Lecture 5: Graphs.

Graphs!
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Graphs!
Definitions: model.
Graphs!
Definitions: model.
Fact!
Graphs!
Definitions: model.
Fact!
Graphs!
Definitions: model.
Fact!
Map Coloring.

Yes! Three colors.

Four colors required!

Theorem: Four colors enough for maps on the plane.
Map Coloring.

Fewer Colors?
Yes! Three colors.

Fewer Colors?
Four colors required!

Theorem: Four colors enough for maps on the plane.
Map Coloring.

Fewer Colors? Yes! Three colors.

Four colors required!

Theorem: Four colors enough for maps on the plane.
Map Coloring.

Fewer Colors?

Yes! Three colors.

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Fewer Colors?
Map Coloring.

Yes! Three colors.
Map Coloring.

Fewer Colors? Yes! Three colors.

Fewer Colors? Four colors required!

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Fewer Colors? Yes! Three colors.

Fewer Colors? Four colors required!

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Theorem: Four colors enough for maps on the plane.
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Fewer Colors?

Yes! Three colors.

Four colors required!

Theorem: Four colors enough for maps on the plane.

Fewer Colors?
Map Coloring.

Yes! Three colors.

Four colors required!

Theorem: Four colors enough for maps on the plane.
Map Coloring.

Four colors required!
Map Coloring.

Four colors required!

Theorem: Four colors enough for maps on the plane.
Scheduling: coloring.

61A  61B  61C

61A  170

70
Scheduling: coloring.

Exam Slot 1.
Exam Slot 2.
Exam Slot 3.

61B
61C

61A
170

70
Scheduling: coloring.

Exam Slot 1.
Exam Slot 2.
Exam Slot 3.
Scheduling: coloring.
Scheduling: coloring.
Scheduling: coloring.
Scheduling: coloring.
Scheduling: coloring.
Scheduling: coloring.
Scheduling: coloring.

Exam Slot 1.

Exam Slot 2.

Exam Slot 3.
Scheduling: coloring.
Scheduling: coloring.

Exam Slot 1.
Exam Slot 2.
Exam Slot 3.
Scheduling: coloring.
Scheduling: coloring.
Scheduling: coloring.

Exam Slot 1.

Exam Slot 2.

Exam Slot 3.
Graphs: formally.

Graph: $G = (V, E)$.
- $V$ - set of vertices. $\{A, B, C, D\}$
- $E \subseteq V \times V$ - set of edges. $\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs. No parallel edges. Multigraph above.
Graphs: formally.

Graph: \( G = (V, E) \).
Graphs: formally.

Graph: \( G = ( V, E ) \).

- \( V \) - set of vertices.
- \( E \subseteq V \times V \) - set of edges.

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- \( V \) - set of vertices.
  - \{A, B, C, D\}
- \( E \subseteq V \times V \) - set of edges.
  - \{(A, B), (A, B), (A, C), (A, C), (B, D), (A, D), (C, D)\}
Graphs: formally.

Graph: \( G = (V, E) \).
- \( V \) - set of vertices.
  - \( \{A, B, C, D\} \)
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Graphs: formally.

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Graphs: formally.

Graph: $G = (V, E)$.
- $V$ - set of vertices.
  - $\{A, B, C, D\}$
- $E \subseteq V \times V$ - set of edges.
  - $\{\{A, B\}\}$
Graphs: formally.

Graph: \( G = (V, E) \).

- \( V \) - set of vertices.
  \( \{A, B, C, D\} \)

- \( E \subseteq V \times V \) - set of edges.
  \( \{\{A, B\}, \{A, B\}\} \)

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  - $\{\{A, B\}, \{A, B\}, \{A, C\}, \}$
Graphs: formally.

Graph: $G = (V, E)$.

$V$ - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$. 
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For CS 70, usually simple graphs.
Graphs: formally.

Graph: \( G = (V, E) \).

- \( V \) - set of vertices.
  \( \{A, B, C, D\} \)

- \( E \subseteq V \times V \) - set of edges.
  \( \{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\} \).

For CS 70, usually simple graphs.
No parallel edges.
Graphs: formally.

Graph: $G = (V, E)$.

- $V$ - set of vertices.
  - $\{A, B, C, D\}$
- $E \subseteq V \times V$ - set of edges.
  - $\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

  - No parallel edges.

Multigraph above.
Directed Graphs

\[ G = (V, E). \]

2 \[\rightarrow\] 1 \[\rightarrow\] 3

4 \[\rightarrow\] 3 \[\rightarrow\] 3

One way streets. Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ...
Directed Graphs

\[ G = (V, E). \]
\[ V \] - set of vertices.

One way streets.

Tournament:
1 beats 2,
...

Precedence:
1 is before 2,
..

Social Network:
Directed?
Undirected?

Friends.
Undirected.
Likes.
Directed.
Directed Graphs

$G = (V, E)$.
$V$ - set of vertices.
$\{1, 2, 3, 4\}$
Directed Graphs

$G = (V, E)$.  
$V$ - set of vertices. 
$\{1, 2, 3, 4\}$  
$E$ ordered pairs of vertices.
Directed Graphs

\[ G = (V, E). \]

\( V \) - set of vertices.
\( \{1, 2, 3, 4\} \)

\( E \) ordered pairs of vertices.
\( \{(1,2), \} \)
Directed Graphs

$G = (V, E)$.

$V$ - set of vertices.

$\{1, 2, 3, 4\}$

$E$ ordered pairs of vertices.

$\{(1, 2), (1, 3), \}$
Directed Graphs

\[ G = (V, E). \]

- **V** - set of vertices.
  \[ \{1, 2, 3, 4\} \]
- **E** ordered pairs of vertices.
  \[ \{(1, 2), (1, 3), (1, 4), \} \]

- One way streets.
- Tournament: \[ 1 \text{ beats } 2, \ldots \]
- Precedence: \[ 1 \text{ is before } 2, \ldots \]
- Social Network: Directed? Undirected?
  - Friends: Undirected.
  - Likes: Directed.
Directed Graphs

\[ G = (V, E). \]

- \( V \) - set of vertices.
  \{1, 2, 3, 4\}

- \( E \) ordered pairs of vertices.
  \{(1,2), (1,3), (1,4), (2,4), (3,4)\}
Directed Graphs

\[ G = (V, E). \]

- \( V \) - set of vertices.
  \[ \{1, 2, 3, 4\} \]
- \( E \) ordered pairs of vertices.
  \[ \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\} \]

One way streets.
Directed Graphs

\[ G = (V, E). \]

- **V** - set of vertices.
  \[ \{1, 2, 3, 4\} \]

- **E** ordered pairs of vertices.
  \[ \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\} \]

One way streets.

Tournament:
Directed Graphs

\[ G = (V, E) \]

- **V** - set of vertices.
  \[ \{1, 2, 3, 4\} \]
- **E** - ordered pairs of vertices.
  \[ \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\} \]

One way streets.
Tournament: 1 beats 2,
Directed Graphs

\[ G = (V, E). \]

*V* - set of vertices.
\{1, 2, 3, 4\}

*E* ordered pairs of vertices.
\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}

One way streets.
Tournament: 1 beats 2, ...
Precedence:
Directed Graphs

\[ G = (V, E) \]

- **V**: set of vertices.
  \[ \{1, 2, 3, 4\} \]
- **E**: ordered pairs of vertices.
  \[ \{(1,2), (1,3), (1,4), (2,4), (3,4)\} \]

- One way streets.
- Tournament: 1 beats 2, ...
- Precedence: 1 is before 2,
Directed Graphs

$G = (V, E)$.

