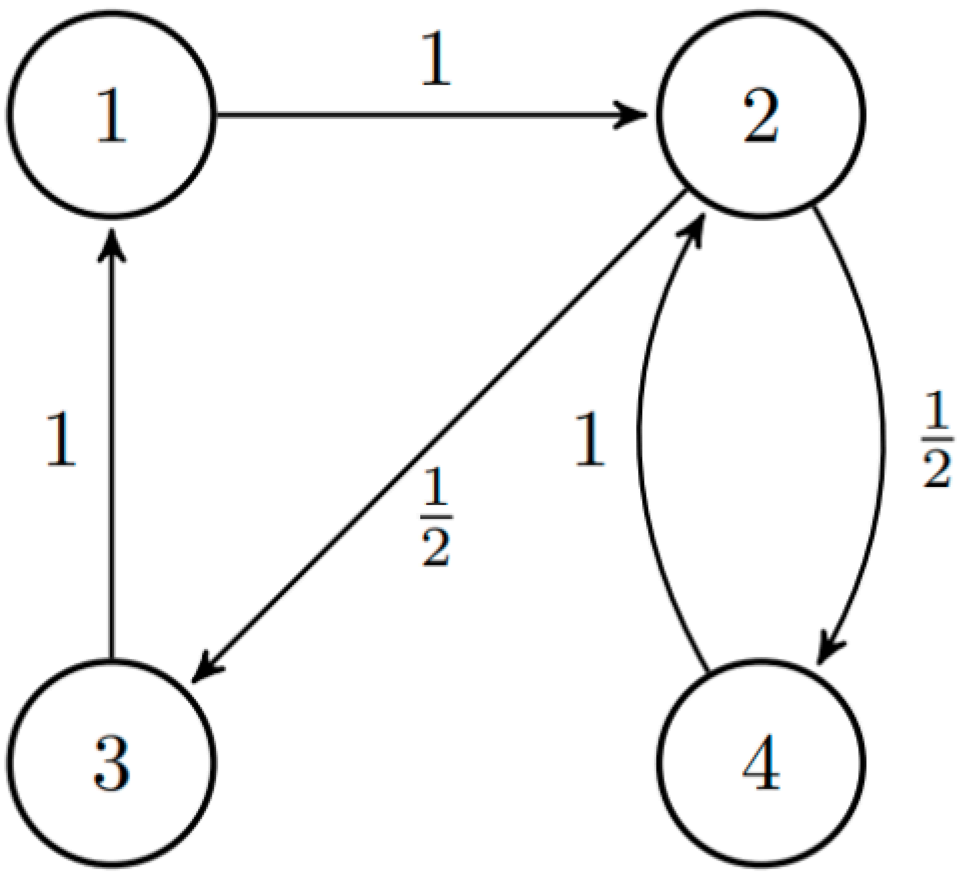
Q1 Markov chain example
4 Points

Consider the following Markov chain:



Q1.1
1 Point

What is the state space?

- (selected) $\{1, 2, 3, 4\}$
- All subsets of $\{1, 2, 3, 4\}$
- $\{1, 1, \frac{1}{2}, 1, \frac{1}{2}\}$
- $\{1, \frac{1}{2}\}$
- $\mathbb{N}$

Explanation

See note.

Q1.2
1 Point

Let P be the transition probability matrix. What is $P(2, 3)$?

- (selected) $\frac{1}{2}$
- 0
- 1

Explanation

$P(2, 3)$ is the transition probability from state 2 to state 3, which is $\frac{1}{2}$ from the figure.

Q1.3
1 Point

Is this chain irreducible?

- (●) Yes
- ( ) No

Explanation
You can go from any state to any other state with at most 5 steps, which means it is irreducible.

Q1.4
1 Point

Is this chain periodic? If so, what is the period?

- (●) No, it is aperiodic
- ( ) Yes, the period is 2
- ( ) Yes, the period is 3
- ( ) Yes, the period is 5

Explanation
See the definition of periodicity in notes.

Q2 Markov chain statements

4 Points

Consider a Markov chain over a finite state space S with transition matrix P. Check if the following statements are correct.

Q2.1

1 Point

A distribution π is an invariant distribution if and only if $\pi = \pi P$.

- (●) True
- ( ) False

Explanation

See note.

Q2.2

1 Point

If the chain has an invariant distribution, then this invariant distribution is unique.

- ( ) True
- (●) False

Explanation

Consider the case where $P = I$ is the identity matrix. Then any distribution is invariant.

Q2.3
1 Point

If the chain is irreducible, then it has an invariant distribution.

- (●) True
- ( ) False

Explanation

See note.

Q2.4
1 Point

If we can reach any state $j \in S$ from any state $i \in S$, then the chain is aperiodic.

- ( ) True
- (●) False

Explanation

The assumption only guarantees that it is irreducible, not necessarily aperiodic. Consider the case where $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Then it has period 2.

Q3 Random walk on an undirected graph

1 Point

Let $G = (V, E)$ be a connected undirected graph with at least 2 vertices. Define a random walk on this graph as follows: each time at a vertex $v \in V$, we choose a uniformly random neighbor u of v then move to u.

Which one of the following is a stationary distribution of the walk? (Recall that $\deg(v)$ is the degree of v.)

- (selected) $\pi(v) = \frac{\deg(v)}{D}$ for each $v \in V$, where $D = \sum_{v \in V} \deg(v)$
- $\pi(v) = \frac{\deg(v)}{D}$ for each $v \in V$, where $D = \sum_{v \in V} \frac{1}{\deg(v)}$
- $\pi(v) = \frac{1}{\deg(v) \cdot D}$ for each $v \in V$, where $D = \sum_{v \in V} \deg(v)$
- $\pi(v) = \frac{1}{\deg(v) \cdot D}$ for each $v \in V$, where $D = \sum_{v \in V} \frac{1}{\deg(v)}$

Explanation

See note