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Woohoo!!

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  Q is consequent or conclusion.

Review puzzle

Theory: If you drink alcohol you must be at least 18.

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Its easier now.

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Write implication and contraposition:

Drink $\implies$ "≥ 18"

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Its easier now. At least for me.

CS70: Lecture 2. Outline.

Today: Proofs!!!

1. By Example.
2. Direct. (Prove $P \implies Q$.)
3. by Contraposition (Prove $P \implies Q$)
4. by Contradiction (Prove P.)
5. by Cases

If time: discuss induction.

Quick Background and Notation.

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A natural number $p > 1$, is **prime** if it is divisible only by 1 and itself.

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Another direct proof.

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Examples:
$n = 121$

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$n = 121$ Alt Sum: $1 - 2 + 1 = 0$.

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Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

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Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

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Examples:

$n = 121$ Alt Sum: $1-2+1 = 0$. Divis. by 11. As is 121.

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Proof: For $n \in D_3$, $n = 100a+10b+c$, for some a, b, c.

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n^2 is even, $n^2 = 2k$, ...$\sqrt{2k}$ even?

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Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

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Did we prove?

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Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.

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Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

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Don't assume what you want to prove!

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Be really careful!

Theorem: $1 = 2$
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$$\begin{aligned} (x^2 - xy) &= x^2 - y^2 \\ x(x-y) &= (x+y)(x-y) \\ x &= (x+y) \end{aligned}$$

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Also: Multiplying inequalities by a negative.

Summary: Note 2.

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Either $\sqrt{2}$ and $\sqrt{2}$ worked.

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Direct Proof:
To Prove: $P \implies Q$. Assume P. Prove Q.

By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
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CS70: Note 3. Induction!

1. The natural numbers.
2. 5 year old Gauss.
3. ..and Induction.
4. Simple Proof.

The natural numbers.

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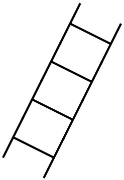
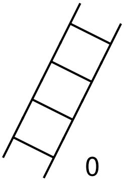


The natural numbers.

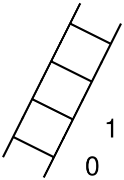
0,



0

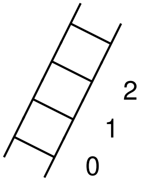
The natural numbers.

0, 1,



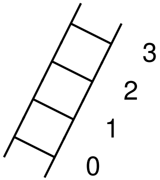
The natural numbers.

0, 1, 2,



The natural numbers.

0, 1, 2, 3,



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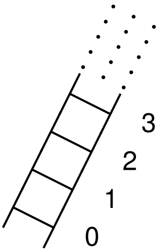

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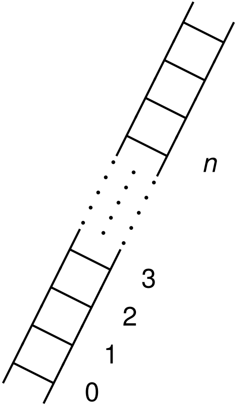
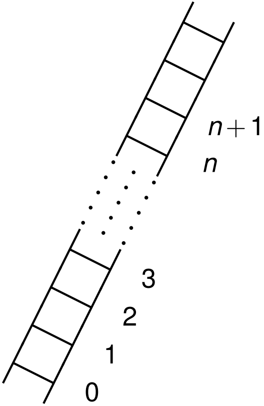


0, 1, 2, 3,

…, n,

The natural numbers.



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$\ldots$, n, $n+1$,

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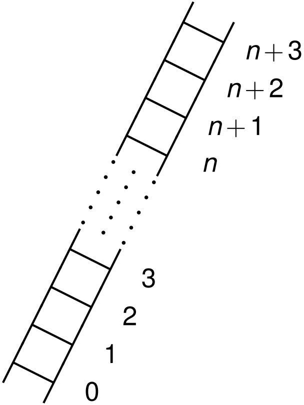

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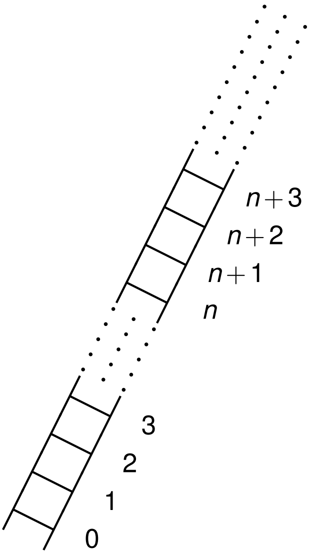

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A formula.

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Teacher: Hello class.

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- Prove $P(0)$.
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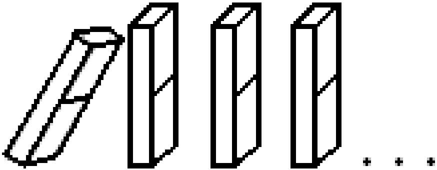
Notes visualization

Note’s visualization: an infinite sequence of dominos.



Prove they all fall down;

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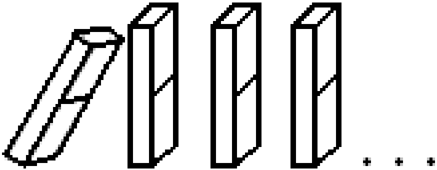

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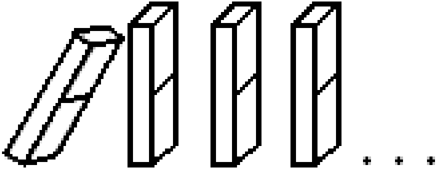

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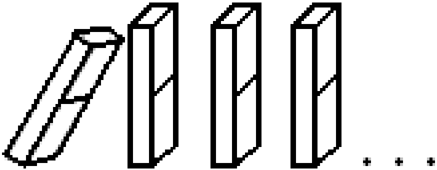

Prove they all fall down;

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  "kth domino falls implies that $k+1$st domino falls"

Climb an infinite ladder?

