

1 Contraposition

Prove the statement "if $a + b < c + d$, then $a < c$ or $b < d$ ".

2 Perfect Square

A *perfect square* is an integer n of the form $n = m^2$ for some integer m . Prove that every odd perfect square is of the form $8k + 1$ for some integer k .

3 Numbers of Friends

Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party.

(Hint: The Pigeonhole Principle states that if n items are placed in m containers, where $n > m$, at least one container must contain more than one item. You may use this without proof.)

4 Fermat's Contradiction

Prove that $2^{1/n}$ is not rational for any integer $n \geq 3$. (*Hint*: Use Fermat's Last Theorem. It states that there exists no positive integers a, b, c s.t. $a^n + b^n = c^n$ for $n \geq 3$.)

5 Prime Form

Prove that every prime number $m > 3$ is either of the form $6k + 1$ or $6k - 1$ for some integer k .