Q1 Counting on Tree

3 Points

Let T be a rooted tree of three levels. Assume the root has n_1 child nodes and each vertex on the second level has n_2 child nodes.

Q1.1

1 Point

How many leaves do we have in T?

- ○ n_2
- ◉ $n_1 \cdot n_2$
- ○ $n_1 + n_1 \cdot n_2$
- ○ $1 + n_1 + n_1 \cdot n_2$

Explanation

n_1 choices for first level, n_2 choices for second level; multiply them together to get $n_1 \cdot n_2$ leaves.

Q1.2

1 Point

How many vertices do we have in T?

- ○ n_2
- ○ $n_1 \cdot n_2$
- ○ $n_1 + n_1 \cdot n_2$
- ◉ $1 + n_1 + n_1 \cdot n_2$

Explanation

The first level has 1 vertex, second level has n_1, and third level (leaves) has $n_1 \cdot n_2$.

Q1.3
1 Point

How many edges do we have in T?

- ○ n_2
- ○ $n_1 \cdot n_2$
- ◉ $n_1 + n_1 \cdot n_2$
- ○ $1 + n_1 + n_1 \cdot n_2$

Explanation

We can pair each non-root vertex with the edge on top of it. Therefore the number of edges in T is the number of vertices minus one (the root).

Q2 Sample without replacement
1 Point

Let n be a positive integer and $S = \{1, 2, \ldots, n\}$. Assume $1 \le k \le n$ is an integer and we iteratively draw k elements from S without replacement (that is, an element is removed from S once it is drawn). What is the number of possible element sequences from this process?

- ○ n^k
- ○ $n!$
- ◉ $\frac{n!}{(n-k)!}$

Explanation

The first draw has n possibilities, then the second draw has $n-1$ since the first element is removed. In general, the i-th draw has $n-i+1$ possibilities. Hence the total number is $n \cdot (n-1) \cdot \dots \cdot (n-k+1) = \frac{n!}{(n-k)!}$.

Q3 Sample with replacement

2 Points

Let n be a positive integer and $S = \{1, 2, \dots, n\}$. Assume $k \geq 1$ is an integer and we iteratively draw an element from S with replacement k times.

Q3.1

1 Point

What is the number of possible element sequences when the order of the elements matters?

- (selected) n^k
- $\binom{n+k-1}{k}$
- $\frac{n!}{(n-k)!}$

Explanation

Each of the draws have n possibilities; since we draw k times, the total number is n^k.

Q3.2

1 Point

What is the number of possible element sequences when the order of the elements do not matter?

- n^k
- (selected) $\binom{n+k-1}{k}$
- $\frac{n!}{(n-k)!}$

Explanation

See note.