## Q1 Types of graphs
**3 Points**

### Q1.1 Edge sum
**1 Point**

What is the sum of all vertex degrees in an $n$ vertex, $m$ edge undirected graph?

- ○ $(n-1)$
- ○ $n$
- ○ $n(n-1)/2$
- ○ $n \log_2 n$
- ○ $2(n \log_2 n)$
- ○ $(n \log_2 n)/2$
- ○ $m$
- ○ $2n$
- ◉ $2m$

**Explanation**

Handshake lemma

**Q1.2 Tree**
**1 Point**

A tree on $n$ vertices has exactly how many edges?

- (selected) $(n-1)$
- $n$
- $n(n-1)/2$
- $n \log_2 n$
- $2(n \log_2 n)$
- $(n \log_2 n)/2$

**Explanation**

$n-1$. A tree is connected and has no cycles, it needs $n-1$ edges to be connected, and adding one would create a cycle.

**Q1.3 HyperHyperHyper**
**1 Point**

A hypercube on $n$ vertices has exactly how many edges? (Hint: Is $\log_2 n$ a natural number?)

- $(n-1)$
- $n$
- $n(n-1)/2$
- $n \log_2 n$
- $2(n \log_2 n)$
- (selected) $(n \log_2 n)/2$

**Explanation**

A hypercube of dimension $d$ has $n = 2^d$ vertices and each vertex has degree $d = \log_2 n$. Thus the number of edges is the sum of degrees divided by 2 or $2^d d/2 = n \log_2 n/2$.

## Q2 Planar graphs
**2 Points**

### Q2.1 Max edge
**1 Point**

What is the maximum number of edges in an $n$-vertex planar graph?

- ○ $(n-1)$
- ○ $1$
- ○ $0$
- ◉ $3n-6$
- ○ $2$

**Explanation**

This is derived in the notes from Euler's formula.

### Q2.2 Tree is flat
**1 Point**

What is the number of faces in a planar drawing of an $n$-vertex tree?

- ○ $(n-1)$
- ◉ $1$
- ○ $0$
- ○ $3n-6$
- ○ $2$

**Explanation**

There are no cycles so the plane is one connected piece.

## Q3 Graph Coloring

**4 Points**

### Q3.1

**1 Point**

$K_5$ is 4-colorable.

- ○ True
- ◉ False

**Explanation**

$K_5$ requires 5 colors; all vertices are connected to one another, so each must be a different color.

### Q3.2

**1 Point**

$K_{3,3}$ is 4-colorable.

- ◉ True
- ○ False

**Explanation**

$K_{3,3}$ is bipartite, so it is 2-colorable and therefore 4-colorable.

**Q3.3**
**1 Point**

There exists a graph $G$ with 9 edges and 5 vertices that is *not* 4-colorable.

- ○ True
- ◉ False

**Explanation**
$G$ must be planar; it cannot contain $K_5$, as it does not have enough edges, and it cannot contain $K_{3,3}$, as it does not have enough vertices. The 4-color theorem says that $G$ must be 4-colorable.

**Q3.4**
**1 Point**

There exists a graph $G$ with 10 edges and 6 vertices that is *not* 4-colorable.

- ◉ True
- ○ False

**Explanation**
Consider $K_5$ with an isolated vertex; it has 10 edges and 6 vertices, but requires 5 colors.