Lecture Today.

To homework (score) or not to homework (score)

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Do proofs of optimality/pessimality again.

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To homework (score) or not to homework (score)

Do proofs of optimality/pessimality again.

Graphs

Poll.

Thoughts on homework or non-homework option?

(A) Thinking about it.
(B) Definitely doing homework for score.
(C) Definitely going for the non-scored homework.

Job Propose and Candidate Reject is optimal!

For jobs?

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For jobs? For candidates?

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Rogue couple for S.

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Used Well-Ordering principle...Induction.

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Structural statement: Job optimality

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Structural statement: Job optimality $\implies$ Candidate pessimality.

Lecture 5: Graphs.

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Graphs!
  Definitions: model.

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- Definitions: model.
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Planar graphs.

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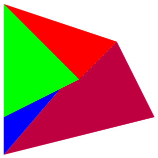
- Graphs!
  - Definitions: model.
  - Fact!
- Planar graphs.
  - Euler Again!!!!

Map Coloring.



Map Coloring.

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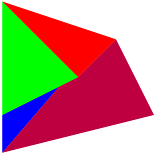
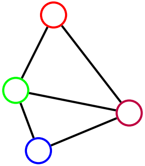


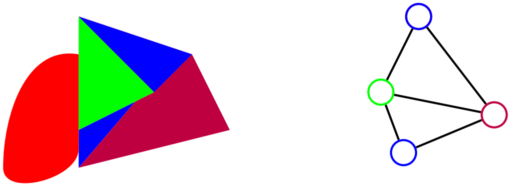


Fewer Colors?

Map Coloring.

Yes! Three colors.

Map Coloring.



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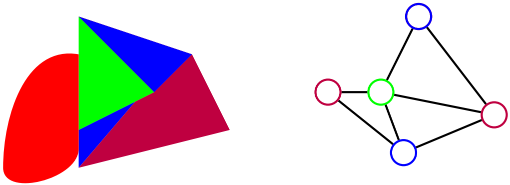

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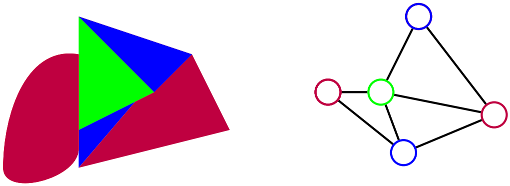

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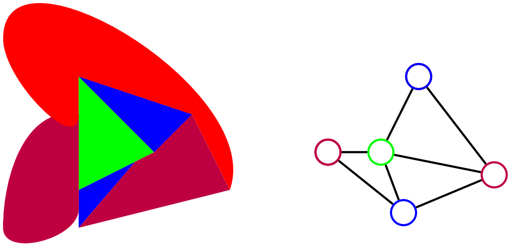

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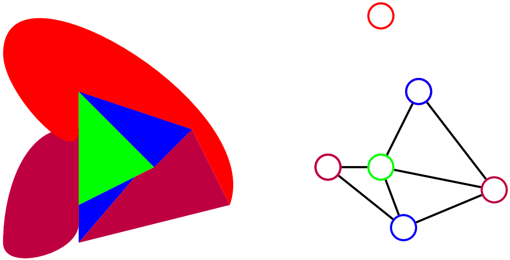

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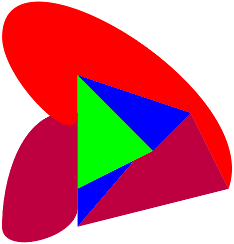

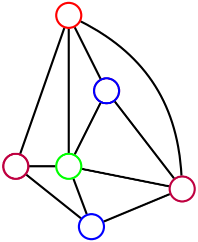

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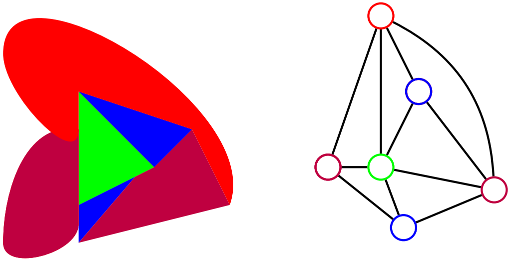

