- degree
- incident
- in-degree
- out-degree

Neighbors of 10? 1, 5, 7, 8.

- $u$ is neighbor of $v$ if $\{u, v\} \in E$.

Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

- Edge $\{u, v\}$ is incident to $u$ and $v$.

Degree of vertex 1? 2

- Degree of vertex $u$ is number of incident edges.
- Equals number of neighbors in simple graph.

Directed graph?

- In-degree of 10? 1
- Out-degree of 10? 3
Graph Concepts and Definitions.

Graph: $G = (V, E)$
Graph Concepts and Definitions.

Graph: $G = (V, E)$
neighbors, adjacent, degree, incident, in-degree, out-degree
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

neighbors, adjacent, degree, incident, in-degree, out-degree

Edge \((8, 5)\) is incident to:

(A) Vertex 8.
(B) Vertex 5.
(C) Edge \((8, 5)\).
(D) Edge \((8, 4)\).
(E) Vertex 10.
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)
neighbors, adjacent, degree, incident, in-degree, out-degree

**Edge** \((8, 5)\) is incident to:

(A) Vertex 8.
(B) Vertex 5.
(C) Edge \((8, 5)\).
(D) Edge \((8, 4)\).
(E) Vertex 10.

(A) and (B) are true.
Graph Concepts and Definitions.

Graph: $G = (V, E)$
neighbors, adjacent, degree, incident, in-degree, out-degree

**Edge (8, 5) is incident to:**

(A) Vertex 8.
(B) Vertex 5.
(C) Edge (8, 5).
(D) Edge (8, 4).
(E) Vertex 10.

(A) and (B) are true.

**The degree of a vertex is:**

(A) The number of edges incident to it.
(B) The number of neighbors of $v$.
(C) Is the number of vertices in its connected component.
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)
neighbors, adjacent, degree, incident, in-degree, out-degree

Edge \((8, 5)\) is incident to:
(A) Vertex 8.
(B) Vertex 5.
(C) Edge \((8, 5)\).
(D) Edge \((8, 4)\).
(E) Vertex 10.

(A) and (B) are true.

The degree of a vertex is:
(A) The number of edges incident to it.
(B) The number of neighbors of \( v \).
(C) Is the number of vertices in its connected component.

(A) and (B) are true.
Sum of degrees?

The sum of the vertex degrees is equal to
Sum of degrees?

The sum of the vertex degrees is equal to
(A) the total number of vertices, $|V|$.
Sum of degrees?

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.

Not (A)!

Triangle.

Not (B)!

Triangle.

What?

For triangle number of edges is 3, the sum of degrees is 6.

Could sum always be...

(A) $2|E|$?

(B) $2|V|$?

(A) is true.
Sum of degrees?

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.  
(B) the total number of edges, $|E|$.  
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.
Sum of degrees?

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)!
The sum of the vertex degrees is equal to

(A) the total number of vertices, \(|V|\).
(B) the total number of edges, \(|E|\).
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)! Triangle.
The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)! Triangle.
Not (B)!
The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)! Triangle.
Not (B)! Triangle.
The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)! Triangle.
Not (B)! Triangle.
The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)! Triangle.
Not (B)! Triangle.

What?
Sum of degrees?

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)! Triangle.
Not (B)! Triangle.

What? For triangle number of edges is 3, the sum of degrees is 6.
Sum of degrees?

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)! Triangle.
Not (B)! Triangle.

What? For triangle number of edges is 3, the sum of degrees is 6.

Could sum always be...
Sum of degrees?

The sum of the vertex degrees is equal to

- (A) the total number of vertices, $|V|$.
- (B) the total number of edges, $|E|$.
- (C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)! Triangle.
Not (B)! Triangle.

What? For triangle number of edges is 3, the sum of degrees is 6.

Could sum always be...

- (A) $2|E|$? ..
Sum of degrees?

The sum of the vertex degrees is equal to

(A) the total number of vertices, \(|V|\).
(B) the total number of edges, \(|E|\).
(C) What?

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(A) \(2|E|\)? ..
(B) \(2|V|\)?
Sum of degrees?

The sum of the vertex degrees is equal to

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Not (B)! Triangle.

What? For triangle number of edges is 3, the sum of degrees is 6.

Could sum always be...

(A) \(2|E|\)? ..
(B) \(2|V|\)?
(A) is true.
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).

degree of \(u\) number of edges incident to \(u\)
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:
- edge, \((u, v)\), is **incident** to endpoints, \(u\) and \(v\).
- degree of \(u\) number of edges **incident** to \(u\)

Let’s count incidences in two ways.
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

- edge, \((u, v)\), is *incident* to endpoints, \(u\) and \(v\).
- degree of \(u\) number of edges *incident* to \(u\)

Let’s count incidences in two ways.
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:
edge, \((u, v)\), is \textit{incident} to endpoints, \(u\) and \(v\).
degree of \(u\) number of edges \textit{incident} to \(u\)

Let's count incidences in two ways.
How many \textit{incidences} does each edge contribute?
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).

degree of \(u\) number of edges incident to \(u\)

Let’s count incidences in two ways.

- How many incidences does each edge contribute? 2.
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

- edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).
- degree of \(u\) number of edges incident to \(u\)

Let’s count incidences in two ways.

- How many incidences does each edge contribute? 2.
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).

degree of \(u\) number of edges incident to \(u\)

Let’s count incidences in two ways.

How many incidences does each edge contribute? 2.

Total Incidences?
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

degree of $u$ number of edges incident to $u$

Let's count incidences in two ways.