- $V$ - set of vertices.
  - $\{1, 2, 3, 4\}$
- $E$ ordered pairs of vertices.
  - $\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.

Tournament: 1 beats 2, ...

Precedence: 1 is before 2, ..
Directed Graphs

\[ G = (V, E). \]
\[ V \text{ - set of vertices.} \]
\[ \{1, 2, 3, 4\} \]
\[ E \text{ ordered pairs of vertices.} \]
\[ \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\} \]

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network:
Directed Graphs

\[ G = (V, E) \]
\[ V - \text{set of vertices.} \]
\[ \{1, 2, 3, 4\} \]
\[ E - \text{ordered pairs of vertices.} \]
\[ \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\} \]

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..
Social Network: Directed?
Directed Graphs

\[ G = (V, E). \]
V - set of vertices.
\{1, 2, 3, 4\}
E ordered pairs of vertices.
\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Directed Graphs

\[ G = (V, E) \]

- Set of vertices: \( V = \{1, 2, 3, 4\} \)
- Ordered pairs of vertices: \( E = \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\} \)

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends.
Directed Graphs

\[ G = (V, E) \]
\[ V - \text{set of vertices.} \]
\[ \{1, 2, 3, 4\} \]
\[ E - \text{ordered pairs of vertices.} \]
\[ \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\} \]

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
  Friends. Undirected.
Directed Graphs

$G = (V, E)$.

$V$ - set of vertices.
\{1, 2, 3, 4\}

$E$ ordered pairs of vertices.
\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes.
Directed Graphs

$G = (V, E)$. 
$V$ - set of vertices. 
{1, 2, 3, 4} 
$E$ ordered pairs of vertices. 
{(1,2), (1,3), (1,4), (2,4), (3,4)}

One way streets. 
Tournament: 1 beats 2, ... 
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected? 
Friends. Undirected. 
Likes. Directed.
Directed Graphs

\[ G = (V, E). \]

- \( V \) - set of vertices.
  \( \{1, 2, 3, 4\} \)
- \( E \) - ordered pairs of vertices.
  \( \{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\} \)

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
  - Friends. Undirected.
  - Likes. Directed.
Graph Concepts and Definitions.

Graph: $G = (V, E)$

Neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10?

$1, 5, 7, 8$. $u$ is neighbor of $v$ if $\{u, v\} \in E$.

Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

Degree of vertex 10?

Degree of vertex $u$ is number of incident edges. Equals number of neighbors in simple graph.

Directed graph?

In-degree of 10?

Out-degree of 10?

$6 / 26$
Graph Concepts and Definitions.

Graph: $G = (V, E)$

- neighbors, adjacent, degree, incident, in-degree, out-degree
Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10?
Graph Concepts and Definitions.

Graph: $G = (V, E)$
neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1,
Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5,
Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7,
Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

\( u \) is neighbor of \( v \) if \( \{u, v\} \in E \).
Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

$u$ is neighbor of $v$ if $\{u, v\} \in E$.

Edge $\{10, 5\}$ is incident to
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1,5,7, 8.

\( u \) is neighbor of \( v \) if \( \{u, v\} \in E \).

Edge \( \{10,5\} \) is incident to vertex 10 and vertex 5.

Edge \( \{u, v\} \) is incident to \( u \) and \( v \).

Degree of vertex 1?
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

\( u \) is neighbor of \( v \) if \( \{u, v\} \in E \).

Edge \( \{10,5\} \) is incident to vertex 10 and vertex 5.

Edge \( \{u, v\} \) is incident to \( u \) and \( v \).

Degree of vertex 1? 2
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

- neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

- \( u \) is neighbor of \( v \) if \( \{u, v\} \in E \).

Edge \( \{10, 5\} \) is incident to vertex 10 and vertex 5.

- Edge \( \{u, v\} \) is incident to \( u \) and \( v \).

Degree of vertex 1? 2

- Degree of vertex \( u \) is number of incident edges.
Graph Concepts and Definitions.

Graph: $G = (V, E)$

- neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

$u$ is neighbor of $v$ if $\{u, v\} \in E$.

Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

Edge $\{u, v\}$ is incident to $u$ and $v$.

Degree of vertex 1? 2

Degree of vertex $u$ is number of incident edges.

Equals number of neighbors in simple graph.
Graph Concepts and Definitions.

Graph: $G = (V, E)$

- neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

$u$ is neighbor of $v$ if $\{u, v\} \in E$.

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Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

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Degree of vertex 1? 2

- Degree of vertex $u$ is number of incident edges.
  - Equals number of neighbors in simple graph.

Directed graph?
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

- neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

- \( u \) is neighbor of \( v \) if \( \{u, v\} \in E \).

Edge \( \{10, 5\} \) is incident to vertex 10 and vertex 5.

- Edge \( \{u, v\} \) is incident to \( u \) and \( v \).

Degree of vertex 1? 2

- Degree of vertex \( u \) is number of incident edges.

  - Equals number of neighbors in simple graph.

Directed graph?

In-degree of 10?
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

- neighbors, adjacent, degree, incident, in-degree, out-degree

1
2
3
4
5
6
7
8
9
10
11

Neighbors of 10? 1, 5, 7, 8.

* u is neighbor of v if \( \{u, v\} \in E \).

Edge \( \{10, 5\} \) is incident to vertex 10 and vertex 5.

Edge \( \{u, v\} \) is incident to u and v.

Degree of vertex 1? 2

- Degree of vertex \( u \) is number of incident edges.
  - Equals number of neighbors in simple graph.

Directed graph?

In-degree of 10? 1
Graph Concepts and Definitions.

Graph: $G = (V, E)$
neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.

$u$ is neighbor of $v$ if $\{u, v\} \in E$.

Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

Edge $\{u, v\}$ is incident to $u$ and $v$.

Degree of vertex 1? 2

Degree of vertex $u$ is number of incident edges.

Equals number of neighbors in simple graph.

Directed graph?

In-degree of 10? 1  Out-degree of 10?
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)
- neighbors, adjacent, degree, incident, in-degree, out-degree

Neighbors of 10? 1, 5, 7, 8.
- \( u \) is neighbor of \( v \) if \( \{u, v\} \in E \).

Edge \( \{10, 5\} \) is incident to vertex 10 and vertex 5.
- Edge \( \{u, v\} \) is incident to \( u \) and \( v \).

Degree of vertex 1? 2
- Degree of vertex \( u \) is number of incident edges.
  - Equals number of neighbors in simple graph.

Directed graph?
- In-degree of 10? 1  
  Out-degree of 10? 3
Graph Concepts and Definitions.

Graph: $G = (V, E)$

- neighbors
- adjacent
- degree
- incident
- in-degree
- out-degree

Neighbors of 10? 1, 5, 7, 8.

$u$ is neighbor of $v$ if $\{u, v\} \in E$.

Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

Edge $\{u, v\}$ is incident to $u$ and $v$.

Degree of vertex 1? 2

Degree of vertex $u$ is number of incident edges.

Equals number of neighbors in simple graph.

Directed graph?

In-degree of 10? 1  Out-degree of 10? 3
Graph Concepts and Definitions.

Graph: $G = (V, E)$

Edge $(8, 5)$ is incident to:

(A) Vertex 8.
(B) Vertex 5.
(C) Edge $(8, 5)$.
(D) Edge $(8, 4)$.
(E) Vertex 10.

(A) and (B) are true.