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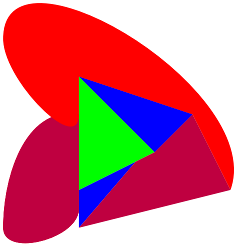

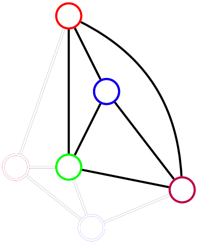

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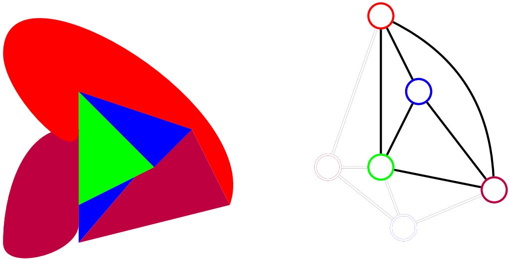
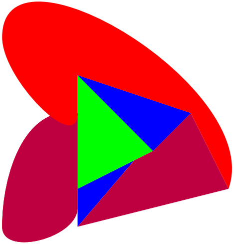
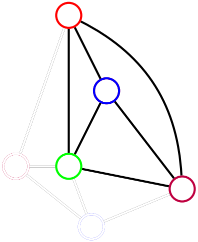


Four colors required!

Map Coloring.

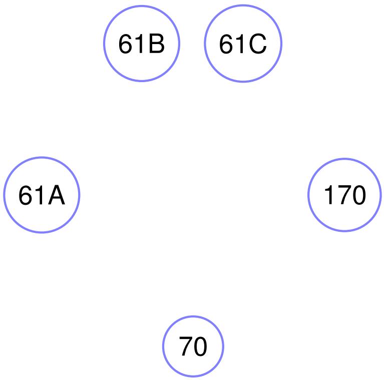




Four colors required!

Theorem: Four colors enough.

Scheduling: coloring.



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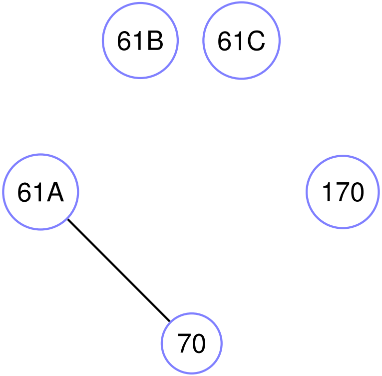

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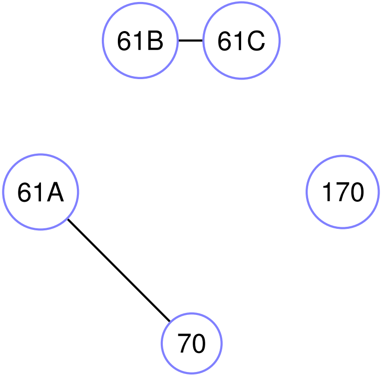

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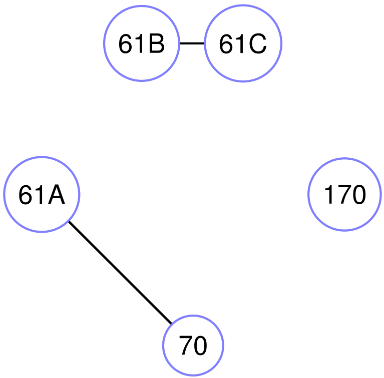

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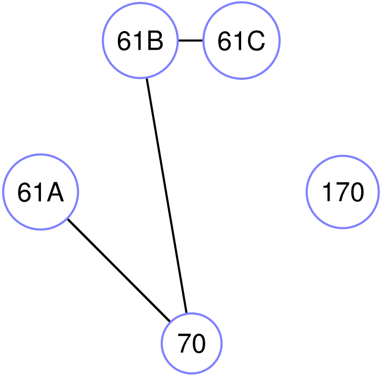

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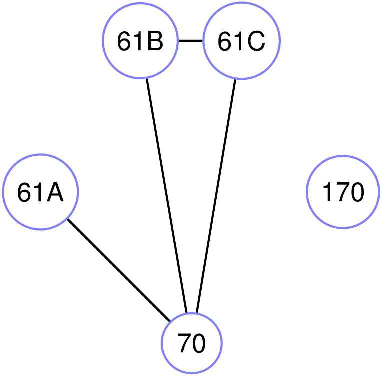