How many incidences does each edge contribute? 2.

Total Incidences? $|E|$ edges, 2 each. $\rightarrow 2|E|$
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).

degree of \(u\) number of edges incident to \(u\)

Let's count incidences in two ways.

How many incidences does each edge contribute? 2.

Total Incidences? \(|E|\) edges, 2 each. \(\rightarrow 2|E|\)
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:
edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).
degree of \(u\) number of edges incident to \(u\)

Let’s count incidences in two ways.

How many incidences does each edge contribute? 2.

Total Incidences? \(|E|\) edges, 2 each. \(\rightarrow 2|E|\)

What is degree \(v\)?
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).

degree of \(u\) number of edges incident to \(u\)

Let's count incidences in two ways.

   How many incidences does each edge contribute? 2.

Total Incidences? \(|E|\) edges, 2 each. \(\rightarrow 2|E|\)

What is degree \(v\)? Incidences corresponding to \(v\)!
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:
edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).
degree of \(u\) number of edges incident to \(u\)

Let's count incidences in two ways.

How many incidences does each edge contribute? 2.

Total Incidences? \(|E|\) edges, 2 each. \(\rightarrow 2|E|\)

What is degree \(v\)? Incidences corresponding to \(v\)!
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:
edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).
degree of \(u\) number of edges incident to \(u\)

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How many incidences does each edge contribute? 2.

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What is degree \(v\)? Incidences corresponding to \(v\)!

Total Incidences?
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).

degree of \(u\) number of edges incident to \(u\)

Let's count incidences in two ways.

How many incidences does each edge contribute? 2.

Total Incidences? \(|E|\) edges, 2 each. \(\rightarrow 2|E|\)

What is degree \(v\)? Incidences corresponding to \(v\)!

Total Incidences? The sum over vertices of degrees!
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:
edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).
degree of \(u\) number of edges incident to \(u\)

Let’s count incidences in two ways.

How many incidences does each edge contribute? 2.

Total Incidences? \(|E|\) edges, 2 each. \(\rightarrow 2|E|\)

What is degree \(v\)? Incidences corresponding to \(v\)!

Total Incidences? The sum over vertices of degrees!
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

- edge, \((u, v)\), is **incident** to endpoints, \(u\) and \(v\).
- degree of \(u\) number of edges **incident** to \(u\)

Let’s count incidences in two ways.

- How many **incidences** does each edge contribute? 2.
  - Total Incidences? \(|E|\) edges, 2 each. \(\rightarrow 2|E|\)

- What is degree \(v\)? Incidences corresponding to \(v\)!
  - Total Incidences? The sum over vertices of degrees!

**Thm:** Sum of vertex degrees is \(2|E|\).
Poll: Proof of “handshake” lemma.

What’s true?

(A) The number of edge-vertex incidences for an edge \( e \) is 2.
(B) The total number of edge-vertex incidences is \( |V| \).
(C) The total number of edge-vertex incidences is \( 2|E| \).
(D) The number of edge-vertex incidences for a vertex \( v \) is its degree.
(E) The sum of degrees is \( 2|E| \).
(F) The total number of edge-vertex incidences is the sum of the degrees.
Poll: Proof of “handshake” lemma.

What’s true?

(A) The number of edge-vertex incidences for an edge e is 2.
(B) The total number of edge-vertex incidences is $|V|$.
(C) The total number of edge-vertex incidences is $2|E|$.
(D) The number of edge-vertex incidences for a vertex v is its degree.
(E) The sum of degrees is $2|E|$.
(F) The total number of edge-vertex incidences is the sum of the degrees.

(A), (C), (D), (E), and (F).
A path in a graph is a sequence of edges.
A path in a graph is a sequence of edges. Path?

Path?

Path?

Path:

A path in a graph is a sequence of edges. Path?

A path in a graph is a sequence of edges. Path?

Path?

Path?

Quick Check!

Length of path?

$k$ vertices or $k - 1$ edges.

Quick Check!

Length of cycle?

$k - 1$ vertices and edges!
A path in a graph is a sequence of edges.

Path? \{1, 10\}, \{8, 5\}, \{4, 5\} ?
A path in a graph is a sequence of edges.

Path? \{1, 10\}, \{8, 5\}, \{4, 5\}? No!
Paths, walks, cycles, tour.

A path in a graph is a sequence of edges.
Path? \{1,10\}, \{8,5\}, \{4,5\}? No!
Path? 

Quick Check!

Length of path? \(k\) vertices or \(k-1\) edges.
Cycle: Path from \(v_1\) to \(v_{k-1}\), + edge \((v_{k-1}, v_1)\)
Length of cycle? \(k-1\) vertices and edges!
Path is usually simple.
No repeated vertex!
Walk is sequence of edges with possible repeated vertex or edge.
Tour is walk that starts and ends at the same node.
Quick Check!
Path is to Walk as Cycle is to ??
Tour!
A path in a graph is a sequence of edges.
Path? \{1,10\}, \{8,5\}, \{4,5\} ? No!
Path? \{1,10\}, \{10,5\}, \{5,4\}, \{4,11\}?
A path in a graph is a sequence of edges.

Path?  
\{1,10\}, \{8,5\}, \{4,5\}  ? No!
Path?  
\{1,10\}, \{10,5\}, \{5,4\}, \{4,11\}? Yes!
A path in a graph is a sequence of edges.

Path? \{1, 10\}, \{8, 5\}, \{4, 5\}? No!
Path? \{1, 10\}, \{10, 5\}, \{5, 4\}, \{4, 11\}? Yes!
Path: (\(v_1, v_2\), (\(v_2, v_3\)), \ldots (\(v_{k-1}, v_k\)).
Paths, walks, cycles, tour.