The degree of a vertex is:

(A) The number of edges incident to it.
(B) The number of neighbors of $v$.
(C) Is the number of vertices in its connected component.

(A) and (B) are true.
Graph Concepts and Definitions.

Graph: $G = (V, E)$
neighbors, adjacent, degree, incident, in-degree, out-degree
Graph Concepts and Definitions.

Graph: \( G = (V, E) \)
neighbors, adjacent, degree, incident, in-degree, out-degree

\begin{itemize}
  \item [\textbf{Edge} (8, 5) is incident to:]
  \begin{enumerate}
    \item [(A)] Vertex 8.
    \item [(B)] Vertex 5.
    \item [(C)] Edge (8, 5).
    \item [(D)] Edge (8, 4).
    \item [(E)] Vertex 10.
  \end{enumerate}
\end{itemize}
Graph Concepts and Definitions.

Graph: $G = (V, E)$
neighbors, adjacent, degree, incident, in-degree, out-degree

Edge $(8, 5)$ is incident to:

(A) Vertex 8.
(B) Vertex 5.
(C) Edge $(8, 5)$.
(D) Edge $(8, 4)$.
(E) Vertex 10.

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Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

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Graph Concepts and Definitions.

Graph: \( G = (V, E) \)

neighbors, adjacent, degree, incident, in-degree, out-degree

Edge \((8, 5)\) is incident to:

(A) Vertex 8.
(B) Vertex 5.
(C) Edge \((8, 5)\).
(D) Edge \((8, 4)\).
(E) Vertex 10.

(A) and (B) are true.

The degree of a vertex is:

(A) The number of edges incident to it.
(B) The number of neighbors of \(v\).
(C) Is the number of vertices in its connected component.

(A) and (B) are true.
The sum of the vertex degrees is equal to
Sum of degrees?

The sum of the vertex degrees is equal to
(A) the total number of vertices, $|V|$. 

(B) the total number of edges, $|E|$. 

Not (A)!

Triangle.

Not (B)!

Triangle.

What?

For triangle number of edges is 3, the sum of degrees is 6.

Could sum always be...

(A) 2 $|E|$?

(B) 2 $|V|$?

(A) is true.
The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.  
(B) the total number of edges, $|E|$. 
The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.  
(B) the total number of edges, $|E|$.  
(C) What?

For a triangle, the number of edges is 3, and the sum of degrees is 6.  
Could sum always be...

(A) 2 $|E|$?

(B) 2 $|V|$?

(A) is true.
The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)!

![Diagram of a triangle with three vertices and three edges]
Sum of degrees?

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)! Triangle.

\begin{figure}
\centering
\includegraphics[width=0.2\textwidth]{triangle.png}
\caption{A triangle graph}
\end{figure}
The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)! Triangle.
Not (B)!
Sum of degrees?

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)! Triangle.
Not (B)! Triangle.
The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

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What?
Sum of degrees?

The sum of the vertex degrees is equal to

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What? For triangle number of edges is 3, the sum of degrees is 6.
The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
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Could sum always be...
The sum of the vertex degrees is equal to

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Not (A)! Triangle.
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What? For triangle number of edges is 3, the sum of degrees is 6.

**Could sum always be...**

(A) $2|E|$? ..
Sum of degrees?

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.  
(B) the total number of edges, $|E|$.  
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What? For triangle number of edges is 3, the sum of degrees is 6.

Could sum always be...

(A) $2|E|$? ..
(B) $2|V|$?
(A) is true.
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

edge, \((u, v)\), is \textit{incident} to endpoints, \(u\) and \(v\).
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

- edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).
- degree of \(u\) number of edges incident to \(u\)
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).

degree of \(u\) number of edges incident to \(u\)

Let’s count incidences in two ways.
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

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Let's count incidences in two ways.

How many incidences does each edge contribute?
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).

degree of \(u\) number of edges incident to \(u\)

Let's count incidences in two ways.

How many incidences does each edge contribute? 2.
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).

degree of \(u\) number of edges incident to \(u\)

Let’s count incidences in two ways.

How many incidences does each edge contribute? 2.
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

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- degree of \(u\) number of edges incident to \(u\).

Let’s count incidences in two ways.

How many incidences does each edge contribute? 2.

Total Incidences?
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

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The sum of the vertex degrees is equal to ??

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What is degree \(v\)?
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:
edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).
degree of \(u\) number of edges incident to \(u\)

Let’s count incidences in two ways.

How many incidences does each edge contribute? 2.

Total Incidences? \(|E|\) edges, 2 each. \(\rightarrow 2|E|\)

What is degree \(v\)? Incidences corresponding to \(v\)!
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).

degree of \(u\) number of edges incident to \(u\)

Let's count incidences in two ways.

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Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

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Let's count incidences in two ways.

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What is degree \(v\)? Incidences corresponding to \(v\)!

Total Incidences?
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

degree of $u$ number of edges incident to $u$

Let’s count incidences in two ways.

How many incidences does each edge contribute? 2.

Total Incidences? $|E|$ edges, 2 each. $\rightarrow 2|E|$

What is degree $v$? Incidences corresponding to $v$!

Total Incidences? The sum over vertices of degrees!
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:
edge, \((u, v)\), is incident to endpoints, \(u\) and \(v\).
degree of \(u\) number of edges incident to \(u\)

Let’s count incidences in two ways.

How many incidences does each edge contribute? 2.

Total Incidences? \(|E|\) edges, 2 each. \(\rightarrow 2|E|\)

What is degree \(v\)? Incidences corresponding to \(v\)!

Total Incidences? The sum over vertices of degrees!
Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:
edge, \((u, v)\), is **incident** to endpoints, \(u\) and \(v\).
degree of \(u\) number of edges **incident** to \(u\)

Let's count incidences in two ways.

How many **incidences** does each edge contribute? 2.

Total Incidences? \(|E|\) edges, 2 each. \(\rightarrow 2|E|\)

What is degree \(v\)? Incidences corresponding to \(v\)!

Total Incidences? The sum over vertices of degrees!

**Thm:** Sum of vertex degress is \(2|E|\).
Poll: Proof of “handshake” lemma.

What’s true?

(A) The number of edge-vertex incidences for an edge e is 2.
(B) The total number of edge-vertex incidences is $|V|$.
(C) The total number of edge-vertex incidences is $2|E|$.
(D) The number of edge-vertex incidences for a vertex v is its degree.
(E) The sum of degrees is $2|E|$.
(F) The total number of edge-vertex incidences is the sum of the degrees.
Poll: Proof of “handshake” lemma.

What’s true?

(A) The number of edge-vertex incidences for an edge e is 2.
(B) The total number of edge-vertex incidences is $|V|$.
(C) The total number of edge-vertex incidences is $2|E|$.
(D) The number of edge-vertex incidences for a vertex v is its degree.
(E) The sum of degrees is $2|E|$.
(F) The total number of edge-vertex incidences is the sum of the degrees.

(A),(C), (D), (E), and (F).
A path in a graph is a sequence of edges.
Paths, walks, cycles, tour.

A path in a graph is a sequence of edges. Path?

Path: $(v_1, v_2), (v_2, v_3), \ldots, (v_{k-1}, v_k)$

Quick Check!

Length of path? $k$ vertices or $k-1$ edges.

Cycle: Path from $v_1$ to $v_k-1$, + edge $(v_k-1, v_1)$

Length of cycle? $k-1$ vertices and edges!

Path is usually simple. No repeated vertex!

Walk is sequence of edges with possible repeated vertex or edge.

Tour is walk that starts and ends at the same node.

Quick Check!

Path is to Walk as Cycle is to ??

