Q1 Combinatorial Equivalence

3 Points

Check if the following equations hold.

Q1.1

1 Point

$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$

- (●) True
- ( ) False

> **Explanation**
>
> Choosing k items from $n+1$ elements is the same as either choosing k elements from the first n elements, or choosing $k-1$ elements from the first n elements and choosing the last one.

Q1.2

1 Point

$\binom{n+1}{k} = \binom{n}{k} + \binom{n-1}{k}$

- ( ) True
- (●) False

> **Explanation**
>
> The form looks similar to the previous one, but this one is false. For example, if $n = 3, k = 1$, then LHS equals 4 and RHS equals 5.

Q1.3
1 Point

$2^n = \sum_{i=1}^{n} \binom{n}{i}$

- ○ True
- ◉ False

Explanation

The RHS misses a $\binom{n}{0} = 1$. You can also check for small number n.

Q2 Inclusion-Exclusion

3 Points

Let A_1, A_2, A_3 be three sets of finite sizes. Check if the following statements are correct.

Q2.1

1 Point

$|A_1| + |A_2| + |A_3| \geq |A_1 \cup A_2 \cup A_3|$ always holds.

- (●) True
- ( ) False

Explanation

The LHS counts the number of elements contained in A_1, A_2, A_3 with multiplicity, whereas the RHS only counts once for each element. Thus the inequality always holds.

Q2.2

1 Point

$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3|$ always holds.

- (●) True
- ( ) False

Explanation

This is the inclusion-exclusion formula. See note.

Q2.3
1 Point

$|A_1| + |A_2| + |A_3| = |A_1 \cup A_2 \cup A_3|$ always holds if A_1, A_2 are disjoint and A_2, A_3 are disjoint.

- ○ True
- ◉ False

Explanation

Consider $A_1 = \{1, 2\}, A_2 = \{3\}, A_3 = \{1, 2\}.$