A path in a graph is a sequence of edges.

Path? \{1,10\}, \{8,5\}, \{4,5\}? No!

Path? \{1,10\}, \{10,5\}, \{5,4\}, \{4,11\}? Yes!

Path: \((v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)\).

Quick Check!
A path in a graph is a sequence of edges.

Path? \( \{1,10\}, \{8,5\}, \{4,5\} \) ? No!
Path? \( \{1,10\}, \{10,5\}, \{5,4\}, \{4,11\} \)? Yes!

Path: \((v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)\).

Quick Check! Length of path?
A path in a graph is a sequence of edges.

Path? \{1, 10\}, \{8, 5\}, \{4, 5\}? No!
Path? \{1, 10\}, \{10, 5\}, \{5, 4\}, \{4, 11\}? Yes!

Path: \((v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)\).
Quick Check! Length of path? \(k\) vertices
A path in a graph is a sequence of edges.
- Path? \{1,10\}, \{8,5\}, \{4,5\} ? No!
- Path? \{1,10\}, \{10,5\}, \{5,4\}, \{4,11\}? Yes!

Path: \((v_1,v_2),(v_2,v_3),\ldots,(v_{k-1},v_k)\).
Quick Check! Length of path? \(k\) vertices or \(k-1\) edges.
A path in a graph is a sequence of edges.

Path? \{1,10\}, \{8,5\}, \{4,5\}? No!
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Quick Check! Length of path? \(k\) vertices or \(k - 1\) edges.

Cycle: Path from \(v_1\) to \(v_{k-1}\), + edge \((v_{k-1}, v_1)\) Length of cycle?
A path in a graph is a sequence of edges.
Path? \{1,10\}, \{8,5\}, \{4,5\}? No!
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**Quick Check!** Length of path? \(k\) vertices or \(k-1\) edges.

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Path is usually simple.
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Paths, walks, cycles, tour.

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Walk is sequence of edges with possible repeated vertex or edge.
A path in a graph is a sequence of edges.

- Path? \{1,10\}, \{8,5\}, \{4,5\}? No!
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**Walk** is sequence of edges with possible repeated vertex or edge.

**Tour** is walk that starts and ends at the same node.
A path in a graph is a sequence of edges.

Path? \{1,10\}, \{8,5\}, \{4,5\}? No!
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Quick Check!
Path is to Walk as Cycle is to ??
A path in a graph is a sequence of edges.

Path?  \{1,10\}, \{8,5\}, \{4,5\}  \?  No!
Path?  \{1,10\}, \{10,5\}, \{5,4\}, \{4,11\}? Yes!

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Cycle: Path from \(v_1\) to \(v_{k-1}\), + edge \((v_{k-1},v_1)\) Length of cycle?  \(k-1\) vertices and edges!

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Quick Check!
Path is to Walk as Cycle is to ?? Tour!
A path in a graph is a sequence of edges.

Path? \(\{1,10\}, \{8,5\}, \{4,5\}\) ? No!
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Walk is sequence of edges with possible repeated vertex or edge.

Tour is walk that starts and ends at the same node.

Quick Check!
Path is to Walk as Cycle is to ?? Tour!
Directed Paths.

Path: \((v_1, v_2), (v_2, v_3), \ldots, (v_{k-1}, v_k)\). Paths, walks, cycles, tours... are analogous to undirected now.
Directed Paths.

Path: \((v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)\).
Directed Paths.

Path: \((v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)\).
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Path: \((v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)\).
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Path: \((v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)\). Paths,
Directed Paths.

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Paths, walks,
Directed Paths.

Path: \((v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)\).

Paths, walks, cycles,
Directed Paths.

Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$. Paths, walks, cycles, tours
Directed Paths.

Path: \((v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)\).

Paths, walks, cycles, tours ... are analogous to undirected now.
Connectivity: undirected graph.

$u$ and $v$ are connected if there is a path between $u$ and $v$. 

Is graph connected? 

Yes? 

No? 

Proof: Use path from $u$ to $x$ and then from $x$ to $v$. 

May not be simple! Either modify definition to walk. Or cut out cycles.
Connectivity: undirected graph.

$u$ and $v$ are connected if there is a path between $u$ and $v$.

A connected graph is a graph where all pairs of vertices are connected.
Connectivity: undirected graph.

$u$ and $v$ are connected if there is a path between $u$ and $v$.

A connected graph is a graph where all pairs of vertices are connected.

If one vertex $x$ is connected to every other vertex.
Connectivity: undirected graph.

\[ u \text{ and } v \] are connected if there is a path between \( u \) and \( v \).

A connected graph is a graph where all pairs of vertices are connected.

If one vertex \( x \) is connected to every other vertex.
Is graph connected?
Connectivity: undirected graph.

$u$ and $v$ are connected if there is a path between $u$ and $v$.

A connected graph is a graph where all pairs of vertices are connected.

If one vertex $x$ is connected to every other vertex.

Is graph connected? Yes?
Connectivity: undirected graph.

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A connected graph is a graph where all pairs of vertices are connected.

If one vertex $x$ is connected to every other vertex. Is graph connected? Yes? No?
Connectivity: undirected graph.

\( u \) and \( v \) are **connected** if there is a path between \( u \) and \( v \).

A connected graph is a graph where all pairs of vertices are connected.

If one vertex \( x \) is connected to every other vertex.

Is graph connected? Yes? No?

Proof:
Connectivity: undirected graph.