Tour!
A path in a graph is a sequence of edges.

Path? \{1, 10\}, \{8, 5\}, \{4, 5\} ?
A path in a graph is a sequence of edges.
Path? \{1, 10\}, \{8, 5\}, \{4, 5\}? No!
Paths, walks, cycles, tour.

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Path?

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Path is to Walk as Cycle is to ??
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Quick Check! Length of path?

Length of path? \(k\) vertices or \(k - 1\) edges.
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Quick Check! Path is to Walk as Cycle is to Tour!
Paths, walks, cycles, tour.

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- Path? \{1, 10\}, \{8, 5\}, \{4, 5\}? No!
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**Path:** \((v_1, v_2), (v_2, v_3), \ldots, (v_{k-1}, v_k)\).

Quick Check! Length of path? \(k\) vertices
A path in a graph is a sequence of edges.

Path? \(\{1, 10\}, \{8, 5\}, \{4, 5\}\) ? No!
Path? \(\{1, 10\}, \{10, 5\}, \{5, 4\}, \{4, 11\}\) ? Yes!

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Quick Check! Length of path? \(k\) vertices or \(k-1\) edges.
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Quick Check!
Path is to Walk as Cycle is to ??
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Quick Check!
Path is to Walk as Cycle is to ?? Tour!
Directed Paths.

Path: (v_1, v_2), (v_2, v_3), ..., (v_{k-1}, v_k).

Paths, walks, cycles, tours... are analogous to undirected now.
Directed Paths.

Path: \((v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)\).
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Directed Paths.

Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$. Paths, walks,
Directed Paths.

Path: \((v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)\).

Paths, walks, cycles,
Directed Paths.

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Paths, walks, cycles, tours
Directed Paths.

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Paths, walks, cycles, tours ... are analogous to undirected now.
Connectivity: undirected graph.

$u$ and $v$ are connected if there is a path between $u$ and $v$. 

A connected graph is a graph where all pairs of vertices are connected.

If one vertex $x$ is connected to every other vertex, is the graph connected? Yes? No?

Proof:
Use path from $u$ to $x$ and then from $x$ to $v$.

May not be simple!
Either modify definition to walk.
Or cut out cycles.
Connectivity: undirected graph.

$u$ and $v$ are **connected** if there is a path between $u$ and $v$.

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Is graph connected?
Connectivity: undirected graph.

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Is graph connected? Yes?
Connectivity: undirected graph.

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Proof: Use path from $u$ to $x$ and then from $x$ to $v$. 
Connectivity: undirected graph.

$u$ and $v$ are connected if there is a path between $u$ and $v$.

A connected graph is a graph where all pairs of vertices are connected.

If one vertex $x$ is connected to every other vertex.

Is graph connected? Yes? No?

Proof: Use path from $u$ to $x$ and then from $x$ to $v$.

May not be simple!
Connectivity: undirected graph.

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Proof: Use path from $u$ to $x$ and then from $x$ to $v$.

May not be simple!
Either modify definition to walk.
Connectivity: undirected graph.

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Proof: Use path from $u$ to $x$ and then from $x$ to $v$.

May not be simple!
Either modify definition to walk.
Or cut out cycles.
Connectivity: undirected graph.

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May not be simple!
Either modify definition to walk.
Or cut out cycles.
Connectivity: undirected graph.

$u$ and $v$ are connected if there is a path between $u$ and $v$.

A connected graph is a graph where all pairs of vertices are connected.

If one vertex $x$ is connected to every other vertex. Is graph connected? Yes? No?

Proof: Use path from $u$ to $x$ and then from $x$ to $v$.

May not be simple!
Either modify definition to walk. Or cut out cycles.
Is the graph above connected?

No!

Connected Components?

\{1\}, \{10, 7, 5, 8, 4, 11\}, \{2, 9, 6\}.

Connected component - maximal set of connected vertices.

Quick Check: Is \{10, 7, 5\} a connected component?

No.
Connected Components: Quiz.

Is graph above connected? Yes!

Connected component - maximal set of connected vertices.

Quick Check: Is \{10, 7, 5\} a connected component? No.
Connected Components: Quiz.

Is graph above connected? Yes!
How about now?

Connected component - maximal set of connected vertices.

Quick Check: Is \{10, 7, 5\} a connected component? No.
Is graph above connected? Yes!

How about now? No!
Connected Components: Quiz.

Is graph above connected? Yes!

How about now? No!

Connected Components?
Connected Components: Quiz.

Is graph above connected? Yes!

How about now? No!

Connected Components? \{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}. 
Connected Components: Quiz.

Is graph above connected? Yes!

How about now? No!

**Connected Components?** \{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}.  
 Connected component - maximal set of connected vertices.
Connected Components: Quiz.

Is graph above connected? Yes!

How about now? No!

**Connected Components?** \{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}.

Connected component - maximal set of connected vertices.

Quick Check: Is \{10, 7, 5\} a connected component?
Is the graph above connected? Yes!

How about now? No!

**Connected Components**? \{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}.

*Connected component - maximal set of connected vertices.*

Quick Check: Is \{10, 7, 5\} a connected component? No.
Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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Konigsberg bridges problem.

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Can you draw a tour in the graph where you visit each edge once?
Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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Can you draw a tour in the graph where you visit each edge once? Yes?
Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

Can you draw a tour in the graph where you visit each edge once? Yes? No?
Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

Can you draw a tour in the graph where you visit each edge once?
Yes?  No?
We will see!
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.
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**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected.
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An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit.
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**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected.
Tour enters and leaves vertex $v$ on each visit.
Uses two incident edges per visit.
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Tour enters and leaves vertex $v$ on each visit.
Uses two incident edges per visit. Tour uses all incident edges.
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**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit. Uses two incident edges per visit. Tour uses all incident edges. Therefore $v$ has even degree.
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**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected.
Tour enters and leaves vertex $v$ on each visit.
Uses two incident edges per visit. Tour uses all incident edges.
Therefore $v$ has even degree.

When you enter,
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

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**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit. Uses two incident edges per visit. Tour uses all incident edges. Therefore $v$ has even degree.

When you enter, you can leave.
Eulerian Tour

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**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

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When you enter, you can leave.
For starting node, tour leaves first
Eulerian Tour

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**Proof of only if:** Eulerian $\implies$ connected and all even degree.

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When you enter, you can leave. For starting node, tour leaves first ....then enters at end.
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**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

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**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected.
Tour enters and leaves vertex $v$ on each visit.
Uses two incident edges per visit. Tour uses all incident edges.
Therefore $v$ has even degree.

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.
Not The Hotel California.
Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

**Theorem:** Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

**Proof of only if:** Eulerian $\implies$ connected and all even degree.

Eulerian Tour is connected so graph is connected. Tour enters and leaves vertex $v$ on each visit. Uses two incident edges per visit. Tour uses all incident edges. Therefore $v$ has even degree.

When you enter, you can leave. For starting node, tour leaves first ....then enters at end. Not The Hotel California.

(Timestamp: 4:02).
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v(1)$ on “unused” edges.
Proof of if: Even + connected \(\implies\) Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from \(v\) (1) on “unused” edges

![Graph Diagram]

1. Start at vertex 1.
2. Move to vertex 2.
5. Move back to vertex 2.
7. Move to vertex 3.
8. Move to vertex 11.
10. Move to vertex 8.
11. Move back to vertex 1.

Splice together: 1,10,7,8,5,10,8,4,3,11,4,5,2,6,9,2 and to 1!
Finding a tour!

