Q1 Basic Continuous Probability

2 Points

Let $F(x)$ be the cumulative distribution function of a random variable X, and $f(x)$ be its probability density function.

Q1.1 CDF

1 Point

True or False: $F(x) = \frac{\mathrm{d}f(x)}{\mathrm{d}x}$ for any $x \in \mathbb{R}$

- ○ True
- ◉ False

Explanation

$F(x) = \mathbb{P}[X \leq x] = \int_{-\infty}^{x} f(z)\mathrm{d}z$, so differentiating both sides with respect to x gives us $f(x) = \frac{\mathrm{d}F(x)}{\mathrm{d}x}$. Note that F and f have been flipped in the question.

Q1.2 Joint distribution

1 Point

True or False: $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\mathrm{d}x\mathrm{d}y = 1$.

- ◉ True
- ○ False

Explanation

Analogous to probability summing to 1 with discrete random variables.

Q2 Continuous Distributions - Uniform

2 Points

Let X be a uniform r.v. on the interval $[0, p]$. Which of the following are true statements?

Q2.1

1 Point

$f_X(x) = \frac{1}{p}, \forall x \in R$

- [ ] True
- [x] False

Explanation

The PDF is incorrect, since it has to be 0 outside of the range $[0, p]$.

Q2.2

1 Point

$\mathbb{E}[X] = \frac{p}{2}$

- [x] True
- [ ] False

Explanation

$\mathbb{E}[X] = \int_0^p \frac{1}{p} x \mathrm{d}x = \frac{1}{p} \frac{p^2}{2} = \frac{p}{2}$

Q3 Continuous Distribution - Exponential

4 Points

Let X be a exponential r.v. with parameter λ. Which of the following are true statements?

Q3.1

1 Point

$f_X(x) = \lambda e^{-\lambda x}$ for $x \geq 0$, and 0 elsewhere

- (●) True
- ( ) False

Explanation

By definition.

Q3.2

1 Point

For $t \geq 0$, $\mathbb{P}[X > t] = e^{-\lambda t}$

- (●) True
- ( ) False

Explanation

$\mathbb{P}[X > t] = \int_t^\infty \lambda e^{-\lambda x} \mathrm{d}x = (-e^{-\lambda x}) \big|_{x=t}^{\infty} = e^{-\lambda t}$

Q3.3
1 Point

For $t \geq 0$, $\mathbb{P}[X \geq t] > \mathbb{P}[X > t]$

- ○ True
- ◉ False

Explanation

The two sides differ by the quantity $\mathbb{P}[X = t]$, which is equal to zero for continuous random variables. So the two sides are actually equal.

Q3.4
1 Point

$\mathbb{E}[X] = \lambda$

- ○ True
- ◉ False

Explanation

The expected value of an exponential random variable is $\frac{1}{\lambda}$.

Q4 Integration Review
3 Points

As we stated in HW 1, we expect you to compute derivatives, integrals, and double integrals. Please take the time to compute these integrals by hand.

Q4.1
1 Point

$\int_0^\infty xe^{-x}dx$

=1+-0

Explanation

We want to find the integral of a product of functions, which Integration by Parts is appropriate for.

Let $u = x, dv = e^{-x}dx$. Then $du = dx, v = -e^{-x}$.
Integration by Parts gives us $\int udv = uv - \int vdu$.

$\int x(e^{-x}dx) = x(-e^{-x}) - \int -e^{-x}(dx)$
$= -xe^{-x} + \int e^{-x}dx$
$= -xe^{-x} - e^{-x}$

Integrating from 0 to ∞:
$\int_0^\infty xe^{-x}dx = \lim_{b\to\infty} \int_0^b xe^{-x}dx$
$= \lim_{b\to\infty}[-xe^{-x} - e^{-x}]_{x=0}^{x=b}$
$= \lim_{b\to\infty}[(-be^{-b} - e^{-b}) - (-0e^{-0} - e^{-0})]$
$= 0 + 1$
$= 1$

Note: Notice that the integral $\int_0^\infty xe^{-x}dx$ is precisely the expectation of an $\mathrm{Exp}(1)$ random variable.

Q4.2
1 Point

$\frac{1}{b^5}\int_0^b 9(bx)^2dx$, for all **constants** $b > 0$

=3+-0

Explanation

First, notice that the variable of integration is x. The other symbol, b, behaves like a **constant**.

$$\frac{1}{b^5}\int_0^b 9(bx)^2dx = \frac{1}{b^5}9b^2\int_0^b x^2dx$$
$$= \frac{9}{b^3}\left[\frac{1}{3}x^3\right]_{x=0}^{x=b}$$
$$= \frac{9}{b^3}\left[\frac{1}{3}b^3 - \frac{1}{3}0^3\right]$$
$$= 3$$

Note: Treating symbols that aren't the variable of integration like **constants** is relevant for computing double integrals!

Q4.3
1 Point

$\int_0^4 \int_0^3 (x^2 + xy)dydx$

If you're stuck, evaluate the inner integral first, keeping x as a **constant**.

=100+-0

Explanation

First, evaluate the inner integral, treating x as a **constant**.

$\int_0^4 \int_0^3 (x^2 + xy)dydx = \int_0^4 [x^2y + \frac{1}{2}xy^2]_{y=0}^{y=3} dx$
$= \int_0^4 [(x^2(3) + \frac{1}{2}x(3)^2) - (x^2(0) + \frac{1}{2}x(0)^2)]dx$
$= \int_0^4 (3x^2 + \frac{9}{2}x)dx$

Now let's evaluate the outer integral.
$\int_0^4 (3x^2 + \frac{9}{2}x)dx = [x^3 + \frac{9}{4}x^2]_{x=0}^{x=4}$
$= [(4^3 + \frac{9}{4}4^2) - (0^3 + \frac{9}{4}0^2)]$
$= 64 + 36$
$= 100$