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A connected graph is a graph where all pairs of vertices are connected.

If one vertex $x$ is connected to every other vertex.

Is graph connected? Yes? No?

Proof: Use path from $u$ to $x$ and then from $x$ to $v$. 

$\square$
Connectivity: undirected graph.

$u$ and $v$ are connected if there is a path between $u$ and $v$.

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Either modify definition to walk.

Or cut out cycles.
Connectivity: undirected graph.

$u$ and $v$ are connected if there is a path between $u$ and $v$.

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Is graph connected? Yes? No?

Proof: Use path from $u$ to $x$ and then from $x$ to $v$.

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Connectivity: undirected graph.

\[ u \text{ and } v \text{ are connected if there is a path between } u \text{ and } v. \]

A connected graph is a graph where all pairs of vertices are connected.

If one vertex \( x \) is connected to every other vertex.

Is graph connected? Yes? No?

Proof: Use path from \( u \) to \( x \) and then from \( x \) to \( v \).

May not be simple!

Either modify definition to walk.

Or cut out cycles.
Connected Components: Quiz.

Is graph above connected?

Yes!

How about now?

No!

Connected Components:

\[
\text{\{1\}}, \text{\{10, 7, 5, 8, 4, 11, 3, 2, 6\}}, \text{\{2, 9\}}.
\]

Connected component - maximal set of connected vertices.

Quick Check: Is \{10, 7, 5\} a connected component?

No.
Connected Components: Quiz.

Is graph above connected? Yes!

Connected components:
1. \{10, 7, 5\}
2. \{4, 3, 8, 11\}
3. \{2, 9, 6\}
Is graph above connected? Yes!

How about now?

Connected Components: Quiz.

Connected component - maximal set of connected vertices.

Quick Check: Is \{10, 7, 5\} a connected component? No.
Connected Components: Quiz.

Is graph above connected? Yes!
How about now? No!

Connected component - maximal set of connected vertices.

Quick Check: Is {10, 7, 5} a connected component? No.
Is graph above connected? Yes!
How about now? No!

Connected Components?
Connected Components: Quiz.

Is graph above connected? Yes!

How about now? No!

Connected Components? \{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}.
Connected Components: Quiz.

Is graph above connected? Yes!

How about now? No!

**Connected Components**? \{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}.

Connected component - maximal set of connected vertices.
Connected Components: Quiz.

Is graph above connected? Yes!

How about now? No!

Connected Components? \{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}.

Connected component - maximal set of connected vertices.

Quick Check: Is \{10, 7, 5\} a connected component?
Connected Components: Quiz.

Is graph above connected? Yes!

How about now? No!

**Connected Components?** \{ 1 \}, \{ 10, 7, 5, 8, 4, 3, 11 \}, \{ 2, 9, 6 \}.

Connected component - maximal set of connected vertices.

Quick Check: Is \{ 10, 7, 5 \} a connected component? No.
Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?
Konigsberg bridges problem.

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Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

Can you draw a tour in the graph where you visit each edge once?
Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

Can you draw a tour in the graph where you visit each edge once? Yes?
Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

Can you draw a tour in the graph where you visit each edge once? Yes? No?
Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

Can you draw a tour in the graph where you visit each edge once? Yes? No? We will see!
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.
An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected.
**Eulerian Tour**

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit.
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit. Uses two incident edges per visit.
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\iff$ connected and all even degree.

Eulerian Tour is connected so graph is connected.
Tour enters and leaves vertex $v$ on each visit.
Uses two incident edges per visit. Tour uses all incident edges.
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected.
Tour enters and leaves vertex $v$ on each visit.
Uses two incident edges per visit. Tour uses all incident edges.
Therefore $v$ has even degree.
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\Rightarrow$ connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit. Uses two incident edges per visit. Tour uses all incident edges. Therefore $v$ has even degree.
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**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian \(\implies\) connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex \(v\) on each visit. Uses two incident edges per visit. Tour uses all incident edges. Therefore \(v\) has even degree.

When you enter,
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit. Uses two incident edges per visit. Tour uses all incident edges. Therefore $v$ has even degree.

When you enter, you can leave.
An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit. Uses two incident edges per visit. Tour uses all incident edges. Therefore $v$ has even degree.

When you enter, you can leave.
Eulerian Tour

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Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit. Uses two incident edges per visit. Tour uses all incident edges. Therefore $v$ has even degree.

When you enter, you can leave.
An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit. Uses two incident edges per visit. Tour uses all incident edges. Therefore $v$ has even degree.

When you enter, you can leave. For starting node,
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit. Uses two incident edges per visit. Tour uses all incident edges. Therefore $v$ has even degree.

When you enter, you can leave. For starting node, tour leaves first.
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit. Uses two incident edges per visit. Tour uses all incident edges. Therefore $v$ has even degree.

When you enter, you can leave. For starting node, tour leaves first ....then enters at end.
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit. Uses two incident edges per visit. Tour uses all incident edges. Therefore $v$ has even degree.

When you enter, you can leave. For starting node, tour leaves first ....then enters at end.
Eulerian Tour

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**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected.
Tour enters and leaves vertex $v$ on each visit.
Uses two incident edges per visit. Tour uses all incident edges.
Therefore $v$ has even degree.

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.
Not The Hotel California.
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.
Finding a tour!

**Proof of if: Even + connected \(\implies\) Eulerian Tour.**
We will give an algorithm. First by picture.
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v(1)$ on “unused” edges
Finding a tour!

