

have arrived in town. Each is of infinite length, and carries a countably infinite number of passengers. How would you accommodate all the new passengers?

2 Countability Practice

- (a) Do $(0, 1)$ and $\mathbb{R}_+ = (0, \infty)$ have the same cardinality? If so, either give an explicit bijection (and prove that it is a bijection) or provide an injection from $(0, 1)$ to $(0, \infty)$ and an injection from $(0, \infty)$ to $(0, 1)$ (so that by Cantor-Bernstein theorem the two sets will have the same cardinality). If not, then prove that they have different cardinalities.

(b) Is the set of strings over the English alphabet countable? (Note that the strings may be arbitrarily long, but each string has finite length. Also the strings need not be real English words.) If so, then provide a method for enumerating the strings. If not, then use a diagonalization argument to show that the set is uncountable.

(c) Consider the previous part, except now the strings are drawn from a countably infinite alphabet \mathcal{A} . Does your answer from before change? Make sure to justify your answer.

3 Counting Functions

Are the following sets countable or uncountable? Prove your claims.

(a) The set of all functions f from \mathbb{N} to \mathbb{N} such that f is non-decreasing. That is, $f(x) \leq f(y)$ whenever $x \leq y$.

(b) The set of all functions f from \mathbb{N} to \mathbb{N} such that f is non-increasing. That is, $f(x) \geq f(y)$ whenever $x \leq y$.