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CS70: Lecture 2. Outline.

Today: Proofs!!!

1. By Example.
2. Direct. (Prove $P \implies Q$.)
3. by Contraposition (Prove $P \implies Q$)
4. by Contradiction (Prove P.)
5. by Cases

If time: discuss induction.

Quick Background and Notation.

Integers closed under addition.

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$a, b \in Z \implies a + b \in Z$

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A natural number $p > 1$, is **prime** if it is divisible only by 1 and itself.

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

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...

Therefore Q.

Another direct proof.

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Examples:
$n = 121$ Alt Sum: $1 - 2 + 1 = 0$.

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Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

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Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

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Add $99a + 11b$ to both sides.

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$$100a + 10b + c = 11k + 99a + 11b$$

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Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

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Examples:
$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

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Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

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- Proof assumed no primes *in between* p_k and q.

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Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.

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Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

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Proof of lemma: Assume a solution of the form a/b.

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Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
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The fourth case is the only one possible, so the lemma follows. □

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One of the cases is true so theorem holds. □

Question: Which case holds? Don't know!!!

Be careful.

Theorem: $3 = 4$

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Proof: Assume $3 = 4$.

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Don't assume what you want to prove!

Be really careful!

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Be really careful!

Theorem: $1 = 2$
Proof: For $x = y$, we have

$$\begin{aligned}(x^2 - xy) &= x^2 - y^2 \\ x(x - y) &= (x + y)(x - y) \\ x &= (x + y)\end{aligned}$$

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$P \implies Q$ does not mean $Q \implies P$.

Summary: Note 2.

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To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
Universal: show that statement holds in all cases.
Existence: used cases where one is true.
Either $\sqrt{2}$ and $\sqrt{2}$ worked.
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Careful when proving!
Don't assume the theorem. Divide by zero.

Summary: Note 2.

Direct Proof:
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CS70: Note 3. Induction!

Poll.

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Poll.

1. The natural numbers.
2. 5 year old Gauss.
3. ..and Induction.
4. Simple Proof.

The natural numbers.

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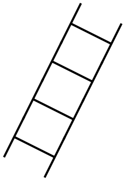
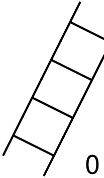


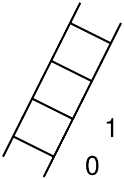
The natural numbers.

0,



The natural numbers.

0, 1,



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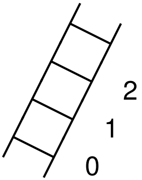
0, 1, 2,



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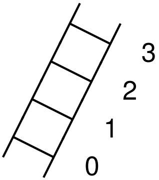

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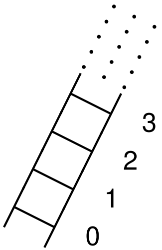

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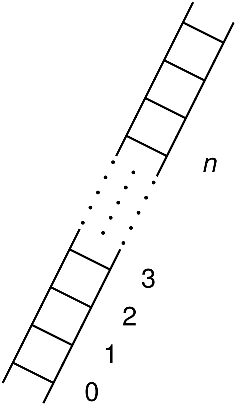

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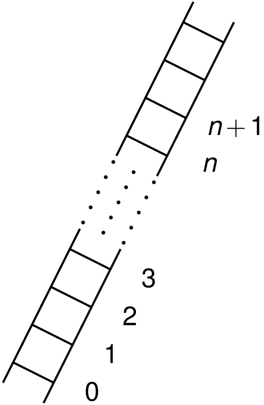

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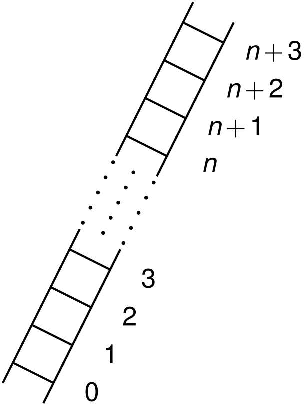

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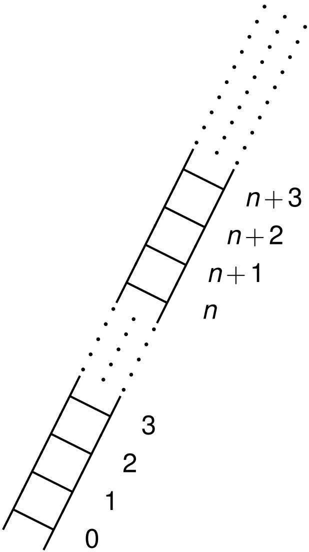

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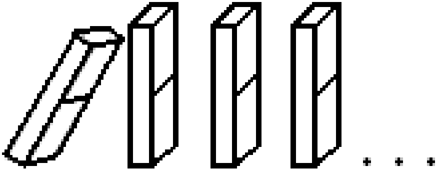
Notes visualization

Note's visualization: an infinite sequence of dominos.