Proof of if: Even + connected $\iff$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v(1)$ on “unused” edges

![Graph Diagram]

1. $v_1 = 1$
2. $v_2 = 10$
3. $v_3 = 4$
4. $v_4 = 2$

5. Splice together.
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges

![Graph diagram]
Finding a tour!

Proof of if: Even + connected \iff\ Eulerian Tour. We will give an algorithm. First by picture.

1. Take a walk starting from \( v(1) \) on “unused” edges
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges

![Diagram of a graph with nodes labeled 1 to 11 and edges connecting them. The walk starts from node 1 and moves through the edges without revisiting any node, ending at node 11.]
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v\ (1)$ on “unused” edges
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.

2. Remove tour, $C$.

3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.

4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$.

5. Splice together.
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.

\begin{center}
\begin{tikzpicture}
\node[circle,draw] (1) at (0,0) {1};
\node[circle,draw] (2) at (1,-1) {2};
\node[circle,draw] (3) at (1,1) {3};
\node[circle,draw] (4) at (2,0) {4};
\node[circle,draw] (5) at (2,2) {5};
\node[circle,draw] (6) at (3,-1) {6};
\node[circle,draw] (7) at (-1,1) {7};
\node[circle,draw] (8) at (-1,-1) {8};
\node[circle,draw] (9) at (2,-2) {9};
\node[circle,draw] (10) at (-2,0) {10};
\node[circle,draw] (11) at (-2,2) {11};
\draw[->] (1) to (2);
\draw[->] (2) to (4);
\draw[->] (4) to (11);
\draw[->] (5) to (11);
\draw[->] (3) to (5);
\draw[->] (5) to (7);
\draw[->] (7) to (8);
\draw[->] (8) to (1);
\draw[->] (10) to (8);
\draw[->] (10) to (6);
\draw[->] (6) to (2);
\draw[->] (9) to (6);
\end{tikzpicture}
\end{center}
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.

1.10 10
8 5
7

4 11

3

9

2 6

1
Proof of if: Even + connected  $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components. Each is touched by $C$. 

1. $8$  
   2. $4$  
   3. $11$  
   4. $5$  
   5. $3$  
   6. $2$  
   7. $1$  
   8. $10$  
   9. $9$  
10. $6$
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why?

\[
\begin{align*}
1 & \quad 2 & \quad 6 & \quad 7 & \quad 8 & \quad 4 & \quad 11 \\
3 & \quad 9 & \quad 5 & \quad 10 & \quad 1 & \quad & \\
\end{align*}
\]
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v_1$ on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
Finding a tour!

Proof of if: Even + connected \( \implies \) Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from \( v \) (1) on “unused” edges
   ... till you get back to \( v \).
2. Remove tour, \( C \).
3. Let \( G_1, \ldots, G_k \) be connected components.
   Each is touched by \( C \).
   Why? \( G \) was connected.
   Let \( v_i \) be (first) node in \( G_i \) touched by \( C \).
   Example: \( v_1 = 1 \),
Proof of if: Even + connected $\Rightarrow$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1$, $v_2 = 10$,
Proof of if: Even + connected \( \implies \) Eulerian Tour.

We will give an algorithm. First by picture.

1. Take a walk starting from \( v \) (1) on “unused” edges
   ... till you get back to \( v \).
2. Remove tour, \( C \).
3. Let \( G_1, \ldots, G_k \) be connected components.
   Each is touched by \( C \).
   Why? \( G \) was connected.

   Let \( v_i \) be (first) node in \( G_i \) touched by \( C \).
   Example: \( v_1 = 1, v_2 = 10, v_3 = 4, \ldots \).
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v_1$ (1) on “unused” edges 
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components. 
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$. 
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$
Finding a tour!

**Proof of if: Even + connected \(\implies\) Eulerian Tour.**

We will give an algorithm. First by picture.

1. Take a walk starting from \(v\) (1) on “unused” edges 
   ... till you get back to \(v\).
2. Remove tour, \(C\).
3. Let \(G_1, \ldots, G_k\) be connected components.
   Each is touched by \(C\).
   Why? \(G\) was connected.
   Let \(v_i\) be (first) node in \(G_i\) touched by \(C\).
   Example: \(v_1 = 1, v_2 = 10, v_3 = 4, v_4 = 2\).
4. Recurse on \(G_1, \ldots, G_k\) starting from \(v_i\)
Finding a tour!

**Proof of if: Even + connected \(\implies\) Eulerian Tour.**

We will give an algorithm. First by picture.

1. Take a walk starting from \(v(1)\) on “unused” edges...
   till you get back to \(v\).

2. Remove tour, \(C\).

3. Let \(G_1, \ldots, G_k\) be connected components.
   Each is touched by \(C\).
   Why? \(G\) was connected.
   Let \(v_i\) be (first) node in \(G_i\) touched by \(C\).
   Example: \(v_1 = 1, v_2 = 10, v_3 = 4, v_4 = 2\).

4. Recurse on \(G_1, \ldots, G_k\) starting from \(v_i\)

5. Splice together.
Proof of if: Even + connected $\implies$ Eulerian Tour.

We will give an algorithm. First by picture.

1. Take a walk starting from $v_1$ on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$
5. Splice together.
   1, 10
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$
5. Splice together.
   $1,10,7,8,5,10$
Finding a tour!

**Proof of if: Even + connected \( \implies \text{Eulerian Tour.} \)**

We will give an algorithm. First by picture.