Proof of if: Even + connected \(\implies\) Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from \(v(1)\) on “unused” edges

```
    1 2 3 4 5 6
    7 8 9 10 11
```

2. Remove tour, \(C\).
3. Let \(G_1, \ldots, G_k\) be connected components. Each is touched by \(C\).
4. Recurse on \(G_1, \ldots, G_k\) starting from \(v_i\)
5. Splice together.
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v(1)$ on “unused” edges
Proof of if: Even + connected $\implies$ Eulerian Tour.

We will give an algorithm. First by picture.

1. Take a walk starting from $v_1$ on "unused" edges
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v\ (1)$ on “unused” edges
   ... till you get back to $v$. 

![Graph Diagram]
Finding a tour!

Proof of if: Even + connected \( \implies \) Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from \( v \) (1) on “unused” edges
   ... till you get back to \( v \).
2. Remove tour, \( C \).

\[
\begin{align*}
1 & \rightarrow 2 \\
7 & \rightarrow 5 \rightarrow 10 \\
8 & \rightarrow 4 \rightarrow 11 \\
5 & \leftarrow 3 \\
9 & \\
11 & \\
6 & \\
\end{align*}
\]
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$.
5. Splice together.
Finding a tour!

Proof of if: Even + connected \(\implies\) Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from \(v\) (1) on “unused” edges
   ... till you get back to \(v\).
2. Remove tour, \(C\).
3. Let \(G_1, \ldots, G_k\) be connected components. Each is touched by \(C\).
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why?
Proof of if: Even + connected $\iff$ Eulerian Tour.

We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.

4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$
5. Splice together.
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$. 
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1$. 
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1$, $v_2 = 10$, ...
Proof of if: Even + connected \( \implies \) Eulerian Tour.

We will give an algorithm. First by picture.

1. Take a walk starting from \( v \) (1) on “unused” edges
   ... till you get back to \( v \).
2. Remove tour, \( C \).
3. Let \( G_1, \ldots, G_k \) be connected components.
   Each is touched by \( C \).
   Why? \( G \) was connected.
   Let \( v_i \) be (first) node in \( G_i \) touched by \( C \).
   Example: \( v_1 = 1, v_2 = 10, v_3 = 4 \),
Finding a tour!

Proof of if: **Even + connected** $\implies$ **Eulerian Tour.**
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components. Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1, v_2 = 10, v_3 = 4, v_4 = 2$. 
Proof of if: Even + connected \( \implies \) Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from \( v \) (1) on “unused” edges 
   ... till you get back to \( v \).
2. Remove tour, \( C \).
3. Let \( G_1, \ldots, G_k \) be connected components. 
   Each is touched by \( C \).
   Why? \( G \) was connected.
   Let \( v_i \) be (first) node in \( G_i \) touched by \( C \).
   Example: \( v_1 = 1, v_2 = 10, v_3 = 4, v_4 = 2 \).
4. Recurse on \( G_1, \ldots, G_k \) starting from \( v_i \).
Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$
Finding a tour!

Proof of if: Even + connected $\Rightarrow$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v$ (1) on “unused” edges
... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$
5. Splice together.
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.

We will give an algorithm. First by picture.

1. Take a walk starting from $v_1$ on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$
5. Splice together.
   1,10
Proof of if: Even + connected \implies Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from \( v_1 \) on “unused” edges
   ... till you get back to \( v \).
2. Remove tour, \( C \).
3. Let \( G_1, \ldots, G_k \) be connected components.
   Each is touched by \( C \).
   Why? \( G \) was connected.
   Let \( v_i \) be (first) node in \( G_i \) touched by \( C \).
   Example: \( v_1 = 1, v_2 = 10, v_3 = 4, v_4 = 2 \).
4. Recurse on \( G_1, \ldots, G_k \) starting from \( v_i \)
5. Splice together.
   \( 1, 10, 7, 8, 5, 10 \)
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v_1$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$
5. Splice together.
   1,10,7,8,5,10 ,8,4
Finding a tour!

**Proof of if: Even + connected \( \implies \) Eulerian Tour.**

We will give an algorithm. First by picture.

1. Take a walk starting from \( v \) (1) on “unused” edges
   ... till you get back to \( v \).
2. Remove tour, \( C \).
3. Let \( G_1, \ldots, G_k \) be connected components.
   Each is touched by \( C \).
   Why? \( G \) was connected.
   Let \( v_i \) be (first) node in \( G_i \) touched by \( C \).
   Example: \( v_1 = 1, v_2 = 10, v_3 = 4, v_4 = 2 \).
4. Recurse on \( G_1, \ldots, G_k \) starting from \( v_i \)
5. Splice together.

\[ 1,10,7,8,5,10,8,4,3,11,4 \]
Proof of if: Even + connected \( \implies \) Eulerian Tour. We will give an algorithm. First by picture.

1. Take a walk starting from \( v \) (1) on “unused” edges
   ... till you get back to \( v \).
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   Example: \( v_1 = 1, v_2 = 10, v_3 = 4, v_4 = 2 \).
4. Recurse on \( G_1, \ldots, G_k \) starting from \( v_i \)
5. Splice together.
   \( 1,10,7,8,5,10,8,4,3,11,4,5,2 \)
Finding a tour!

**Proof of if: Even + connected \( \implies \) Eulerian Tour.**
We will give an algorithm. First by picture.

1. Take a walk starting from \( v (1) \) on “unused” edges
   ... till you get back to \( v \).
2. Remove tour, \( C \).
3. Let \( G_1, \ldots, G_k \) be connected components.
   Each is touched by \( C \).
   Why? \( G \) was connected.
   Let \( v_i \) be (first) node in \( G_i \) touched by \( C \).
   Example: \( v_1 = 1, v_2 = 10, v_3 = 4, v_4 = 2 \).
4. Recurse on \( G_1, \ldots, G_k \) starting from \( v_i \)
5. Splice together.
   \( 1,10,7,8,5,10,8,4,3,11,4,5,2,6,9,2 \)
Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.

1. Take a walk starting from $v_1$ (1) on “unused” edges
   ... till you get back to $v$.
2. Remove tour, $C$.
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   Each is touched by $C$.
   Why? $G$ was connected.
   Let $v_i$ be (first) node in $G_i$ touched by $C$.
   Example: $v_1 = 1, v_2 = 10, v_3 = 4, v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from $v_i$
5. Splice together.
   $1, 10, 7, 8, 5, 10, 8, 4, 3, 11, 4, 5, 2, 6, 9, 2$ and to 1!
Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node $v$, until you get back to $v$. 

Claim: Do get back to $v$!

Proof of Claim: Even degree. If enter, can leave except for $v$.

2. Remove cycle, $C$, from $G$. Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$. Let $v_i$ be first vertex of $C$ that is in $G_i$.

Why is there a $v_i$ in $C$? $G$ was connected $\Rightarrow$ a vertex in $G_i$ must be incident to a removed edge in $C$.

Claim: Each vertex in each $G_i$ has even degree and is connected.

Prf: Tour $C$ has even incidences to any vertex $v$.

3. Find tour $T_i$ of $G_i$ starting/ending at $v_i$.

Induction.

4. Splice $T_i$ into $C$ where $v_i$ first appears in $C$.

Visits every edge once: Visits edges in $C$ exactly once. By induction for all edges in each $G_i$. 

Recursive/Inductive Algorithm.

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**Claim:** Do get back to $v$!
Recursive/Inductive Algorithm.

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2. Remove cycle, $C$, from $G$.

Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

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By induction for all edges in each $G_i$. 
Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node $v$, until you get back to $v$.