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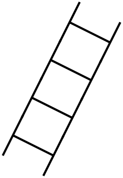

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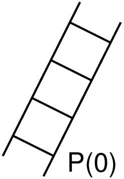

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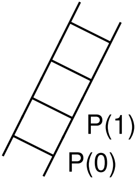

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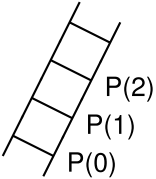

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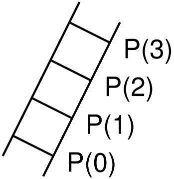

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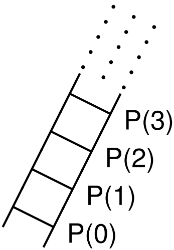

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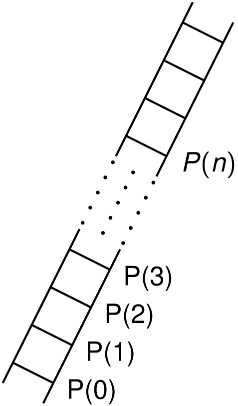

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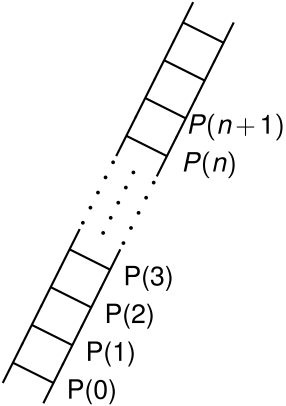

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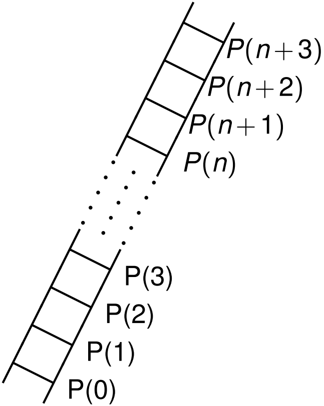

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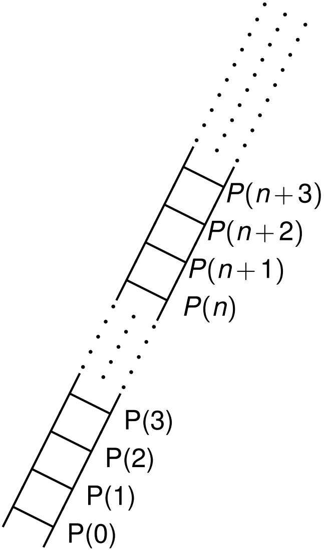

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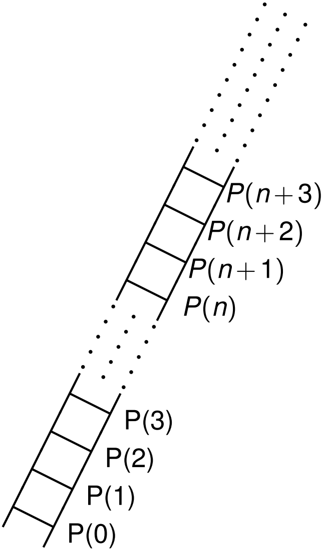

Your favorite example of forever..

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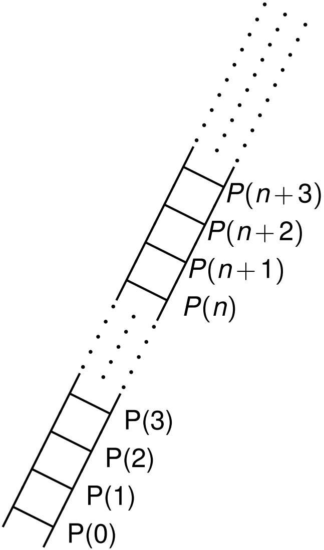

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Your favorite example of forever..or the natural numbers...

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$.

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

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$\sum_{i=1}^{k+1} i$

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How about $k+2$.

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Induction Step.

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Need to start somewhere.

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Statement is true for $n = 0$

Gauss and Induction

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Statement is true for $n = 0$ $P(0)$ is true

Gauss and Induction

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plus inductive step

Gauss and Induction

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Gauss and Induction

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plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

Gauss and Induction

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plus inductive step $\implies$ true for $n = 2$

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$\sum_{i=1}^{k+1} i = (\sum_{i=1}^{k} i) + (k+1) = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}$.

How about $k+2$. Same argument starting at $k+1$ works!

Induction Step. $P(k) \implies P(k+1)$.

Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 1 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

plus inductive step $\implies$ true for $n = 2$ $(P(1) \wedge (P(1) \implies P(2))) \implies P(2)$

...

true for $n = k$ $\implies$ true for $n = k+1$ $(P(k) \wedge (P(k) \implies P(k+1))) \implies P(k+1)$

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Predicate, $P(n)$, True for all natural numbers!

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$ Proof?

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Predicate, $P(n)$, True for all natural numbers! **Proof by Induction.**

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

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Next Time.

More induction!

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See you on Tuesday!