1. Take a walk starting from \( v \) (1) on “unused” edges
   ... till you get back to \( v \).
2. Remove tour, \( C \).
3. Let \( G_1, \ldots, G_k \) be connected components.
   Each is touched by \( C \).
   **Why?** \( G \) was connected.
   Let \( v_i \) be (first) node in \( G_i \) touched by \( C \).
   Example: \( v_1 = 1, v_2 = 10, v_3 = 4, v_4 = 2 \).
4. Recurse on \( G_1, \ldots, G_k \) starting from \( v_i \)
5. Splice together.
   \( 1,10,7,8,5,10,8,4 \)
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v(1)$ on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1, v_2 = 10, v_3 = 4, v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$.
5. Splice together.
   $1,10,7,8,5,10,8,4,3,11,4$
Finding a tour!

Proof of if: Even + connected \( \implies \) Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from \( v \) (1) on “unused” edges
   ... till you get back to \( v \).
2. Remove tour, \( C \).
3. Let \( G_1, \ldots, G_k \) be connected components. Each is touched by \( C \).
   Why? \( G \) was connected.
   Let \( v_i \) be (first) node in \( G_i \) touched by \( C \).
   Example: \( v_1 = 1, v_2 = 10, v_3 = 4, v_4 = 2 \).
4. Recurse on \( G_1, \ldots, G_k \) starting from \( v_i \)
5. Splice together.
   1,10,7,8,5,10,8,4,3,11,4,5,2
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$
5. Splice together.
   $1,10,7,8,5,10,8,4,3,11,4,5,2,6,9,2$
Finding a tour!

Proof of if: Even + connected \(\implies\) Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from \(v\) (1) on “unused” edges
   ... till you get back to \(v\).
2. Remove tour, \(C\).
3. Let \(G_1, \ldots, G_k\) be connected components.
   Each is touched by \(C\).
   Why? \(G\) was connected.
   Let \(v_i\) be (first) node in \(G_i\) touched by \(C\).
   Example: \(v_1 = 1, v_2 = 10, v_3 = 4, v_4 = 2\).
4. Recurse on \(G_1, \ldots, G_k\) starting from \(v_i\)
5. Splice together.
   \(1,10,7,8,5,10,8,4,3,11,4,5,2,6,9,2\) and to 1!
Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node $v$, until you get back to $v$. 
Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node $v$, until you get back to $v$.

**Claim:** Do get back to $v$!
Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node $v$, until you get back to $v$.

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**Proof of Claim:** Even degree.
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Let components be $G_1, \ldots, G_k$. 

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Mark correct statements for a connected graph where all vertices have even degree. (Below, we use tour to mean uses an edge exactly once, but may involve a vertex several times.)

(A) Removing a tour leaves a graph of even degree.
(B) A tour connecting a set of connected components, each with a Eulerian tour is really cool! Eulerian even.
(C) There is no hotel california in this graph.
(D) After removing a set of edges $E'$ in a connected graph, every connected component is incident to an edge in $E'$.
(E) If one walks on new edges, starting at $v$, one must eventually get back to $v$.
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Only (F) is false.
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A Tree, a tree.

Graph \( G = (V, E) \).

Binary Tree!

More generally.
Trees.

Definitions:

A connected graph without a cycle.

A connected graph with $|V| - 1$ edges.

A connected graph where any edge removal disconnects it.

A connected graph where any edge addition creates a cycle.

Some trees.

No cycle and connected? Yes.

$|V| - 1$ edges and connected? Yes.

Removing any edge disconnects it. Harder to check. But yes.

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To tree or not to tree!
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![Tree 1](image1.png) ![Tree 2](image2.png) ![Tree 3](image3.png)

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To tree or not to tree!
Equivalence of Definitions.

**Theorem:**
“G connected and has $|V| - 1$ edges” $\equiv$
“G is connected and has no cycles.”
Equivalence of Definitions.

**Theorem:**
“$G$ connected and has $|V| - 1$ edges” $\equiv$
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**Lemma:** If $v$ is degree 1 in connected graph $G$, $G - v$ is connected.

**Proof:**
For $x \neq v, y \neq v \in V$, 

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For $x \neq v, y \neq v \in V$, there is path between $x$ and $y$ in $G$ since connected.
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```
```

\[ v \quad y \]
\[ x \]

```

Proof of only if.

Thm:
“G connected and has $|V| - 1$ edges” $\implies$
“G is connected and has no cycles.”

Proof of $\implies$:

"G connected and has $|V| - 1$ edges" $\implies$
"G is connected and has no cycles."
Proof of only if.

Thm:
“\( G \) connected and has \( |V| - 1 \) edges” \( \implies \)
“\( G \) is connected and has no cycles.”

Proof of \( \implies \) : By induction on \( |V| \).
Proof of only if.

**Thm:**
“G connected and has $|V| - 1$ edges” $\implies$ “G is connected and has no cycles.”

**Proof of $\implies$:** By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.
Proof of only if.

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*Base Case:* $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

*Induction Step:*
Proof of only if.

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**Proof of $\implies$:** By induction on $|V|$.

**Base Case:** $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

**Induction Step:**

**Claim:** There is a degree 1 node.
Proof of only if.

Thm:
“G connected and has $|V| - 1$ edges” $\implies$
“G is connected and has no cycles.”

Proof of $\implies$: By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
  Proof: First, connected $\implies$ every vertex degree $\geq 1$. 
Proof of only if.

Thm:
“G connected and has $|V| - 1$ edges” $\implies$
“G is connected and has no cycles.”

Proof of $\implies$ : By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree $\geq 1$.
Sum of degrees is $2|E| = 2(|V| - 1) = 2|V| - 2$
Proof of only if.

