Q1 Markov's inequality

2 Points

Q1.1

1 Point

Markov inequality only applies to nonnegative random variables.

- (●) True
- ( ) False

Explanation

This is required in the proof of Markov's inequality.

Q1.2

1 Point

Suppose $X \geq 0$ is a nonnegative random variable, and $\mathbb{E}[X] = 5$. Use Markov's inequality to bound $\mathbb{P}[X \geq 10]$.

- ( ) $\mathbb{P}[X \geq 10] \leq \frac{1}{5}$
- (●) $\mathbb{P}[X \geq 10] \leq \frac{1}{2}$
- ( ) $\mathbb{P}[X \geq 10] \geq \frac{1}{5}$
- ( ) $\mathbb{P}[X \geq 10] \leq \frac{2}{3}$

Explanation

By Markov's inequality, $\mathbb{P}[X \geq 10] \leq \frac{\mathbb{E}[X]}{10} = \frac{1}{2}$.

Q2 Chebyshev's inequality

1 Point

Suppose I flip 100 fair coins. Let X denote the number of heads. Use Chebyshev's inequality to bound $\mathbb{P}[|X-50| \geq 20]$.

- (selected) $\mathbb{P}[|X-50| \geq 20] \leq \frac{1}{16}$
- $\mathbb{P}[|X-50| \geq 20] \leq \frac{1}{2}$
- $\mathbb{P}[|X-50| \geq 20] \leq \frac{1}{20}$
- $\mathbb{P}[|X-50| \geq 20] \leq \frac{5}{2}$

Explanation

The variance of one fair coin flip is $\frac{1}{4}$, so $\mathrm{Var}(X) = \frac{100}{4} = 25$ by independence of the coin flips. Also, $\mathbb{E}[X] = 50$. Thus, $\mathbb{P}[|X-50| \geq 20] \leq \frac{\mathrm{Var}(X)}{20^2} = \frac{25}{400} = \frac{1}{16}$.

Q3 Bounding Variances

1 Point

Let X be a Bernoulli random variable with parameter p. What is the maximum possible value for $\mathrm{Var}(X)$?

- 0
- (selected) 0.25
- 0.5
- 1

Explanation

If X is a Bernoulli random variable, then $\mathrm{Var}(X) = p(1-p)$. Intuitively, if $p = 0.5$, then $p = 1-p$, yielding the highest possible value when multiplied, which is 0.25. This can also be solved analytically - taking the derivative yields $1 - 2p = 0$, so $p = 0.25$ and $p(1-p) = 0.5 \cdot 0.5 = 0.25$.

Q4 Chebyshev's inequality conditions
1 Point

Chebyshev's inequality holds for any constant c.

- ○ True
- ◉ False

Explanation

Chebyshev's only holds for positive constants c.