Q1 The Geometric Distribution w/ Coupons

4 Points

Q1.1

1 Point

What is the expected value of a geometric random variable with parameter p?

- (selected) $\frac{1}{p}$
- $\frac{1}{p^2}$
- p
- $p(1-p)$

Explanation

This is the expectation for geometric which was proven. One way to prove it is to note that $\mathbb{E}[X] = p(1) + (1-p)(1+\mathbb{E}[X])$ since after the first trial, you are either done with probability p or you start over with probability $1-p$. If you are done, the value of the random variable is 1, if not the expected value of the random variable is 1 for the trial you did plus the expected value of the remaining trials.

Q1.2
1 Point

For the next two subparts, suppose there are n coupons and each "trial," you get one random coupon with equal probability among the n coupons. Let X_i denote the number of trials used to obtain the ith new coupon (a coupon you do not already have), since obtaining the $(i-1)$th new coupon.

$X_1 = 1$ with probability 1.

- (●) True
- ( ) False

Explanation

We do not have any coupons at the start, so any coupon we receive will be a new coupon.

Q1.3
1 Point

What is the expectation of X_i?

- (●) $\frac{n}{n-i+1}$
- ( ) $\frac{1}{i}$
- ( ) $\frac{n}{i}$
- ( ) $\frac{n}{n-i}$

Explanation

Since $i-1$ coupons have already been collected, the number of trials until the ith coupon is $\text{Geom}\left(\frac{n-(i-1)}{n}\right)$, which has mean $\frac{n}{n-i+1}$.

Q1.4
1 Point

What is the $\sum_{i=1}^{n} 1/i$ closest to?

- ○ $n(n+1)/2$
- ○ n^2
- ◉ $\ln n$

Explanation

One can approximate the sum with $\int_1^n 1/x dx$ as that is the area under the curve which can be approximated with rectangles each of height $1/i.$ The integral evaluates to $\ln n - \ln 1 = \ln n.$ It's a bit off as the notes say.

Q2 Poisson's Waiting Game

4 Points

Q2.1 Office Hours

1 Point

Sam and Andy are waiting for students in their office hours. They've observed that students arrive at a rate of one student every 30 minutes, and decide to model student arrivals as a Poisson random variable.

One day, nobody has shown up after 50 minutes. They're debating whether they should leave early to eat pineapple pizza. What is the probability that at least one student arrives in the last ten minutes?

- ( ) $\frac{1}{3}$
- ( ) $e^{-1/3}$
- (•) $1 - e^{-1/3}$
- ( ) $1 - e^{-1/2}$

Explanation

Let $\lambda_{10} = \frac{1}{3}$. Let S_1 be the number of students who arrive in ten minutes. Then $S_1 \sim \text{Poisson}(\lambda_{10})$. We have $P[S_1 = 0] = e^{-\lambda_{10}} = e^{-1/3}$, so $P[S_1 > 0] = 1 - e^{-1/3}$.

Q2.2
1 Point

Suppose Jay observes the same rate of student arrivals. However, due to his excellent teaching skills, once a student arrives at his OH, they do not leave. If nobody is in the room at the start of the hour, what is the expected number of students in the room at the end of the hour?

- ○ e^{-2}
- ○ 1
- ○ $\frac{1}{2}$
- ◉ 2

Explanation

Let $\lambda_{60} = 2$. Let S_2 be the number of students who arrive in one hour. Then $S_2 \sim \text{Poisson}(\lambda_{60})$ and $\mathbb{E}[S_2] = \lambda_{60} = 2$.

Q2.3 BART
1 Point

Suppose we model the departure of BART trains as an exponential random variable. On average, BART trains leave Richmond every 20 minutes.

If Ishan shows up at a random time, what is his expected wait for a BART train to leave?

- ○ 7.5 minutes
- ○ 15 minutes
- ○ 30 minutes
- ◉ 20 minutes

Explanation

20 minutes. By the memoryless property, this is equal to the expected wait time for a departure from any arbitrary point.

Q2.4
1 Point

A station agent informs Ishan that the most recent train left 5 minutes ago; what is his expected wait for a BART train?

- ( ) 10 minutes
- ( ) 7.5 minutes
- (•) 20 minutes
- ( ) 15 minutes

Explanation

20 minutes. Same as the previous part, since whatever happens in a previous time interval is irrelevant for a memoryless random variable.