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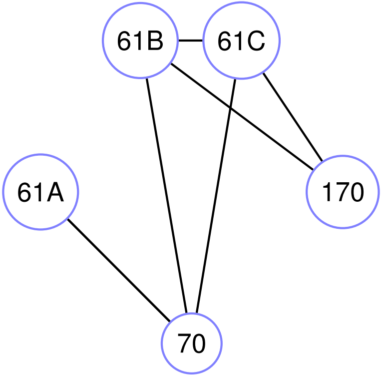

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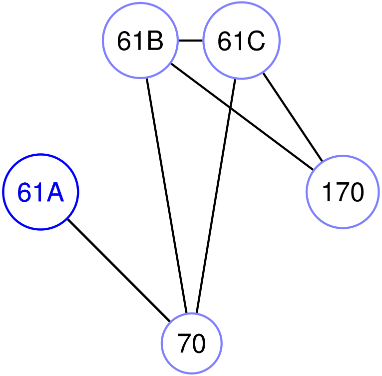

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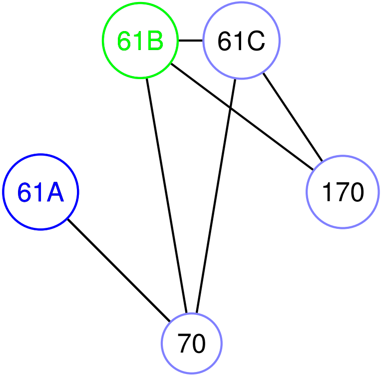

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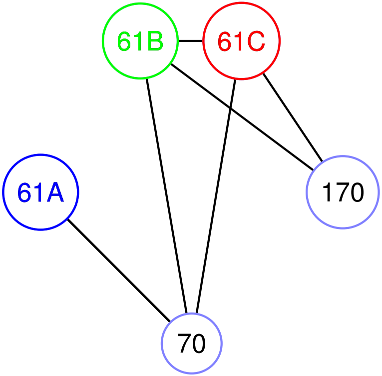
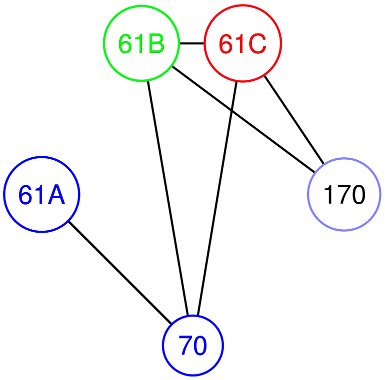
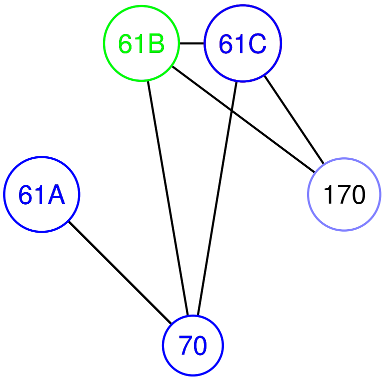


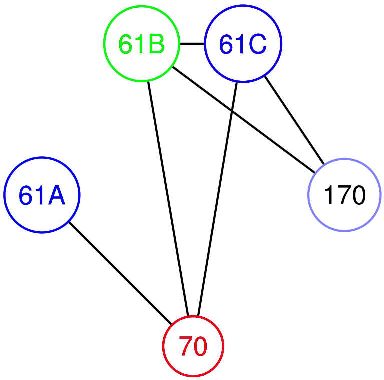
Scheduling: coloring.



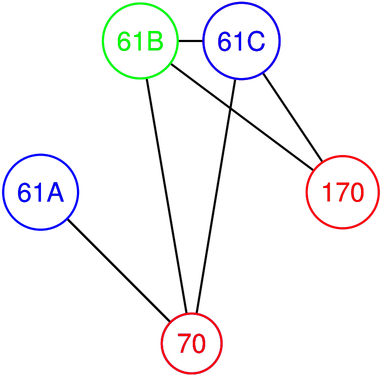
Scheduling: coloring.