**Claim:** Do get back to $v$!

**Proof of Claim:** Even degree. If enter, can leave
Recursive/Inductive Algorithm.

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Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node $v$, until you get back to $v$.

   **Claim:** Do get back to $v$!

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2. Remove cycle, $C$, from $G$.
   Resulting graph may be disconnected. (Removed edges!)
   Let components be $G_1$, $\ldots$, $G_k$.

3. Find tour $T_i$ of $G_i$ starting/ending at $v_i$.

4. Splice $T_i$ into $C$ where $v_i$ first appears in $C$.

Visits every edge once:
Visits edges in $C$ exactly once.
By induction for all edges in each $G_i$. □
Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node $v$, until you get back to $v$.

**Claim:** Do get back to $v$!

**Proof of Claim:** Even degree. If enter, can leave except for $v$.

2. Remove cycle, $C$, from $G$.

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Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node \( v \), until you get back to \( v \).

**Claim:** Do get back to \( v \)!

**Proof of Claim:** Even degree. If enter, can leave except for \( v \).

2. Remove cycle, \( C \), from \( G \).
   Resulting graph may be disconnected. (Removed edges!)

- **Tour** \( C \) has even incidences to any vertex \( v \).

3. Find tour \( T_i \) of \( G_i \) starting/ending at \( v_i \).

4. Splice \( T_i \) into \( C \) where \( v_i \) first appears in \( C \).

Visits every edge once: Visits edges in \( C \) exactly once.

By induction for all edges in each \( G_i \).
Recursive/Inductive Algorithm.

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2. Remove cycle, $C$, from $G$.
   Resulting graph may be disconnected. (Removed edges!)
   Let components be $G_1, \ldots, G_k$. 
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   Resulting graph may be disconnected. (Removed edges!)

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   Why is there a $v_i$ in $C$?

   $G$ was connected $\Rightarrow$ a vertex in $G_i$ must be incident to a removed edge in $C$.

   **Claim:** Each vertex in each $G_i$ has even degree and is connected.

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   Let components be \( G_1, \ldots, G_k \).
   
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   Why is there a \( v_i \) in \( C \)?
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Let components be \( G_1, \ldots, G_k \).
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**Claim:** Each vertex in each $G_i$ has even degree
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Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

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$G$ was connected $\implies$ a vertex in $G_i$ must be incident to a removed edge in $C$.

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3. Find tour \( T_i \) of \( G_i \) starting/ending at \( v_i \). Induction.
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   Visits every edge once:
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   Let components be \( G_1, \ldots, G_k \).
   Let \( v_i \) be first vertex of \( C \) that is in \( G_i \).
   Why is there a \( v_i \) in \( C \)?
   
   \( G \) was connected \( \Rightarrow \)
   
   a vertex in \( G_i \) must be incident to a removed edge in \( C \).

**Claim:** Each vertex in each \( G_i \) has even degree and is connected.
   
   **Prf:** Tour \( C \) has even incidences to any vertex \( v \).

3. Find tour \( T_i \) of \( G_i \) starting/ending at \( v_i \). Induction.
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Visits every edge once:
   
   Visits edges in \( C \) exactly once.
   
   By induction for all edges in each \( G_i \).
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4. Splice $T_i$ into $C$ where $v_i$ first appears in $C$.

Visits every edge once:
   Visits edges in $C$ exactly once.
   By induction for all edges in each $G_i$. 
Poll: Euler concepts.

Mark correct statements for a connected graph where all vertices have even degree. (Below, tours uses edges at most once, but may involve a vertex several times.)

(A) Removing a tour leaves a graph of even degree.
(B) A tour connecting a set of connected components, each with an Eulerian tour, is really cool! Eulerian even.
(C) There is no hotel california in this graph.
(D) After removing a set of edges $E'$ in a connected graph, every connected component is incident to an edge in $E'$.
(E) If one walks on new edges, starting at $v$, one must eventually get back to $v$.

Only (F) is false.
Mark correct statements for a connected graph where all vertices have even degree. (Below, tours uses edges at most once, but may involve a vertex several times.

(A) Removing a tour leaves a graph of even degree.
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(D) After removing a set of edges $E'$ in a connected graph, every connected component is incident to an edge in $E'$
(E) If one walks on new edges, starting at $v$, one must eventually get back to $v$.
(F) Removing a tour leaves a connected graph.
Mark correct statements for a connected graph where all vertices have even degree. (Below, tours uses edges at most once, but may involve a vertex several times.)

(A) Removing a tour leaves a graph of even degree.
(B) A tour connecting a set of connected components, each with a Eulerian tour is really cool! Eulerian even.
(C) There is no hotel california in this graph.
(D) After removing a set of edges $E'$ in a connected graph, every connected component is incident to an edge in $E'$
(E) If one walks on new edges, starting at $v$, one must eventually get back to $v$.
(F) Removing a tour leaves a connected graph.

Only (F) is false.
A Tree, a tree.

Graph $G = (V, E)$. Binary Tree!

More generally.
Trees.

Definitions:

- A connected graph without a cycle.
- A connected graph with $|V| - 1$ edges.
- A connected graph where any edge removal disconnects it.
- A connected graph where any edge addition creates a cycle.

Trees.

Definitions:

A connected graph without a cycle.
Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.
Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.
A connected graph where any edge removal disconnects it.

Some trees.

no cycle and connected? Yes.
$|V| - 1$ edges and connected? Yes.
Removing any edge disconnects it. Harder to check. But yes.

Adding any edge creates cycle. Harder to check. But yes.

To tree or not to tree!
Trees.

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no cycle and connected?
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Some trees.

no cycle and connected? Yes.
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Some trees.

no cycle and connected? Yes.
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Some trees.

- No cycle and connected? Yes.
- $|V| - 1$ edges and connected? Yes.
- Removing any edge disconnects it. Harder to check.
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A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.
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Some trees.

- No cycle and connected? Yes.
- $|V| - 1$ edges and connected? Yes.
- Removing any edge disconnects it. Harder to check. But yes.
- Adding any edge creates cycle.
Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V| − 1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

Some trees.

no cycle and connected? Yes.
$|V| − 1$ edges and connected? Yes.
removing any edge disconnects it. Harder to check. but yes.
Adding any edge creates cycle. Harder to check.
Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.
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A connected graph where any edge addition creates a cycle.

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no cycle and connected? Yes.
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Trees.

Definitions:

- A connected graph without a cycle.
- A connected graph with $|V| - 1$ edges.
- A connected graph where any edge removal disconnects it.
- A connected graph where any edge addition creates a cycle.

Some trees.

- A graph with no cycle and connected? Yes.
- A graph with $|V| - 1$ edges and connected? Yes.
- Removing any edge disconnects it. Harder to check. But yes.
- Adding any edge creates a cycle. Harder to check. But yes.
Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.
A connected graph where any edge removal disconnects it.
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Some trees.

- ![Tree 1](image1.png)
- ![Tree 2](image2.png)
- ![Tree 3](image3.png)

no cycle and connected? Yes.
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To tree or not to tree!
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Some trees.

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- A connected graph with $|V| - 1$ edges.
- A connected graph where any edge removal disconnects it.
- A connected graph where any edge addition creates a cycle.

Some trees.

![Trees](image)

no cycle and connected? Yes.
$|V| - 1$ edges and connected? Yes.
removing any edge disconnects it. Harder to check. but yes.
Adding any edge creates cycle. Harder to check. but yes.

To tree or not to tree!
Equivalence of Definitions.

