

1 Short Answers - Graphs

- (a) Bob removed a degree 3 node from an n -vertex tree. How many connected components are there in the resulting graph?
- (b) Given an n -vertex tree, Bob added 10 edges to it and then Alice removed 5 edges. If the resulting graph has 3 connected components, how many edges must be removed in order to remove all cycles from the resulting graph?

2 Planarity

- (a) Prove that $K_{3,3}$ is nonplanar.
- (b) Consider graphs with the property T : For every three distinct vertices v_1, v_2, v_3 of graph G , there are at least two edges among them. Use a proof by contradiction to show that if G is a graph on ≥ 7 vertices, and G has property T , then G is nonplanar.

3 Graph Coloring

Prove that a graph with maximum degree at most k is $(k + 1)$ -colorable.