Q1 Conditional Probability

3 Points

For a sample space Ω, with $\mathbb{P} : \Omega \to \mathbb{R}$ and event A, and a sample point ω, answer the following question:

Q1.1 TF1

1 Point

True or false: $\mathbb{P}[\omega \mid A] = \frac{\mathbb{P}[\omega]}{\mathbb{P}[A]}$

- ○ True
- ● False

Explanation

Aligns with the definition of conditional probability but needs the condition that $\omega \in A$ as the conditional probability will be zero if $\omega \notin A$.

Q1.2 TF2

1 Point

True or false: $\mathbb{P}[\omega \cap A] = \mathbb{P}[A \mid \omega]\mathbb{P}[\omega]$

- ● True
- ○ False

Explanation

If $\omega \in A$, then $\mathbb{P}[\omega \cap A] = \mathbb{P}[\omega] = \mathbb{P}[A \mid \omega]\mathbb{P}[\omega]$ as $\mathbb{P}[A \mid \omega] = 1$. Otherwise, $\mathbb{P}[\omega \cap A] = \mathbb{P}[\emptyset] = 0$ and $\mathbb{P}[A \mid \omega] = 0$, so both sides will be zero.

Q1.3 TF3
1 Point

True or false: If $B \subset A$, then $\mathbb{P}[B \mid A] = \frac{\mathbb{P}[B]}{\mathbb{P}[A]}$

- (selected) True
- False

Explanation

Apply the definition of conditional probability, noting that $A \cap B = B$ since B is a subset of A.

Q2 More conditional probability

2 Points

Given that $\Omega = \{0,1\}^3$ is the set of all bitstrings of length 3, and Ω is an uniform probability space, answer the following questions.

Q2.1

1 Point

What is $\mathbb{P}[000 \mid \{000, 001, 111\}]$?

- ○ 1
- ○ $\frac{1}{2}$
- ● $\frac{1}{3}$
- ○ $\frac{1}{4}$

Explanation

This problem attempts to give intuition on conditional probability. That is, the view that it is simply probability in the sample space defined by the event. Here the probability is uniform over the 3 points in the event and one of the sample points is 000, so it is $\frac{1}{3}$.

Q2.2
1 Point

Let A be the event consisting of length-3 bit-strings with at least 1 one, and let B be the event consisting of length-3 bit-strings with at least 2 one's.
What is $\mathbb{P}[B \mid A]$?

- ( ) $\frac{1}{2}$
- ( ) $\frac{3}{8}$
- ( ) $\frac{3}{7}$
- (●) $\frac{4}{7}$

Explanation

Of the 8 total outcomes, 7 of them fall in event A (have at least a single one). Note that 4 out of these 7 sample points in the conditioning event fall in event B.

Q3 Bayes Rule

2 Points

Consider a probability space, with sample space, Ω and $\mathbb{P} : \Omega \to \mathbb{R}$.

Q3.1

1 Point

What is $\mathbb{P}[A \cap B]$ in terms of $\mathbb{P}[A \mid B]$ and $\mathbb{P}[B]$?

- ○ $\mathbb{P}[A \mid B]/\mathbb{P}[B]$
- ○ $\mathbb{P}[B \mid A]/\mathbb{P}[A]$
- ○ $\mathbb{P}[B \mid A]\mathbb{P}[A]$
- ◉ $\mathbb{P}[A \mid B]\mathbb{P}[B]$

Explanation

Product rule.

Q3.2

1 Point

What is $\mathbb{P}[A \mid B]$ in terms of $\mathbb{P}[B \mid A]$, $\mathbb{P}[A]$ and $\mathbb{P}[B]$.

- ○ $\mathbb{P}[A \mid B]\mathbb{P}[B]/\mathbb{P}[A]$
- ○ $\mathbb{P}[B \mid A]\mathbb{P}[B]/\mathbb{P}[A]$
- ○ $\mathbb{P}[A \mid B]\mathbb{P}[A]/\mathbb{P}[B]$
- ◉ $\mathbb{P}[B \mid A]\mathbb{P}[A]/\mathbb{P}[B]$

Explanation

Put together product rules to make Bayes rule: $\mathbb{P}[A \cap B] = \mathbb{P}[A \mid B]\mathbb{P}[B] = \mathbb{P}[B \mid A]\mathbb{P}[A] \implies \mathbb{P}[A \mid B] = \mathbb{P}[B \mid A]\mathbb{P}[A]/\mathbb{P}[B]$.

Q4 Independence 1

1 Point

Suppose event A and B are independent, and $\mathbb{P}[A] = \mathbb{P}[B] = \frac{1}{3}$. Compute $\mathbb{P}[A \cap B]$.

- ( ) $\frac{1}{3}$
- ( ) $\frac{2}{3}$
- (•) $\frac{1}{9}$
- ( ) $\frac{4}{9}$

Explanation

By the definition of independence, $\mathbb{P}[A \cap B] = \mathbb{P}[A]\mathbb{P}[B] = \frac{1}{9}$.