**Theorem:**
“G connected and has $|V| - 1$ edges” $\equiv$
“G is connected and has no cycles.”
Equivalence of Definitions.

**Theorem:**
“G connected and has $|V| - 1$ edges” \(\equiv\)
“G is connected and has no cycles.”

**Lemma:** If \(v\) is degree 1 in connected graph \(G\), \(G - v\) is connected.

**Proof:**
For \(x \neq v, y \neq v \in V\),
Equivalence of Definitions.

**Theorem:**
“$G$ connected and has $|V| - 1$ edges” $\equiv$
“$G$ is connected and has no cycles.”

**Lemma:** If $v$ is degree 1 in connected graph $G$, $G - v$ is connected.

**Proof:**
For $x \neq v, y \neq v \in V$,
there is path between $x$ and $y$ in $G$ since connected.
Equivalence of Definitions.

**Theorem:**
“G connected and has $|V| - 1$ edges” $\equiv$
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**Lemma:** If $v$ is degree 1 in connected graph $G$, $G - v$ is connected.

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For $x \neq v, y \neq v \in V$,
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and does not use $v$ (degree 1)
Equivalence of Definitions.

**Theorem:**
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Equivalence of Definitions.

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“G connected and has $|V| - 1$ edges” $\equiv$
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Theorem: “G connected and has $|V| - 1$ edges” ≡ “G is connected and has no cycles.”

Lemma: If $v$ is degree 1 in connected graph $G$, $G - v$ is connected.

Proof:
For $x \neq v, y \neq v \in V$,
there is path between $x$ and $y$ in $G$ since connected.
and does not use $v$ (degree 1)
$\implies G - v$ is connected.
Proof of only if.

**Thm:**
“G connected and has $|V| - 1$ edges” $\implies$ “G is connected and has no cycles.”

**Proof of $\implies$:**
Proof of only if.

Thm:
“G connected and has $|V| - 1$ edges” $\implies$
“G is connected and has no cycles.”

Proof of $\implies$: By induction on $|V|$.
Proof of only if.

Thm:
“G connected and has $|V| – 1$ edges” $\implies$
“G is connected and has no cycles.”

Proof of $\implies$: By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| – 1$ edges and has no cycles.
Proof of only if.

Thm: “G connected and has $|V| - 1$ edges” $\implies$ “G is connected and has no cycles.”

Proof of $\implies$: By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.
Proof of only if.

**Thm:**
“G connected and has $|V| - 1$ edges” $\implies$
“G is connected and has no cycles.”

**Proof of $\implies$:** By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:
Proof of only if.

**Thm:**
“$G$ connected and has $|V| - 1$ edges” $\implies$ “$G$ is connected and has no cycles.”

**Proof of $\implies$:** By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:
**Claim:** There is a degree 1 node.
Proof of only if.

Thm:
“G connected and has $|V| - 1$ edges” $\implies$
“G is connected and has no cycles.”

Proof of $\implies$: By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree $\geq 1$. 
Proof of only if.

**Thm:**
“G connected and has $|V| - 1$ edges” $\implies$
“G is connected and has no cycles.”

**Proof of $\implies$:** By induction on $|V|$.

**Base Case:** $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

**Induction Step:**

**Claim:** There is a degree 1 node.

**Proof:** First, connected $\implies$ every vertex degree $\geq 1$.

Sum of degrees is $2|E| = 2(|V| - 1) = 2|V| - 2$
Proof of only if.

**Thm:**
“G connected and has $|V| - 1$ edges” $\implies$ “G is connected and has no cycles.”

**Proof of $\implies$:** By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:

**Claim:** There is a degree 1 node.

**Proof:** First, connected $\implies$ every vertex degree $\geq 1$.
Sum of degrees is $2|E| = 2(|V| - 1) = 2|V| - 2$
Average degree $(2|V| - 2)/|V| = 2 - (2/|V|)$. 
Proof of only if.

Thm: “G connected and has $|V| - 1$ edges” $\implies$ “G is connected and has no cycles.”

Proof of $\implies$ : By induction on $|V|$.
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Average degree $(2|V| - 2)/|V| = 2 - (2/|V|)$. Must be a degree 1 vertex.
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   Cuz not everyone is bigger than average!
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Cuz not everyone is bigger than average!

By degree 1 removal lemma, $G - v$ is connected.
Proof of only if.

Thm:
“G connected and has $|V| - 1$ edges” $\implies$
“G is connected and has no cycles.”

Proof of $\implies$: By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.

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Sum of degrees is $2|E| = 2(|V| - 1) = 2|V| - 2$
Average degree $(2|V| - 2)/|V| = 2 - (2/|V|)$. Must be a degree 1 vertex.

Cuz not everyone is bigger than average!

By degree 1 removal lemma, $G - v$ is connected.
$G - v$ has $|V| - 1$ vertices and $|V| - 2$ edges so by induction
**Proof of only if.**

**Thm:**
“G connected and has \(|V| - 1\) edges” \(\implies\) “G is connected and has no cycles.”

**Proof of \(\implies\):** By induction on \(|V|\).

**Base Case:** \(|V| = 1. 0 = |V| - 1\) edges and has no cycles.

**Induction Step:**

**Claim:** There is a degree 1 node.

**Proof:** First, connected \(\implies\) every vertex degree \(\geq 1\).

Sum of degrees is \(2|E| = 2(|V| - 1) = 2|V| - 2\).

Average degree \((2|V| - 2)/|V| = 2 - (2/|V|)\). Must be a degree 1 vertex.

Cuz not everyone is bigger than average!

By degree 1 removal lemma, \(G - v\) is connected.

\(G - v\) has \(|V| - 1\) vertices and \(|V| - 2\) edges so by induction
\(\implies\) no cycle in \(G - v\).
Proof of only if.

**Thm:**
```
“G connected and has $|V| - 1$ edges” $\iff$
“G is connected and has no cycles.”
```

**Proof of $\implies$:** By induction on $|V|$.

**Base Case:** $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

**Induction Step:**

**Claim:** There is a degree 1 node.

**Proof:** First, connected $\implies$ every vertex degree $\geq 1$.

- Sum of degrees is $2|E| = 2(|V| - 1) = 2|V| - 2$
- Average degree $(2|V| - 2)/|V| = 2 - (2/|V|)$. Must be a degree 1 vertex.

Cuz not everyone is bigger than average!

By degree 1 removal lemma, $G - v$ is connected.

$G - v$ has $|V| - 1$ vertices and $|V| - 2$ edges so by induction
$\implies$ no cycle in $G - v$.

And no cycle in $G$ since degree 1 cannot participate in cycle.
Proof of only if.

Thm: “G connected and has \(|V| - 1\) edges” \(\implies\) “G is connected and has no cycles.”

