Q1 Principles of Induction

4 Points

Q1.1

1 Point

$(P(0) \wedge (\forall n \in \mathbb{N})(P(n) \implies P(n+1))) \equiv (\forall n \in \mathbb{N})(P(n))$

- (selected) True
- False

Explanation

This is the principle of induction.

Q1.2

1 Point

$(P(1) \wedge (\forall n \in \mathbb{N})(P(n) \implies P(n+1)))$ implies that $P(0)$ is true.

- True
- (selected) False

Explanation

The left hand side does imply that $P(n)$ is true for all $n \geq 1$. It says nothing about $P(0)$.

Q1.3
1 Point

$(P(k) \wedge (\forall n \in \mathbb{N})(P(n) \implies P(n+1)))$ for a natural number k implies that $(\forall n \in \mathbb{N})(n < k \vee P(n))$.

- [x] True
- [ ] False

Explanation

This follows from the principle of induction as every successor of k has $P(k)$ being true. Formally, one can apply the principle of induction directly to the statement $Q(n) = P(n+k)$, and then $P(k) = Q(0)$ and $\forall n \in \mathbb{N}, P(n) \implies P(n+1)$ implies the statement $\forall n \in \mathbb{N}, Q(n) \implies Q(n+1)$, so we have $\forall n \in \mathbb{N}, Q(n)$ which is equivalent to $\forall n \in \mathbb{N}, P(n+k) \equiv \forall n \in \mathbb{N}, n < k \vee P(n)$.

Q1.4
1 Point

$(\forall n \in \mathbb{N})(P(n))$ implies that $\neg(\exists n \in \mathbb{N})(\neg P(n))$.

- [x] True
- [ ] False

Explanation

If $P(n)$ is always true, there does not exist an n where $P(n)$ is false.

Q2 Properties of objects.

3 Points

Q2.1

1 Point

$\sum_{i=1}^{n} 1/i^2$ is strictly less than 2 for all natural numbers $n \geq 1$.

- (●) True
- ( ) False

Explanation

This is implied by Theorem 3.5 of the note 3.

Q2.2

1 Point

All maps (not necessarily formed using straight lines) are two-colorable.

- ( ) True
- (●) False

Explanation

The two-color theorem only applies to maps that can be formed with straight lines.

Q2.3
1 Point

For all $n \in \mathbb{N}$, there is $k \in \mathbb{Z}$ where $n^3 - n = 3k$.

- (●) True
- ( ) False

Explanation

This is a restatement of Theorem 3.2