

### Q1 Bayesian thinking

1 Point

Rahil buys a single lottery ticket from store  $A$  with probability  $\frac{1}{3}$ , otherwise he buys a ticket from store  $B$ . Tickets from store  $A$  have a winning probability of  $\frac{1}{4}$ , and tickets from store  $B$  has a winning probability of  $\frac{1}{2}$ . Given that Rahil wins the lottery, compute the probability that he bought the lottery ticket from store  $A$ .

- ☐  $\frac{1}{2}$
- ☐  $\frac{1}{3}$
- ☐  $\frac{1}{4}$
- ☒  $\frac{1}{5}$
- ☐  $\frac{3}{4}$

#### Explanation

Let  $W$  denote the event that Rahil wins the lottery. Then, by Bayes Rule, we

$$\text{get } \mathbb{P}[A \mid W] = \frac{\mathbb{P}[W|A]\mathbb{P}[A]}{\mathbb{P}[W]} = \frac{\frac{1}{4} \cdot \frac{1}{3}}{\frac{1}{4} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{3}} = \frac{1}{5}$$

## Q2 Independence

2 Points

### Q2.1

1 Point

For any three events  $A_1, A_2, A_3$  in the same probability space: if  $A_1, A_2$  are independent and  $A_2, A_3$  are independent, then  $A_1, A_3$  are independent.

☐ True

☒ False

#### Explanation

Consider  $A_1 = A_3$ . We can have  $A_1, A_2$  to be independent, which means that  $A_2, A_3$  are also independent (since  $A_3, A_1$  are the same event). But an event is (usually) not independent with itself!

### Q2.2

1 Point

If the series of events  $A_1, A_2, \dots, A_n$  is mutually independent, so is the series  $\overline{A_1}, \overline{A_2}, \dots, \overline{A_n}$ .

☒ True

☐ False

#### Explanation

This follows from the second definition of mutual independence given in the notes.

### Q3 Intersections and Unions

3 Points

#### Q3.1

1 Point

For any sequence of events  $A_1, \dots, A_n$ , the following equation holds true:

$$\mathbb{P} \left[ \bigcap_{i=1}^n A_i \right] = \mathbb{P}[A_1] \mathbb{P}[A_2 \mid A_1] \mathbb{P}[A_3 \mid A_1 \cap A_2] \cdots \mathbb{P}[A_n \mid \bigcap_{i=1}^{n-1} A_i]$$

☒ True

☐ False

Explanation

The product rule holds true for any events.

#### Q3.2

1 Point

If a sequence of events  $A_1, \dots, A_n$  satisfies  $\mathbb{P}[\bigcup_{i=1}^n A_i] = \sum_{i=1}^n \mathbb{P}[A_i]$ , then  $A_1, \dots, A_n$  are mutually independent.

☐ True

☒ False

Explanation

They are mutually exclusive, not mutually independent.

**Q3.3****1 Point**

For any events  $A, B$  in a uniform probability space, the probability that neither of the events happens is  $1 - \mathbb{P}[A] - \mathbb{P}[B]$ .

☐ True

☒ False

**Explanation**

This is only true if  $A$  and  $B$  are disjoint. In general, the probability in question is  $\mathbb{P}[\overline{A \cup B}] = 1 - \mathbb{P}[A \cup B] = 1 - \mathbb{P}[A] - \mathbb{P}[B] + \mathbb{P}[A \cap B]$ .