

1 Administrivia

- (a) Make sure you are on the course Piazza (for Q&A) and Gradescope (for submitting homeworks, including this one). Find and familiarize yourself with the course website. What is its homepage's URL?
- (b) Read the policies page on the course website.
 - (i) What is the breakdown of how your grade is calculated?
 - (ii) What is the attendance policy for small discussions? Big discussions?
 - (iii) When are vitamins released and when are they due?
 - (iv) How many vitamins do you have to complete to get full credit?

Solution:

- (a) The course website is located at <http://www.eecs70.org/>.
- (b)
 - (i) Vitamins are worth 15 points, homework is worth 45 points, the midterm is worth 60 points, and the final is worth 80 points. There is also a partial clobber policy for exams: if your score on the final is higher than your score on the midterm then your final will be worth 110 points and your midterm will be worth 30 points.
 - (ii) If you sign up for a small discussion, you must attend at least two sections per week. If you miss more than two sessions in a week without informing your TA, then you will be dropped. Big discussions have no restrictions however - anyone can show and attendance is not mandatory.
 - (iii) The vitamin for the current day will be released on Gradescope immediately after lecture (at 4:30 pm) and is due the next day by the start of lecture (3:00 pm). The solutions will be provided immediately after that.
 - (iv) You only need to complete 18 out of the 26 vitamins to get full credit. It is in your best interests to do as many as you can however, as every vitamin after the first 18 counts as a half point of extra credit!

2 Course Policies

Go to the course website and read the course policies carefully. Leave a followup in the Homework 0, Question 2 thread on Piazza if you have any questions. Are the following situations violations of course policy? Write "Yes" or "No", and a short explanation for each.

- (a) Alice and Bob work on a problem in a study group. They write up a solution together and submit it, noting on their submissions that they wrote up their homework answers together.
- (b) Carol goes to a homework party and listens to Dan describe his approach to a problem on the board, taking notes in the process. She writes up her homework submission from her notes, crediting Dan.
- (c) Erin comes across a proof that is part of a homework problem while studying course material. She reads it and then, after she has understood it, writes her own solution using the same approach. She submits the homework with a citation to the website.
- (d) Frank is having trouble with his homework and asks Grace for help. Grace lets Frank look at her written solution. Frank copies it onto his notebook and uses the copy to write and submit his homework, crediting Grace.
- (e) Heidi has completed her homework using \LaTeX . Her friend Irene has been working on a homework problem for hours, and asks Heidi for help. Heidi sends Irene her PDF solution, and Irene uses it to write her own solution with a citation to Heidi.
- (f) Joe found homework solutions before they were officially released, and every time he got stuck, he looked at the solutions for a hint. He then cited the solutions as part of his submission.

Solution:

- (a) Yes, this is a violation of course policy. All solutions must be written entirely by the student submitting the homework. Even if students collaborate, each student must write a unique, individual solution. In this case, both Alice and Bob would be culpable.
- (b) No, this is not a violation of course policy. While sharing *written solutions* is not allowed, sharing *approaches* to problems is allowed and encouraged. Because Carol only copied down *notes*, not *Dan's solution*, and properly cited Dan's contribution, this is an actively encouraged form of collaboration.
- (c) No, this is not a violation of course policy. Using external sources to help with homework problems, while less encouraged than peer collaboration, is fine as long as (i) the student makes sure to understand the solution; (ii) the student uses understanding to write a new solution, and does not copy from the external source; and (iii) the student credits the external source. However, looking up a homework problem online is a violation of course policies; the correct course of action upon finding homework solutions online is to close the tab.
- (d) Yes, this is a violation of course policy, and both Frank and Grace would be culpable. Even though Frank credits Grace, written solutions should never be shared in the first place, and certainly not copied down. This is to ensure that each student learns how to write and present clear and convincing arguments. To be safe, try not to let anybody see your written solutions at any point in the course—restrict your collaboration to *approaches* and *verbal communication*.

- (e) Yes, this is a violation of course policy. Once again, a citation does not make up for the fact that written solutions should never be shared, in written or typed form. In this case, both Heidi and Irene would be culpable.
- (f) Yes, this is a violation of course policy. Joe should not be reading solutions before they are officially released. Instead, Joe should ask for help when he is stuck through Piazza or Office Hours.

3 L^AT_EX

If you have ample time on your hands and would like to learn a new skill, you may be interested in learning L^AT_EX! L^AT_EX is a document preparation system that puts mathematical formulae into nicely formatted documents. Using L^AT_EX can help you organize your thought process and make lives easier for readers. We have provided some resources on the course website to help you get started with using L^AT_EX. Feel free to ask questions on Piazza if you have any questions.

