Q1 Random variables

2 Points

Consider a sample space Ω with $\mathbb{P}\colon \Omega \to \mathbb{R}$ and a a random variable $X\colon \Omega \to \mathbb{R}$.

Q1.1

1 Point

What is $\mathbb{P}[X = x]$?

- ○ $\sum_{\omega \in \Omega} \mathbb{P}[\omega]$
- ◉ $\sum_{\omega \in \Omega : X(\omega) = x} \mathbb{P}[\omega]$

Explanation

The first expression is 1. The second is summing over all sample points where $X(\omega) = x$, which is the probability that $X(\omega) = x$ over the possible choices of ω.

Q1.2

1 Point

If $x \neq y$, then $\mathbb{P}[X = x] + \mathbb{P}[X = y] = \sum_{\omega \in \Omega : X(\omega) \in \{x,y\}} \mathbb{P}[\omega]$.

- ◉ True
- ○ False

Explanation

This is because $X(\omega)$ is a function, and no single ω can have two different values of $X(\omega)$.

Q2 Distribution
5 Points

Consider flipping a fair coin three times. Define the random variable X corresponding to the number of heads we flip.

Q2.1
1 Point

What is the number of sample points in the sample space?

- ( ) 0
- ( ) 1
- ( ) 2
- ( ) 4
- (•) 8

Explanation

The number of sample points is $2 \times 2 \times 2 = 8$.

Q2.2
1 Point

What is $\mathbb{P}[X = 1]$?

- ( ) 0
- ( ) $1/7$
- ( ) $3/7$
- (•) $3/8$
- ( ) $1/2$
- ( ) $5/7$
- ( ) 1

Explanation

$X = 1$ is the event that there is one heads. This has 3 possibilities out of 8 in total.

Q2.3
1 Point

What is $\mathbb{P}[X = 2 \mid X \geq 1]$?

- ○ 0
- ○ $1/7$
- ◉ $3/7$
- ○ $3/8$
- ○ $1/2$
- ○ $5/7$
- ○ 1

Explanation

There are 3 outcomes with two heads out of 7 outcomes with at least one heads.

Q2.4
1 Point

What is $\mathbb{P}[X = 0 \mid X \geq 1]$?

- ◉ 0
- ○ $1/7$
- ○ $3/7$
- ○ $3/8$
- ○ $1/2$
- ○ $5/7$
- ○ 1

Explanation

If there is at least one heads, then we cannot have $X = 0$.

Q2.5
1 Point

Does X follow a binomial distribution?

- (●) Yes
- ( ) No

Explanation
We are repeating 3 trials with probability of success 0.5.

Q3 Binomial vs Bernoulli distribution

1 Point

Let n be a large integer and $p \in [0, 1]$ be a parameter. Assume X is a discrete random variable with probability density function $\mathbb{P}[X = i] = \binom{n}{i} p^i (1-p)^{n-i}$ for each $i = 0, 1, \ldots, n$.
Then the distribution of X is

- (●) Binomial distribution
- ( ) Bernoulli distribution

Explanation
See note.

Q4 Joint distribution and independence

1 Point

Let X, Y, Z be random variables on $\{0, 1\}$. Assume

- the marginal distribution of X is uniform over $\{0, 1\}$, and same for Y, Z;
- the joint distribution of (X, Y) is uniform over $\{0, 1\}^2$, and same for $(X, Z), (Y, Z)$.

Then the joint distribution of (X, Y, Z) must be uniform over $\{0, 1\}^3$.

- ○ True
- ◉ False

Explanation

Let X and Y be independent unbiased coins and define $Z = X + Y \pmod 2$. Then you can verify that the conditions hold, but (X, Y, Z) can never be $(0, 0, 1)$.

Q5
2 Points

Let X be the outcome of one six-sided die roll and Y be the outcome of a coin flip (i.e., $Y = 1$ corresponds to heads and $Y = 0$ corresponds to tails).

Q5.1
1 Point

What is $\mathbb{E}[X]$?

- ( ) 1
- ( ) 2.75
- (•) 3.5
- ( ) 4
- ( ) 5.5
- ( ) 6

Explanation

The outcomes of a fair die roll have equal probabilities over $1, 2, 3, 4, 5, 6$. Therefore the expectation is $(1 + 2 + 3 + 4 + 5 + 6)/2 = 3.5$.

Q5.2
1 Point

What is $\mathbb{E}[X + 4Y]$?

- ○ 1
- ○ 2.75
- ○ 3.5
- ○ 4
- ◉ 5.5
- ○ 6

Explanation

$\mathbb{E}[Y] = 0.5$. Then $\mathbb{E}[X + 4Y] = \mathbb{E}[X] + 4\mathbb{E}[Y] = 3.5 + 4 \cdot 0.5 = 5.5$.