**Thm:**
“G connected and has \(|V| − 1\) edges” \(\implies\)
“G is connected and has no cycles.”

**Proof of \(\implies\):** By induction on \(|V|\).

- **Base Case:** \(|V| = 1\). \(0 = |V| − 1\) edges and has no cycles.

- **Induction Step:**
  **Claim:** There is a degree 1 node.
  **Proof:** First, connected \(\implies\) every vertex degree \(\geq 1\).
  Sum of degrees is \(2|E| = 2(|V| − 1) = 2|V| − 2\)
  Average degree \(2 − 2/|V|\)
Proof of only if.

Thm:
“G connected and has \(|V| - 1\) edges” \(\implies\) “G is connected and has no cycles.”

Proof of \(\implies\): By induction on \(|V|\).
Base Case: \(|V| = 1\). 0 = \(|V| - 1\) edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected \(\implies\) every vertex degree \(\geq 1\).
Sum of degrees is \(2|E| = 2(|V| - 1) = 2|V| - 2\)
Average degree \(2 - 2/|V|\)
Not everyone is bigger than average!
Proof of only if.

Thm:
“G connected and has $|V| - 1$ edges” $\implies$ “G is connected and has no cycles.”

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Thm:
“G connected and has $|V| - 1$ edges” $\implies$ “G is connected and has no cycles.”

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Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree $\geq 1$.
Sum of degrees is $2|E| = 2(|V| - 1) = 2|V| - 2$
Average degree $2 - 2/|V|$
Not everyone is bigger than average!
By degree 1 removal lemma, $G - v$ is connected.
Proof of only if.

**Thm:**
“G connected and has \(|V|−1\) edges” \(\iff\) “G is connected and has no cycles.”

**Proof of \(\iff\):** By induction on \(|V|\).
Base Case: \(|V| = 1\). \(0 = |V|−1\) edges and has no cycles.

Induction Step:

**Claim:** There is a degree 1 node.

**Proof:** First, connected \(\iff\) every vertex degree \(\geq 1\).

- Sum of degrees is \(2|E| = 2(|V|−1) = 2|V|−2\)
- Average degree \(2−2/|V|\)
  - Not everyone is bigger than average!

By degree 1 removal lemma, \(G−v\) is connected.
\(G−v\) has \(|V|−1\) vertices and \(|V|−2\) edges so by induction
Proof of only if.

Thm: “G connected and has $|V| - 1$ edges” $\implies$ “G is connected and has no cycles.”

Proof of $\implies$: By induction on $|V|$.

Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:

Claim: There is a degree 1 node.

Proof: First, connected $\implies$ every vertex degree $\geq 1$.

Sum of degrees is $2|E| = 2(|V| - 1) = 2|V| - 2$

Average degree $2 - 2/|V|$

Not everyone is bigger than average!

By degree 1 removal lemma, $G - v$ is connected.

$G - v$ has $|V| - 1$ vertices and $|V| - 2$ edges so by induction $\implies$ no cycle in $G - v$. 

$\blacksquare$
Proof of only if.

Thm:
“\( G \) connected and has \(|V| - 1\) edges” \( \implies \)
“\( G \) is connected and has no cycles.”

Proof of \( \implies \): By induction on \(|V|\).
Base Case: \(|V| = 1\). \(0 = |V| - 1\) edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected \( \implies \) every vertex degree \( \geq 1\).
Sum of degrees is \(2|E| = 2(|V| - 1) = 2|V| - 2\)
Average degree \(2 - 2/|V|\)
Not everyone is bigger than average!

By degree 1 removal lemma, \(G - v\) is connected.
\(G - v\) has \(|V| - 1\) vertices and \(|V| - 2\) edges so by induction
\( \implies \) no cycle in \(G - v\).
And no cycle in \(G\) since degree 1 cannot participate in cycle.
Proof of only if.

Thm:
“G connected and has $|V| - 1$ edges” $\implies$
“G is connected and has no cycles.”

Proof of $\implies$: By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
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Sum of degrees is $2|E| = 2(|V| - 1) = 2|V| - 2$
Average degree $2 - 2/|V|$
Not everyone is bigger than average!

By degree 1 removal lemma, $G - v$ is connected.
$G - v$ has $|V| - 1$ vertices and $|V| - 2$ edges so by induction
$\implies$ no cycle in $G - v$.
And no cycle in $G$ since degree 1 cannot participate in cycle.
Proof of if

**Thm:**
“G is connected and has no cycles”

⇒ “G connected and has $|V| - 1$ edges”

**Proof:**

Walk from a vertex using untraversed edges.
Until get stuck.

Claim:
Degree 1 vertex.

Proof of Claim:
Can’t visit more than once since no cycle.
Entered.
Didn’t leave.
Only one incident edge.

Removing node doesn’t create cycle.
New graph is connected.

Removing degree 1 node doesn’t disconnect from Degree 1 lemma.
By induction $G - v$ has $|V| - 2$ edges.

$G$ has one more or $|V| - 1$ edges.
Proof of if

Thm:
“G is connected and has no cycles”
⇒ “G connected and has $|V| - 1$ edges”

Proof:
Walk from a vertex using untraversed edges.
Thm:
“G is connected and has no cycles”
\[ \implies \text{“G connected and has } |V| - 1 \text{ edges”} \]

Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Proof of if

**Thm:**
"G is connected and has no cycles"

$$\implies \text{“G connected and has } |V| - 1 \text{ edges”}$$

**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.

**Claim:** Degree 1 vertex.
Proof of if

**Thm:**
“G is connected and has no cycles”
\[ \implies \text{“G connected and has } |V| - 1 \text{ edges”} \]

**Proof:**
Walk from a vertex using untraversed edges. Until get stuck.

**Claim:** Degree 1 vertex.

**Proof of Claim:**
Can’t visit more than once since no cycle.
Thm:
"G is connected and has no cycles"
⇒ "G connected and has |V| − 1 edges"

Proof:
Walk from a vertex using untraversed edges.
Until get stuck.

