## Q1 Variance of 4-sided die

**2 Points**

Let $X$ represent the result of 1 roll of a fair 4-sided die.

### Q1.1

**1 Point**

What is $\mathbb{E}[X^2]$?

- (selected) $\frac{15}{2}$
- $3$
- $\frac{9}{2}$
- $15$

**Explanation**

This is equal to $\sum_{i=1}^{4} i^2 \mathbb{P}[X = i] = \frac{1}{4}(1^2 + 2^2 + 3^2 + 4^2) = \frac{15}{2}$.

### Q1.2

**1 Point**

What is $\text{Var}(X)$?

- $5$
- $\frac{5}{2}$
- $\frac{15}{8}$
- (selected) $\frac{5}{4}$

**Explanation**

This is equal to $\mathbb{E}[X^2] - \mathbb{E}[X]^2 = \frac{15}{2} - (\frac{5}{2})^2 = \frac{30}{4} - \frac{25}{4} = \frac{5}{4}$.

Alternatively, you could derive that the variance of a discrete $\text{Uniform}[1, n]$ random variable is $\frac{n^2-1}{12}$ so the variance here is $\frac{4^2-1}{12} = \frac{15}{12} = \frac{5}{4}$.

## Q2 Variance, and Covariance

**3 Points**

Let $Y = Y_1 + Y_2 + Y_3 + Y_4$ represent the number of heads from tossing 4 fair coins, where each $Y_i$ is an indicator representing whether the $i$th toss was a head (1 for head, 0 for tail).

### Q2.1

**1 Point**

What is $\text{Var}(Y)$?

- (selected) $1$
- $\frac{3}{2}$
- $2$
- $\frac{1}{2}$

**Explanation**

Since all the $Y_i$ are independent, $\text{Var}[Y] = \text{Var}[\sum_{i=1}^{4} Y_i] = \sum_{i=1}^{4} \text{Var}[Y_i] = \sum_{i=1}^{4} \text{Var}[Y_i] = 4 \cdot \text{Var}[Y_1] = 4 \cdot (\frac{1}{4}) = 1.$

**Important note: "linearity" of variance does not hold in general!** This is a special case since all the $Y_i$ are independent, so their *covariances* are all 0. Usually, you will have to use bilinearity of covariance to "FOIL" out sums of random variables, and calculate the individual covariances between terms.

**Q2.2**
**1 Point**

What is $\mathrm{Cov}(Y_1, Y_2)$?

- (selected) $0$
- $\frac{1}{2}$
- $\frac{1}{4}$
- $-\frac{1}{4}$

**Explanation**

Since all the $Y_i$ are independent, $\mathrm{Cov}(Y_1, Y_2) = 0$. Mathematically, we have that $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$ for independent RVs, so $\mathrm{Cov}(Y_1, Y_2) = \mathbb{E}[Y_1 Y_2] - \mathbb{E}[Y_1]\mathbb{E}[Y_2] = 0$.

**Q2.3**
**1 Point**

What is $\mathrm{Cov}(Y, Y_1)$?

- $0$
- $\frac{1}{2}$
- (selected) $\frac{1}{4}$
- $-\frac{1}{4}$

**Explanation**

By bilinearity of covariance, we have $\mathrm{Cov}(Y, Y_1) = \mathrm{Cov}(Y_1, Y_1) + \mathrm{Cov}(Y_2, Y_1) + \mathrm{Cov}(Y_3, Y_1) + \mathrm{Cov}(Y_4, Y_1) = \mathrm{Cov}(Y_1, Y_1) = \mathrm{Var}(Y_1) = \frac{1}{4}$ since $Y_1$ is independent from all other $Y_i$'s.