Q1 Gone fishing

5 Points

Q1.1

1 Point

You go fishing a very large number of times.

The number of fish you catch on a giving fishing trip is a random variable of some distribution.

Which distribution best approximates the total number of fish you catch (throughout all the fishing trips)?

- ( ) Binomial
- ( ) Geometric
- ( ) Poisson
- ( ) Exponential
- (•) Normal

Explanation

The sum of a set of independent and identically distributed random variables approaches a normal distribution by CLT.

Q1.2
1 Point

You catch an expected number of 1.5 fish per hour. You can catch a fish at any instant of time.

Which distribution best characterizes the number of fish you catch in one hour of fishing?

- ○ Binomial
- ○ Geometric
- ◉ Poisson
- ○ Exponential
- ○ Normal

Explanation
You're measuring the discrete number of fish you catch based on the rate at which fish-catching happens in a continuous time interval.

Q1.3
1 Point

You catch an expected number of 1.5 fish per hour. You can catch a fish at any instant of time.

Which distribution best characterizes how the amount of time it takes to catch a fish after casting the line?

- ○ Binomial
- ○ Geometric
- ○ Poisson
- ◉ Exponential
- ○ Normal

Explanation
You're measuring the continuous duration of time until a fish is caught.

Q1.4
1 Point

Each day you fish for 2 hours. Assume that the numbers of fish you catch each day are independent.

Which distribution best characterizes the number of days until you catch more than 10 fish in one day?

- ( ) Binomial
- (•) Geometric
- ( ) Poisson
- ( ) Exponential
- ( ) Normal

Explanation

The probabilities of catching more than 10 fish on each day are independently and identically distributed, and you're measuring the number of trials until success.

Q1.5
1 Point

Each day you fish for 2 hours. Assume that the numbers of fish you catch each day are independent.

Which distribution best characterizes the number of days you catch no fish over a week of fishing?

- (•) Binomial
- ( ) Geometric
- ( ) Poisson
- ( ) Exponential
- ( ) Normal

Explanation

The probabilities of catching no fish on each day are independently and identically distributed, and you're measuring the total number of successful trials.

Q2 Normal distribution

3 Points

Let X be a Normal random variable with mean μ and variance σ^2. Which of the following are true statements?

Q2.1

1 Point

$\frac{X-\mu}{\sigma} \sim N(0,1)$

- (●) True
- ( ) False

Explanation

Let $Y = \frac{X-\mu}{\sigma}$. $\mathbb{P}[a \leq Y \leq b] = \mathbb{P}[a\sigma + \mu \leq X \leq b\sigma + \mu] = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{a\sigma+\mu}^{b\sigma+\mu} e^{-(x-\mu)^2/(2\sigma^2)} = \frac{1}{\sqrt{2\pi}} \int_a^b e^{-y^2/2} \mathrm{d}y.$

Q2.2

1 Point

The sum of any two normal random variables is also normally distributed

- ( ) True
- (●) False

Explanation

This is true if and only if the variables are independent. Proof under Corollary 21.1 in Note 21

Q2.3
1 Point

The sum of n i.i.d. random variables follows a normal distribution as n approaches infinity.

- (●) True
- ( ) False

Explanation

Let S_n be the sum of n i.i.d random variables each with mean μ, variance σ^2. From CLT, $S_n \sim N(n\mu, n\sigma^2)$, which is normal.

Q3 CLT
1 Point

Suppose $S_n = X_1 + \cdots + X_n$, where X_i's are i.i.d random variables with $\mathbb{E}[X_i] = \mu$, $\mathrm{Var}(X_i) = \sigma^2$. As $n \to \infty$, which of the following random variables converges to the standard normal distribution?

- ( ) $\frac{S_n}{n}$
- ( ) $\frac{S_n}{n} - \mu$
- (●) $\frac{S_n - n\mu}{\sigma\sqrt{n}}$
- ( ) $\frac{S_n - n\mu}{\sigma n}$

Explanation

Note that by linearity of expectation, $\mathbb{E}[\frac{S_n - n\mu}{\sigma\sqrt{n}}] = \frac{1}{\sigma\sqrt{n}}(\mathbb{E}[S_n] - n\mu) = \frac{1}{\sigma\sqrt{n}}(\mathbb{E}[X_1] + \cdots + \mathbb{E}[X_n] - n\mu) = 0$, and since X_i's are independent, $\mathrm{Var}(\frac{S_n - n\mu}{\sigma\sqrt{n}}) = \frac{1}{\sigma^2 n}\mathrm{Var}(S_n - n\mu) = \frac{1}{\sigma^2 n}\mathrm{Var}(S_n) = \frac{1}{\sigma^2 n}(\mathrm{Var}(X_1) + \cdots + \mathrm{Var}(X_n)) = \frac{1}{\sigma^2 n} \cdot \sigma^2 n = 1$. By CLT, $\frac{S_n - n\mu}{\sigma\sqrt{n}}$ converges to $N(0,1)$ as $n \to \infty$.