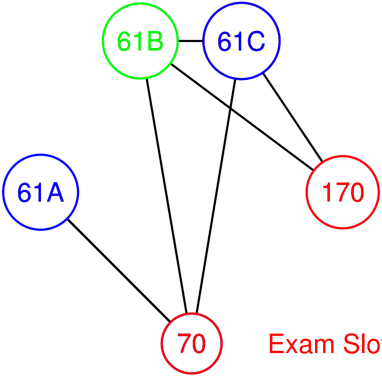
Scheduling: coloring.



Scheduling: coloring.



Scheduling: coloring.

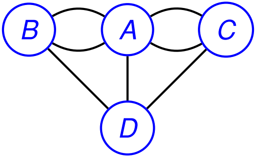


Exam Slot 1.

Exam Slot 2.

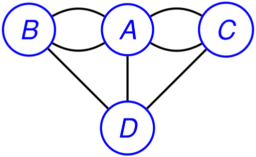
Exam Slot 3.

Graphs: formally.



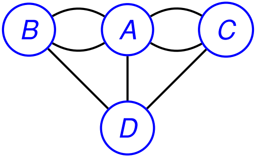
Graph:

Graphs: formally.



Graph: $G = (V, E)$.

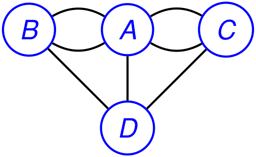
Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

Graphs: formally.

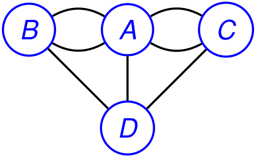


Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

Graphs: formally.



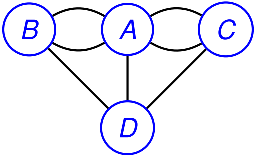
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ -

Graphs: formally.



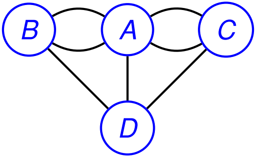
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

Graphs: formally.



Graph: $G = (V, E)$.

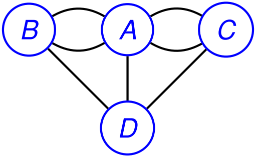
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

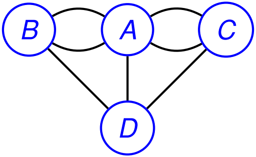
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

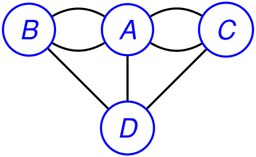
$\{\{A, B\}, \{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.
V - set of vertices.
$\{A, B, C, D\}$
$E \subseteq V \times V$ - set of edges.
$\{\{A, B\}, \{A, B\}, \{A, C\},$

Graphs: formally.



Graph: $G = (V, E)$.

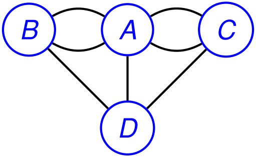
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

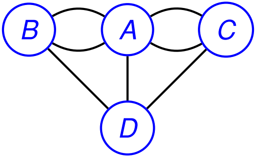
$\{A, B, C, D\}$

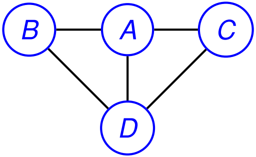
$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

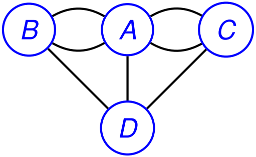
Graphs: formally.

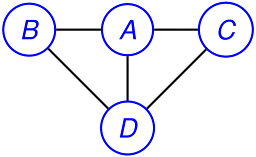




Graph: $G = (V, E)$.
V - set of vertices.
$\{A, B, C, D\}$
$E \subseteq V \times V$ - set of edges.
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.
For CS 70, usually simple graphs.
No parallel edges.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

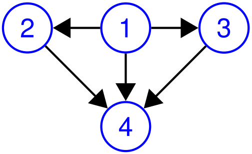
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}.$

For CS 70, usually simple graphs.

No parallel edges.

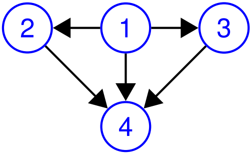
Multigraph above.

