Q1 Bayesian thinking

1 Point

Rahil buys a single lottery ticket from store A with probability $\frac{1}{3}$, otherwise he buys a ticket from store B. Tickets from store A have a winning probability of $\frac{1}{4}$, and tickets from store B has a winning probability of $\frac{1}{2}$. Given that Rahil wins the lottery, compute the probability that he bought the lottery ticket from store A.

- ( ) $\frac{1}{2}$
- ( ) $\frac{1}{3}$
- ( ) $\frac{1}{4}$
- (•) $\frac{1}{5}$
- ( ) $\frac{3}{4}$

Explanation

Let W denote the event that Rahil wins the lottery. Then, by Bayes Rule, we get $\mathbb{P}[A \mid W] = \frac{\mathbb{P}[W|A]\mathbb{P}[A]}{\mathbb{P}[W]} = \frac{\frac{1}{4}\cdot\frac{1}{3}}{\frac{1}{4}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{2}{3}} = \frac{1}{5}$

Q2 Independence
2 Points

Q2.1
1 Point

For any three events A_1, A_2, A_3 in the same probability space: if A_1, A_2 are independent and A_2, A_3 are independent, then A_1, A_3 are independent.

- ○ True
- ◉ False

Explanation

Consider $A_1 = A_3$. We can have A_1, A_2 to be independent, which means that A_2, A_3 are also independent (since A_3, A_1 are the same event). But an event is (usually) not independent with itself!

Q2.2
1 Point

If the series of events $A_1, A_2, \ldots, A_n$ is mutually independent, so is the series $\overline{A_1}, \overline{A_2}, \ldots, \overline{A_n}$.

- ◉ True
- ○ False

Explanation

This follows from the second definition of mutual independence given in the notes.

Q3 Intersections and Unions

3 Points

Q3.1

1 Point

For any sequence of events $A_1, \ldots, A_n$, the following equation holds true:
$\mathbb{P}\left[\bigcap_{i=1}^{n} A_i\right] = \mathbb{P}[A_1]\mathbb{P}[A_2 \mid A_1]\mathbb{P}[A_3 \mid A_1 \cap A_2] \cdots \mathbb{P}[A_n \mid \bigcap_{i=1}^{n-1} A_i]$

- (●) True
- ( ) False

> **Explanation**
>
> The product rule holds true for any events.

Q3.2

1 Point

If a sequence of events $A_1, \ldots, A_n$ satisfies $\mathbb{P}[\bigcup_{i=1}^{n} A_i] = \sum_{i=1}^{n} \mathbb{P}[A_i]$, then $A_1, \ldots, A_n$ are mutually independent.

- ( ) True
- (●) False

> **Explanation**
>
> They are mutually exclusive, not mutually independent.

Q3.3
1 Point

For any events A, B in a uniform probability space, the probability that neither of the events happens is $1 - \mathbb{P}[A] - \mathbb{P}[B]$.

- ○ True
- ◉ False

Explanation

This is only true if A and B are disjoint. In general, the probability in question is $\mathbb{P}[\overline{A \cup B}] = 1 - \mathbb{P}[A \cup B] = 1 - \mathbb{P}[A] - \mathbb{P}[B] + \mathbb{P}[A \cap B]$.