Q1 Bijections and Cardinality

3 Points

Check if the following statements are correct.

Q1.1

1 Point

Let S and T be two sets. Let $f\colon S \to T$ be a mapping.
Then f is a bijection if and only if (1) for every $y \in T$, there is some $x \in S$ such that $f(x) = y$, and (2) for every distinct $x, x' \in S$, we have $f(x) \neq f(x')$.

- (●) True
- ( ) False

Explanation

By the definition of bijection (injective and surjective).

Q1.2

1 Point

If two sets are finite and there is a bijection between them, then they may still have different size.

- ( ) True
- (●) False

Explanation

A bijection between two sets S, T means $|S| = |T|$.

Q1.3
1 Point

Let S be the set of even integers. Let T be the set of integers congruent to 2 modulo 4.
Then $T \subsetneq S$ and there is no bijection between them.

- ( ) True
- (•) False

Explanation

Define $f\colon S \to T$ by $f(x) = 2x + 2$. This is a bijection.

Q2 Countability

3 Points

Q2.1

1 Point

Every finite set is countable.

- (selected) True
- False

Explanation

It's always possible to bijectively map a finite set to a subset of $\mathbb{N}$, so any finite set is always countable..

Q2.2

1 Point

$\mathbb{N}$ and $2\mathbb{N}$ have the same cardinality.

- (selected) True
- False

Explanation

The map $f(x) = 2x$ is a bijective map.

Q2.3
1 Point

The set $A = \{(x, y) | x \in \mathbb{Q}, y \in \mathbb{Q}\}$ is uncountable.

- ◯ True
- ◉ False

Explanation

The Cartesian product of two countable sets is always countable.