Proof of \(\implies\): By induction on \(|V|\).
Base Case: \(|V| = 1\). \(0 = |V| - 1\) edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected \(\implies\) every vertex degree \(\geq 1\).
Sum of degrees is \(2|E| = 2(|V| - 1) = 2|V| - 2\)
Average degree \((2|V| - 2)/|V| = 2 - (2/|V|)\). Must be a degree 1 vertex.
Cuz not everyone is bigger than average!
By degree 1 removal lemma, \(G - v\) is connected.
\(G - v\) has \(|V| - 1\) vertices and \(|V| - 2\) edges so by induction
\(\implies\) no cycle in \(G - v\).
And no cycle in \(G\) since degree 1 cannot participate in cycle.
Proof of if

Thm:
“G is connected and has no cycles”
\[ \implies \text{“G connected and has } |V| - 1 \text{ edges”} \]

Proof:
Proof of if

**Thm:**
“G is connected and has no cycles”
⇒ “G connected and has \(|V| - 1\) edges”

**Proof:**
Walk from a vertex using untraversed edges.
Proof of if

**Thm:**
“G is connected and has no cycles”
\[\implies\] “G connected and has \(|V| - 1\) edges”

**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.
Proof of if

**Thm:**
“G is connected and has no cycles”
\[ \implies \text{ “G connected and has } |V| - 1 \text{ edges”} \]

**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.

**Claim:** Degree 1 vertex.
Proof of if

**Thm:**
“G is connected and has no cycles”
\[ \implies \text{“G connected and has } |V| - 1 \text{ edges”} \]

**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.

**Claim:** Degree 1 vertex.

**Proof of Claim:**
Can’t visit more than once since no cycle.
Proof of if

**Thm:**
“G is connected and has no cycles”
⇒ “G connected and has $|V| - 1$ edges”

**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.

**Claim:** Degree 1 vertex.

**Proof of Claim:**
Can’t visit more than once since no cycle.
Entered.
Thm:
“G is connected and has no cycles”
\[ \Rightarrow \text{“G connected and has } |V| - 1 \text{ edges”} \]

Proof:
Walk from a vertex using untraversed edges.
Until get stuck.

Claim: Degree 1 vertex.

Proof of Claim:
Can’t visit more than once since no cycle.
Entered. Didn’t leave.
Proof of if

**Thm:**
"G is connected and has no cycles"
\[\implies \text{"G connected and has } |V| - 1 \text{ edges"} \]

**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.

**Claim:** Degree 1 vertex.

**Proof of Claim:**
Can’t visit more than once since no cycle.
Entered. Didn’t leave. Only one incident edge.
Proof of if

**Thm:**
“G is connected and has no cycles”
⇒ “G connected and has \(|V| − 1\) edges”

**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.

**Claim:** Degree 1 vertex.

**Proof of Claim:**
Can’t visit more than once since no cycle.
Entered. Didn’t leave. Only one incident edge.

Removing node doesn’t create cycle.
Thm:
“G is connected and has no cycles”
    ⇒ “G connected and has |V| − 1 edges”

Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
**Proof of Claim:**
- Can’t visit more than once since no cycle.
- Entered. Didn’t leave. Only one incident edge.
Removing node doesn’t create cycle.
New graph is connected.
Proof of if

Thm:  
“G is connected and has no cycles”  
\[\implies \text{“G connected and has } |V| - 1 \text{ edges”} \]

Proof:  
Walk from a vertex using untraversed edges.  
Until get stuck.

Claim: Degree 1 vertex.

Proof of Claim:  
Can’t visit more than once since no cycle.  
Entered. Didn’t leave. Only one incident edge.

Removing node doesn’t create cycle.  
New graph is connected.  
Removing degree 1 node doesn’t disconnect from Degree 1 lemma.
Proof of if

**Thm:**
“$G$ is connected and has no cycles”

$\implies$ “$G$ connected and has $|V| - 1$ edges”

**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.

**Claim:** Degree 1 vertex.

**Proof of Claim:**
Can’t visit more than once since no cycle.
Entered. Didn’t leave. Only one incident edge.

Removing node doesn’t create cycle.
New graph is connected.
Removing degree 1 node doesn’t disconnect from Degree 1 lemma.
By induction $G - v$ has $|V| - 2$ edges.
Proof of if

Thm:
“G is connected and has no cycles”
\[\implies \text{“G connected and has } |V| - 1 \text{ edges”}\]

Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can’t visit more than once since no cycle.
Entered. Didn’t leave. Only one incident edge.
Removing node doesn’t create cycle.
New graph is connected.
Removing degree 1 node doesn’t disconnect from Degree 1 lemma.
By induction \(G - v\) has \(|V| - 2\) edges.
\(G\) has one more or \(|V| - 1\) edges.
Proof of if

**Thm:**
“G is connected and has no cycles”
⇒ “G connected and has |V| − 1 edges”

**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.

**Claim:** Degree 1 vertex.

**Proof of Claim:**
Can’t visit more than once since no cycle.
Entered. Didn’t leave. Only one incident edge.
Removing node doesn’t create cycle.
New graph is connected.
Removing degree 1 node doesn’t disconnect from Degree 1 lemma.
By induction \( G - v \) has \(|V| - 2\) edges.
\( G \) has one more or \(|V| - 1\) edges.
Let $G$ be a connected graph with $|V| - 1$ edges.
Poll: Oh tree, beautiful tree.

Let $G$ be a connected graph with $|V| - 1$ edges.

(A) Removing a degree 1 vertex can disconnect the graph.
(B) One can use induction on smaller objects.
(C) The average degree is $2 - 2/|V|$.
(D) There is a hotel california: a degree 1 vertex.
(E) Everyone can be bigger than average.
Let $G$ be a connected graph with $|V| - 1$ edges.

(A) Removing a degree 1 vertex can disconnect the graph.
(B) One can use induction on smaller objects.
(C) The average degree is $2 - 2/|V|$.
(D) There is a hotel california: a degree 1 vertex.
(E) Everyone can be bigger than average.

(B), (C), (D) are true
Lecture Summary.

Graphs.
Lecture Summary.

Graphs.
Basics.
Graphs.

Basics.

Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.
Lecture Summary.

Graphs.
Basics.
Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.
Connected Component.
Lecture Summary.

Graphs.
Basics.
Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.
Connected Component.
maximal set of vertices that are connected.
Lecture Summary.

Graphs.
Basics.
Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.
Connected Component.
maximal set of vertices that are connected.
Algorithm for Eulerian Tour.
Graphs.
Basics.
Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.
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maximal set of vertices that are connected.
Algorithm for Eulerian Tour.
Take a walk until stuck to form tour.
Lecture Summary.

Graphs.
Basics.
Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.
Connected Component.
maximal set of vertices that are connected.
Algorithm for Eulerian Tour.
Take a walk until stuck to form tour.
Remove tour.
Lecture Summary.

Graphs.
Basics.
Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.
Connected Component.
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Algorithm for Eulerian Tour.
Take a walk until stuck to form tour.
Remove tour.
Recurse on connected components.
Graphs.
Basics.
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  Take a walk until stuck to form tour.
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Basics.
Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.
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Trees: degree 1 lemma $\implies$ equivalence of several definitions.
Graphs.
  Basics.
  Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.
  Connected Component.
    maximal set of vertices that are connected.
Algorithm for Eulerian Tour.
  Take a walk until stuck to form tour.
  Remove tour.
  Recurse on connected components.

Trees: degree 1 lemma $\iff$ equivalence of several definitions.
  $G$: $n$ vertices and $n-1$ edges and connected.
    remove degree 1 vertex.
  $n-1$ vertices, $n-2$ edges and connected $\iff$ acyclic.
    (Ind. Hyp.)
  degree 1 vertex is not in a cycle.
  $G$ is acyclic.
Lecture Summary.

Graphs.

Basics.

Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.

Connected Component.

maximal set of vertices that are connected.

Algorithm for Eulerian Tour.

Take a walk until stuck to form tour.

Remove tour.

Recurse on connected components.

Trees: degree 1 lemma $\implies$ equivalence of several definitions.

$G$: $n$ vertices and $n - 1$ edges and connected.

remove degree 1 vertex.

$n - 1$ vertices, $n - 2$ edges and connected $\implies$ acyclic.

(Ind. Hyp.)

degree 1 vertex is not in a cycle.

$G$ is acyclic.