Q1 FSEs and Balance Equations

2 Points

Q1.1 (Optional, will be covered in Thursday 4/30 lecture)

0 Points

Recall that a system of balance equations $\pi = \pi P$ are used to solve for the invariant distribution of a Markov chain.

Conceptually, how are these equations derived?

- ○ The equation for $\pi(i)$ is derived by using the **law of total probability** and looking at all possible transitions **out of** each state i.
- ○ The equation for $\pi(i)$ is derived by using the **law of total expectation** and looking at all possible transitions **out of** each state i.
- ◉ The equation for $\pi(i)$ is derived by using the **law of total probability** and looking at all possible transitions **into** each state i.
- ○ The equation for $\pi(i)$ is derived by using the **law of total expectation** and looking at all possible transitions **into** each state i.

Explanation

The balance equations assert that if we started with some distribution of states π, moving one timestep into the future will not change the distribution. As such, to find the probability of being in state i at any given timestep, we'd need to look at all possible ways of *arriving* at state i, and add these probabilities together using the law of total probability.

Q1.2
1 Point

Recall the first step equations for computing the probability of reaching a state A before reaching a state B; we define $\alpha(i)$ to be the probability of reaching A before B given that we are currently in state i.

Conceptually, how are these equations derived?

- (selected) The equation for $\alpha(i)$ is derived by using the **law of total probability** and looking at all possible transitions **out of** each state i.
- The equation for $\alpha(i)$ is derived by using the **law of total expectation** and looking at all possible transitions **out of** each state i.
- The equation for $\alpha(i)$ is derived by using the **law of total probability** and looking at all possible transitions **into** each state i.
- The equation for $\alpha(i)$ is derived by using the **law of total expectation** and looking at all possible transitions **into** each state i.

Explanation

Here, we want to find the probability of reaching A before B, given that we're at state i; to do so, we look at all possible steps *leaving* state i, and use the law of total probability to recursively compute the probabilities of reaching A before B when starting at these future states.

Q1.3
1 Point

Recall the first step equations for computing the expected number of steps before reaching a state A; we define $\beta(i)$ to be the expected number of steps to reach state A given that we are currently in state i.

Conceptually, how are these equations derived?

- ○ The equation for $\beta(i)$ is derived by using the **law of total probability** and looking at all possible transitions **out of** each state i.
- ◉ The equation for $\beta(i)$ is derived by using the **law of total expectation** and looking at all possible transitions **out of** each state i.
- ○ The equation for $\beta(i)$ is derived by using the **law of total probability** and looking at all possible transitions **into** each state i.
- ○ The equation for $\beta(i)$ is derived by using the **law of total expectation** and looking at all possible transitions **into** each state i.

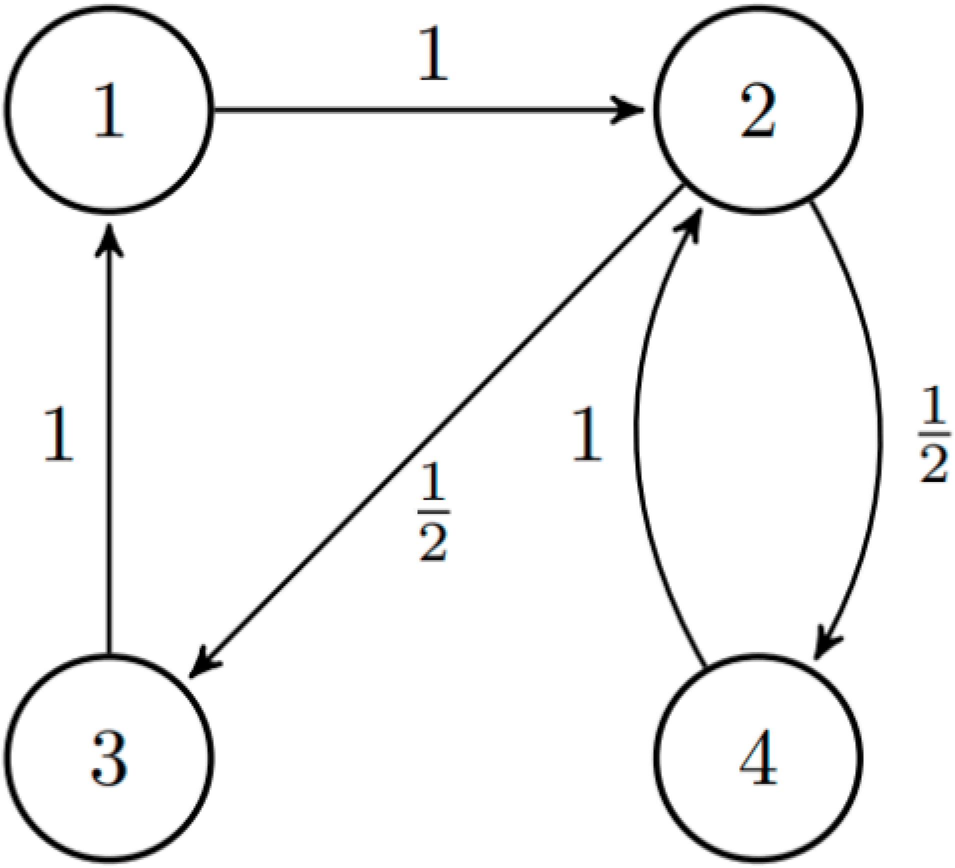
Explanation

Here, we want to find the expected hitting time for state A, given that we're at state i; to do so, we look at all possible steps *leaving* state i, and use the law of total expectation to recursively compute the expected number of steps to reach state A when starting at these future states.

Q2 Markov chain example

2 Points

Consider the following Markov chain:



Q2.1
1 Point

Starting at 1, what is the probability of reaching state 4 before reaching state 3?

- ○ 0
- ● $\frac{1}{2}$
- ○ $\frac{1}{3}$
- ○ $\frac{1}{4}$
- ○ 1

Explanation

Notice that state 1 always goes to state 2. So the problem is equivalent to analyzing the probability starting at 2. Since 3 and 4 are of equal probability as the outgoing edges of 2, the desired probability is just $1/2$.

Q2.2
1 Point

What is the average number of steps to reach state 1 starting at state 4?

- ○ 2
- ○ 3
- ○ 3.5
- ○ 4
- ◉ 5

Explanation

We have the following first-step equations:
$\beta(1) = 0$
$\beta(2) = 1 + \frac{1}{2}\beta(3) + \frac{1}{2}\beta(4)$
$\beta(3) = 1 + \beta(1)$
$\beta(4) = 1 + \beta(2)$
Solving above gives $\beta(4) = 5$.