For this question, try to typeset the following formulas. This will give you some practice writing mathematical formulas properly. **If you choose to hand-write and scan your solutions, just write out the formulas by hand.**

Hint: You may find the *amsmath* L^AT_EX package helpful.

(a) $\forall x \exists y ((P(x) \wedge Q(x, y)) \implies x \leq \sqrt{y})$

(b)

$$\sum_{i=0}^k i = \frac{k(k+1)}{2}$$

Solution:

(a) `\forall x \exists y \left(\left(P(x) \wedge Q(x, y) \right) \implies x \leq \sqrt{y} \right)`

(b) `\left[\sum_{i=0}^k i = \frac{k(k+1)}{2} \right]`

4 Logical Equivalence?

Decide whether each of the following logical equivalences is correct and justify your answer.

(a) $\forall x (P(x) \wedge Q(x)) \equiv \forall x P(x) \wedge \forall x Q(x)$

$$(b) \forall x (P(x) \vee Q(x)) \equiv \forall x P(x) \vee \forall x Q(x)$$

$$(c) \exists x (P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$$

$$(d) \exists x (P(x) \wedge Q(x)) \equiv \exists x P(x) \wedge \exists x Q(x)$$

Solution:

(a) **Correct.**

Assume that the left hand side is true. Then we know for an arbitrary x $P(x) \wedge Q(x)$ is true. This means that both $\forall x P(x)$ and $\forall x Q(x)$. Therefore the right hand side is true. Now for the other direction assume that the right hand side is true. Since for any x $P(x)$ and for any y $Q(y)$ holds, then for an arbitrary x both $P(x)$ and $Q(x)$ must be true. Thus the left hand side is true.

(b) **Incorrect.**

Note, there are many possible counterexamples - here we present only one. Suppose that the universe (i.e. the values that x can take on) is $\{1, 2\}$ and that P and Q are truth functions defined on this universe. If we set $P(1)$ to be true, $Q(1)$ to be false, $P(2)$ to be false and $Q(2)$ to be true, the left-hand side will be true, but the right-hand side will be false. Hence, we can find a universe and truth functions P and Q for which these two expressions have different values, so they must be different.

(c) **Correct**

Assuming that the left hand side is true, we know there exists some x such that one of $P(x)$ and $Q(x)$ is true. Thus $\exists x P(x)$ or $\exists x Q(x)$ and the right hand side is true. To prove the other direction, assume the left hand side is false. Then there does not exist an x for which $P(x) \vee Q(x)$ is true, which means there is no x for which $P(x)$ or $Q(x)$ is true. Therefore the right hand side is false.

(d) **Incorrect.**

Note, there are many possible counterexamples - here we present only one. Suppose that the universe (i.e. the values that x can take on) is the natural numbers \mathbb{N} , and that P and Q are truth functions defined on this universe. If we set $P(1)$ to be true and $P(x)$ to be false for all other x , and $Q(2)$ to be true and $Q(x)$ to be false for all other x , then the right hand side would be true. However, there would be no value of x at which both $P(x)$ and $Q(x)$ would be simultaneously true, so the left hand side would be false. Hence, we can find a universe and truth functions P and Q for which these two expressions have different values, so they must be different.

5 Propositional Practice

In parts (a)-(b), convert the English sentences into propositional logic. In parts (c) - (d), convert the propositions into English. For parts (b) and (d), use the notation $a \mid b$ to denote the statement “ a divides b ”, and use the notation $P(x)$ to denote the statement “ x is a prime number”.

- (a) For every real number k , there is a unique real solution to $x^3 = k$.
- (b) If p is a prime number, then for any two natural numbers a and b , if p doesn't divide a and p divides ab , then p divides b .
- (c) $(\forall x, y \in \mathbb{R}) [(xy = 0) \implies ((x = 0) \vee (y = 0))]$
- (d) $\neg((\exists y \in \mathbb{N}) [(\forall x \in \mathbb{N}) [(x > y) \implies ((y | x) \vee P(x))]])$

Solution:

- (a) The trickiest part of this problem is the word 'unique'. We can express the existence of a unique solution in propositional logic with two statements connected with an 'and': (1) A solution exists, and (2) Any two solutions have to be the same. Hence, we can rewrite this statement as "For every real number k , there exists a real number x such that $x^3 = k$ and for all reals y and z , if both $y^3 = k$ and $z^3 = k$, then $y = z$." This, in propositional logic, is below:

$$(\forall k \in \mathbb{R}) [(\exists x \in \mathbb{R})(x^3 = k) \wedge (\forall y, z \in \mathbb{R})((y^3 = k) \wedge (z^3 = k)) \implies (y = z)].$$

- (b) This sentence can be written in propositional logic as

$$(\forall p \in \mathbb{N}) [(P(p)) \implies ((\forall a, b \in \mathbb{N}) [((p | ab) \wedge \neg(p | a)) \implies (p | b)])].$$

- (c) If the product of two real numbers is 0, then one of them must be 0.
- (d) There is no natural number that divides every composite number greater than it.