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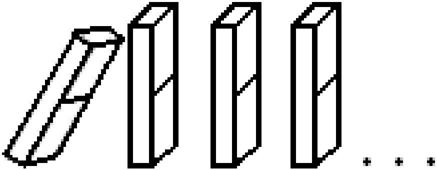

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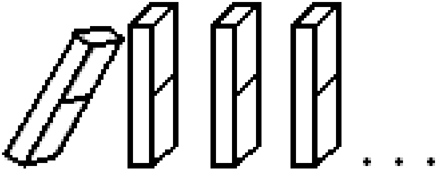

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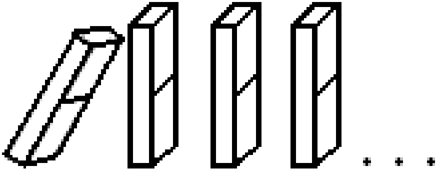

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Climb an infinite ladder?

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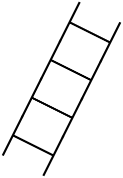

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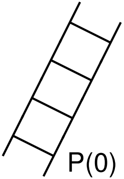

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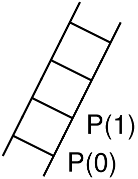

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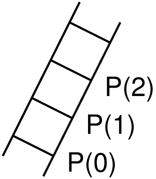

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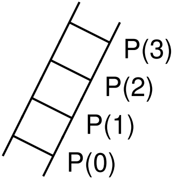

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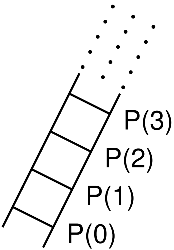

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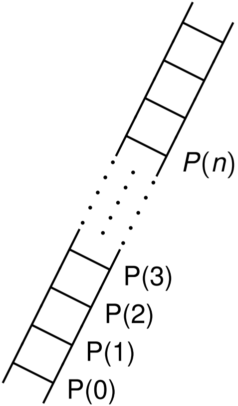

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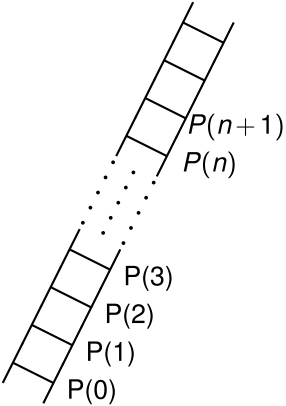

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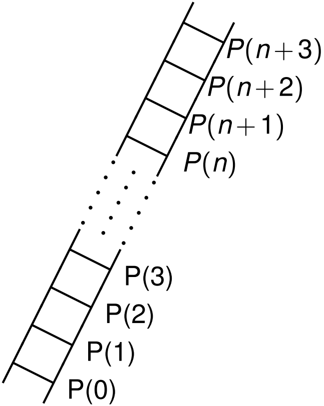

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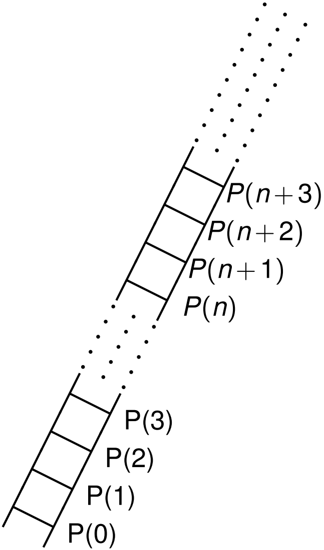

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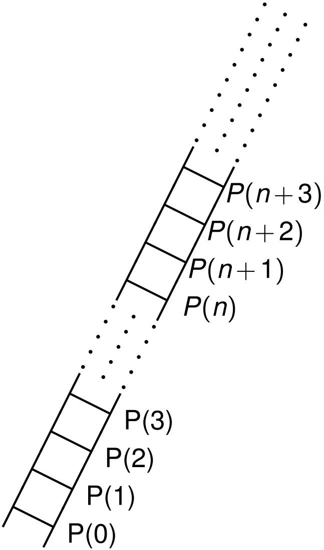

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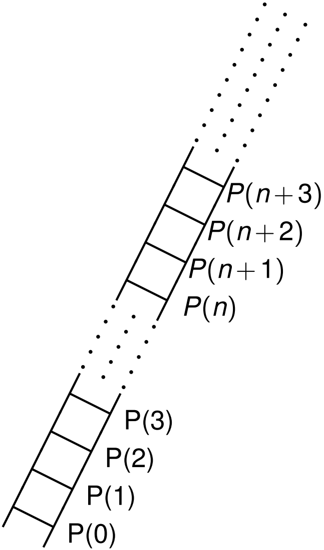

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Your favorite example of forever..or the natural numbers...

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Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$

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Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Is predicate, $P(n)$ true for $n = k+1$?

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Predicate, $P(n)$, True for all natural numbers!

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Is this a proof? It shows that we can always move to the next step.

Need to start somewhere. $P(0)$ is $\sum_{i=0}^{0} i = 0 = \frac{(0)(0+1)}{2}$ **Base Case.**

Statement is true for $n = 0$ $P(0)$ is true

plus inductive step $\implies$ true for $n = 1$ $(P(0) \wedge (P(0) \implies P(1))) \implies P(1)$

plus inductive step $\implies$ true for $n = 2$ $(P(1) \wedge (P(1) \implies P(2))) \implies P(2)$

...

true for $n = k$ $\implies$ true for $n = k+1$ $(P(k) \wedge (P(k) \implies P(k+1))) \implies P(k+1)$

...

Predicate, $P(n)$, True for all natural numbers! **Proof by Induction.**

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

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- Prove $P(0)$. “Base Case”.

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Next Time.

More induction!

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"See you" on Tuesday!