Q1
3 Points

For each of the following statements, determine whether each are propositions.

Q1.1
1 Point

2 is prime.

- (●) Proposition
- ( ) Not a proposition

Explanation

This is proposition.

Q1.2
1 Point

n is even.

- ( ) Proposition
- (●) Not a proposition

Explanation

This is not a proposition, since n is a free variable, and is unquantified.

Q1.3
1 Point

$\forall n \in \mathbb{N}$, n is even.

- (●) Proposition
- ( ) Not a proposition

Explanation

This is proposition, since n is quantified.

Q2 Connectives

4 Points

Which of the following are always true, when P and Q are propositions?

Q2.1

1 Point

$$P \vee \neg P$$

- (●) Always true
- ( ) Not always true

Explanation

P must either be true or false, so this expression is always true.

Q2.2

1 Point

$$(P \vee Q) \implies P$$

- ( ) Always true
- (●) Not always true

Explanation

If P is false while Q is true, this expression is false.

Q2.3
1 Point

$$P \equiv (P \vee Q) \wedge (P \vee \neg Q)$$

- (●) Always true
- ( ) Not always true

Explanation

If Q is true, the RHS simplifies to $\text{True} \wedge P \equiv P$, and if Q is false, the RHS simplifies to $P \wedge \text{True} \equiv P$. Equivalently, we can rewrite the RHS as $P \wedge (Q \vee \neg Q) \equiv P$, since $Q \vee \neg Q$ is always true.

Q2.4
1 Point

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

- (●) Always true
- ( ) Not always true

Explanation

This is true by DeMorgan's laws.

Q3 Quantifiers.

4 Points

Which of the following are always true for predicates $P(n)$ and $Q(n)$ and non-empty universe U?

Q3.1

1 Point

$$(\forall x \in U)(\neg P(x)) \implies (\exists x \in U)(\neg P(x))$$

- (●) Always true
- ( ) Not always true

Explanation

This is always true because if the predicate $P(x)$ is false for all x, then there's some x for which it's false.

Q3.2

1 Point

$$\neg(\forall x \in U)(P(x)) \equiv (\exists x \in U)(\neg P(x))$$

- (●) Always true
- ( ) Not always true

Explanation

This is true because saying "it is not the case that for all x, $P(x)$ is true" is equivalent to claiming "there is some x where $P(x)$ is false".

Q3.3
1 Point

$$(\forall x \in U)(P(x) \wedge Q(x)) \equiv \Big((\forall x \in U)(P(x)) \wedge (\forall x \in U)(Q(x))\Big)$$

- (selected) Always true
- Not always true

Explanation

This is true because in order for $P(x)$ and $Q(x)$ to be true then both individual predicates must be true.

Q3.4
1 Point

$$(\forall x \in U)(P(x) \vee Q(x)) \equiv \Big((\forall x \in U)(P(x)) \vee (\forall x \in U)(Q(x))\Big)$$

- Always true
- (selected) Not always true

Explanation

This is false because for $P(x)$ or $Q(x)$ to be true, only one of the two need to be true. One clear counter example is to let U be the set of positive integers, $P(x)$ be the claim "x is odd" and $Q(x)$ be the claim "x is even". We can see that the left hand side is true since every positive integer is either even or odd but the right hand side of the equivalence is false.