Claim: Degree 1 vertex.

Proof of Claim:
Can’t visit more than once since no cycle.
Entered.
Proof of if

**Thm:**
“G is connected and has no cycles”
⇒ “G connected and has $|V| - 1$ edges”

**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.

**Claim:** Degree 1 vertex.

**Proof of Claim:**
Can’t visit more than once since no cycle.
Entered. Didn’t leave.
Proof of if

**Thm:**
“G is connected and has no cycles”
\[ \implies \text{“G connected and has } |V| - 1 \text{ edges”} \]

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Walk from a vertex using untraversed edges.
Until get stuck.

**Claim:** Degree 1 vertex.

**Proof of Claim:**
Can’t visit more than once since no cycle.
Entered. Didn’t leave. Only one incident edge.
Proof of if

**Thm:**
“G is connected and has no cycles”
\[ \implies \text{“G connected and has } |V| - 1 \text{ edges”} \]

**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.

**Claim:** Degree 1 vertex.

**Proof of Claim:**
Can’t visit more than once since no cycle.
Entered. Didn’t leave. Only one incident edge.
Removing node doesn’t create cycle.
Proof of if

Thm:
“G is connected and has no cycles”
\[ \implies \text{“G connected and has } |V| - 1 \text{ edges”} \]

Proof:
Walk from a vertex using untraversed edges.
Until get stuck.

Claim: Degree 1 vertex.

Proof of Claim:
Can’t visit more than once since no cycle.
Entered. Didn’t leave. Only one incident edge.

Removing node doesn’t create cycle.
New graph is connected.
Proof of if

Thm:
“G is connected and has no cycles”
\[ \implies \text{“G connected and has } |V| - 1 \text{ edges”} \]

Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.

Proof of Claim:
Can’t visit more than once since no cycle.
Entered. Didn’t leave. Only one incident edge.
Removing node doesn’t create cycle.
New graph is connected.
Removing degree 1 node doesn’t disconnect from Degree 1 lemma.
Proof of if

**Thm:**
“G is connected and has no cycles”

\[ \implies \text{“G connected and has } |V| - 1 \text{ edges”} \]

**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.

**Claim:** Degree 1 vertex.

**Proof of Claim:**
Can’t visit more than once since no cycle.
Entered. Didn’t leave. Only one incident edge.

Removing node doesn’t create cycle.
New graph is connected.
Removing degree 1 node doesn’t disconnect from Degree 1 lemma.
By induction \( G - v \) has \( |V| - 2 \) edges.
Thm:
“G is connected and has no cycles”
⇒ “G connected and has \(|V| – 1\) edges”

Proof:
Walk from a vertex using untraversed edges.
Until get stuck.

Claim: Degree 1 vertex.

Proof of Claim:
Can’t visit more than once since no cycle.
Entered. Didn’t leave. Only one incident edge.

Removing node doesn’t create cycle.
New graph is connected.
Removing degree 1 node doesn’t disconnect from Degree 1 lemma.
By induction \(G – v\) has \(|V| – 2\) edges.
\(G\) has one more or \(|V| – 1\) edges.
Proof of if

Thm:
“G is connected and has no cycles”
\[ \implies \text{“G connected and has } |V| - 1 \text{ edges”} \]

Proof:
Walk from a vertex using untraversed edges.
Until get stuck.

Claim: Degree 1 vertex.

Proof of Claim:
Can’t visit more than once since no cycle.
Entered. Didn’t leave. Only one incident edge.
Removing node doesn’t create cycle.
New graph is connected.
Removing degree 1 node doesn’t disconnect from Degree 1 lemma.
By induction \( G - v \) has \( |V| - 2 \) edges.
\( G \) has one more or \( |V| - 1 \) edges.
Let $G$ be a connected graph with $|V| - 1$ edges.
Let $G$ be a connected graph with $|V| - 1$ edges.

(A) Removing a degree 1 vertex can disconnect the graph.
(B) One can use induction on smaller objects.
(C) The average degree is $2 - 2/|V|$.
(D) There is a hotel california: a degree 1 vertex.
(E) Everyone can be bigger than average.
Poll: Oh tree, beautiful tree.

Let $G$ be a connected graph with $|V| - 1$ edges.

(A) Removing a degree 1 vertex can disconnect the graph.
(B) One can use induction on smaller objects.
(C) The average degree is $2 - 2/|V|$.
(D) There is a hotel california: a degree 1 vertex.
(E) Everyone can be bigger than average.

(B), (C), (D) are true
Lecture in a minute.

Graphs.
Lecture in a minute.

Graphs.
Basics.
Lecture in a minute.

Graphs.
Basics.
Connectivity.
Lecture in a minute.

Graphs.
Basics.
Connectivity.
Algorithm for Eulerian Tour.
Lecture in a minute.

Graphs.
  Basics.
Connectivity.
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Lecture in a minute.

Graphs.
Basics.
Connectivity.
Algorithm for Eulerian Tour.

Trees: degree 1 lemma $\implies$ several definitions.
Lecture in a minute.

Graphs.
   Basics.
Connectivity.
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Trees: degree 1 lemma $\implies$ several definitions.
Planar Graphs: intro.