6 Prove or Disprove

For each of the following statements, (1) decide whether or not the statement is false, and (2) provide a rigorous proof backing up your answer.

- (a) For all natural numbers n , $n^2 + 3n$ is even if and only if n is odd.
- (b) For all real numbers r , if r is irrational then $r + 1$ is irrational.

Solution:

- (a) This is tricky, but the statement is actually false because of the "if and only if". Note that the expression $n^2 + 3n$ is always even - not just when n is odd. Because of this, the fact that $n^2 + 3n$ is even doesn't imply that n is odd, so the two statements are not equivalent. To disprove this rigorously, we provide a counterexample. Observe that when $n = 2$, the statement " n is odd" is false, but the statement " $n^2 + 3n = 10$ is even" is true. Thus, these two statements are not logically equivalent for every n , so the statement "For all natural numbers n , $n^2 + 3n$ is even if and only if n is odd" is not true.

- (b) This statement is true, and to prove it, we will use a proof by contraposition. Assume that $r + 1$ is rational. Since $r + 1$ is rational, it can be written in the form a/b where a and b are integers. Then r can be written as $(a - b)/b$. By the definition of rational numbers, r is a rational number, since both $a - b$ and b are integers. By contraposition, if r is irrational, then $r + 1$ is irrational.

7 Proof Checker

Your friend has sent you three proofs, listed below. As a CS 70 student, your job is to determine whether each proof is correct or incorrect. If the proof is correct, then just write “the proof is correct”. If the proof is incorrect, clearly and precisely describe what the logical error is. Note: We are not looking for answers such as “the claim is false”; we want you to point out the fault in the reasoning!

- (a) **Claim:** for all $n \in \mathbb{N}$, $(2n + 1 \text{ is a multiple of } 3) \implies (n^2 + 1 \text{ is a multiple of } 3)$.

Proof: We will use proof by contraposition. Assume $2n + 1$ is not a multiple of 3. We have the following cases.

- If $n = 3k + 1$ for $k \in \mathbb{N}$, then $n^2 + 1 = 9k^2 + 6k + 2$ is not a multiple of 3.
- If $n = 3k + 2$ for $k \in \mathbb{N}$, then $n^2 + 1 = 9k^2 + 12k + 5$ is not a multiple of 3.
- If $n = 3k + 3$ for $k \in \mathbb{N}$, then $n^2 + 1 = 9k^2 + 18k + 10$ is not a multiple of 3.

In all cases, we have concluded $n^2 + 1$ is not a multiple of 3, so we have proved the claim.

- (b) **Claim:** For all real numbers a, b , if $a + b \geq 20$ then $a \geq 17$ or $b \geq 3$.

Proof We will again use a proof by contraposition. Suppose that $a < 17$ and $b < 3$ (note that this is equivalent to $\neg(a \geq 17 \vee b \geq 3)$). Since $a < 17$ and $b < 3$, $a + b < 20$ (note that $a + b < 20$ is equivalent to $\neg(a + b \geq 20)$). Thus, if $a + b \geq 20$, then $a \geq 17$ or $b \geq 3$ (or both, as “or” is not “exclusive or” in this case). By contraposition, for all real numbers a, b , if $a + b \geq 20$ then $a \geq 17$ or $b \geq 3$.

Solution:

- (a) The proof is incorrect. You want to prove an implication of the form $P(n) \implies Q(n)$ for every n , where $P(n)$ is “ $2n + 1$ is a multiple of 3” and $Q(n)$ is “ $n^2 + 1$ is a multiple of 3”. The contrapositive is $\neg Q(n) \implies \neg P(n)$. Your proof begins with $\neg P(n)$ and concludes with $\neg Q(n)$, so you have shown $\neg P(n) \implies \neg Q(n)$, which is the converse, not contrapositive. Besides, $n = 0$ is not covered in the proof. Note: when $n = 3k + 1$, $2n + 1 = 6k + 3$ is a multiple of 3, so the case is redundant to prove $\neg P(n) \implies \neg Q(n)$.
- (b) The proof is correct.