1 Zerg Player

A Zerg player wants to produce an army to fight against Protoss in early game, and he wants to have a small army which consumes exactly 10 supply. And he has the following choices:

- Zerglings: consumes 1 supply
- Hydralisk: consumes 2 supply
- Roach: consumes 2 supply

How many different compositions can the player’s army have? Note that Zerglings are indistinguishable, as are Hydralisks and Roachs.

**Solution:** Let there are $i$ 2-supply units have been made. For the rest of supply, we can fill it with zerglings.

And if there are $i$ 2-supply units, there are $i + 1$ different compositions: 0 Hydra $i$ Roach $10 - 2i$ zerglings, 1 Hydra $i - 1$ Roach $10 - 2i$ zerglings, ..., $i$ Hydra 0 Roach $10 - 2i$ zerglings.

Then we have $\sum_{i=0}^{5} (i+1) = 1 + 2 + 3 + 4 + 5 + 6 = 21$.

2 Strings

What is the number of strings you can construct given:

(a) $n$ ones, and $m$ zeroes?

(b) $n_1$ A’s, $n_2$ B’s and $n_3$ C’s?

(c) $n_1, n_2, \ldots, n_k$ respectively of $k$ different letters?

**Solution:**

(a) $\binom{n+m}{n}$

(b) $(n_1 + n_2 + n_3)!/(n_1! \cdot n_2! \cdot n_3!)$

(c) $(n_1 + n_2 + \cdots + n_k)!/(n_1! \cdot n_2! \cdots n_k!)$.
3  Counting Game

RPG games are all about exploring different mazes. Here is a weird maze: there are $2^n$ rooms, where each room is the vertex on a the $n$-dimensional hypercube, labeled by a $n$ bit binary string.

For each room, there are $n$ different doors, each door corresponding to an edge on the hypercube. If you are at room $i$, and choose door $j$, then you will go to room $i \oplus 2^j$ (flips the $j+1$-th bit in number $i$).

(a) How many different shortest path are there from room 0 to room $2^n - 1$?

(b) How many different paths of $n + 2$ steps are there to go from room 0 to room $2^n - 1$?

(c) If $n = 8$, how many different shortest paths are there from room 0 to room 63 that pass through 3 and 19?

Solution:

(a) $n!$, the shortest path is $n$, and for the $i$-th step, there are only $n - i$ doors flips a zero to one.

(b) The player made one mistake during his trip, so suppose he made the mistake at step $i$, $i > 0$, so there are $i$ different ways to make the mistake. Then he will start from a room with $n - i + 1$ zeros. So the total number is $\sum_{i=1}^{n} \left( \binom{n}{i} \cdot i! \cdot i \cdot (n - i + 1)! \right)$.

Optional for further steps:

\[
\sum_{i=1}^{n} \left( \binom{n}{i} \cdot i! \cdot i \cdot (n - i + 1)! \right) = \sum_{i=1}^{n} \frac{n! \cdot i \cdot (n-i+1)!}{(n-i)! \cdot i!} = \sum_{i=1}^{n} \frac{n! \cdot i \cdot (n-i+1)}{(n-i)!} = n! \sum_{i=1}^{n} i(n-i+1) = n! \sum_{i=1}^{n} i(n-i+1) = n! \sum_{i=1}^{n} i \cdot (i^2 + i) = n! \sum_{i=1}^{n} i - \sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i = \frac{n(n+1)(n+2)}{6}
\]

(c) From 0 to 3, 2 different paths. From 3 to 19: notice $3 \oplus 19 = 16$ so there is only one way. From 19 to 63, there are 3 zeros in $63 \oplus 19$ so total 3! different paths. In total $2 \times 3!$ different paths.