Directed Graphs



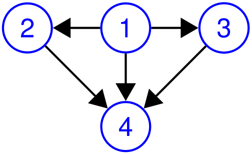
$G = (V, E)$.

Directed Graphs



$G = (V, E)$.
V - set of vertices.

Directed Graphs

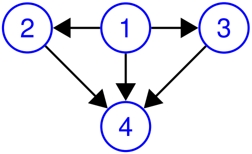


$G = (V, E)$.

V - set of vertices.

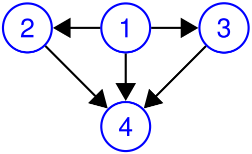
$\{1, 2, 3, 4\}$

Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.

Directed Graphs



$G = (V, E)$.

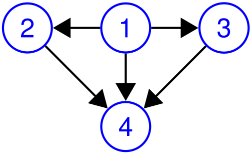
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2),$

Directed Graphs



$G = (V, E)$.

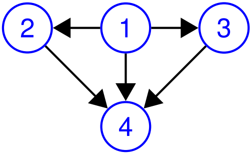
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2), (1, 3),$

Directed Graphs



$G = (V, E)$.

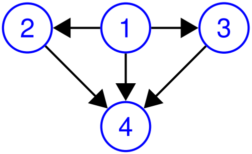
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4),$

Directed Graphs



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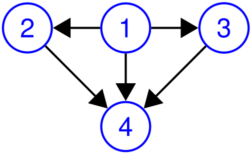
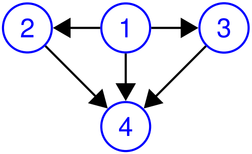
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.

Directed Graphs



$G = (V, E)$.

V - set of vertices.

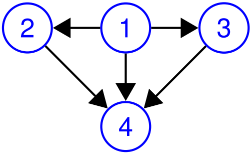
$\{1,2,3,4\}$

E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament:

Directed Graphs



$G = (V, E)$.

V - set of vertices.

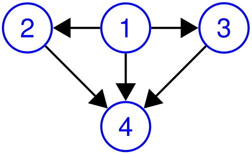
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2,

Directed Graphs



$G = (V, E)$.

V - set of vertices.

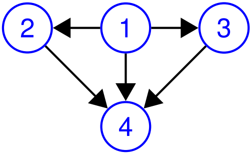
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence:

Directed Graphs



$G = (V, E)$.
V - set of vertices.
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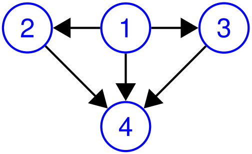
E ordered pairs of vertices.
$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

Directed Graphs



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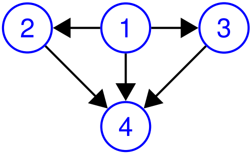
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Directed Graphs



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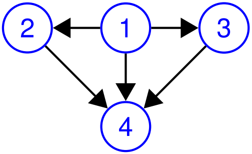
E ordered pairs of vertices.
$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network:

Directed Graphs

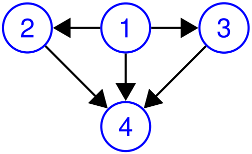


$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed?

Directed Graphs

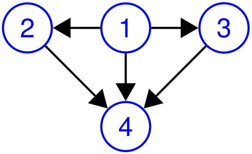


$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?

Directed Graphs



$G = (V, E)$.
V - set of vertices.
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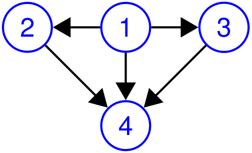
E ordered pairs of vertices.
$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends.

Directed Graphs

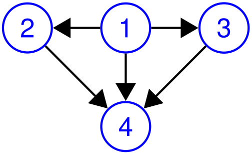


$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.

Directed Graphs

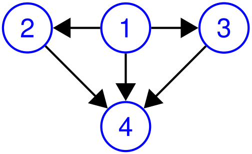


$G = (V, E)$.
V - set of vertices.
$\{1,2,3,4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes.

Directed Graphs



$G = (V, E)$.
V - set of vertices.
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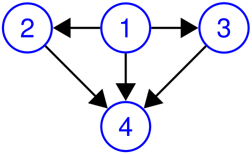
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Directed Graphs



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One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

Graph Concepts and Definitions.

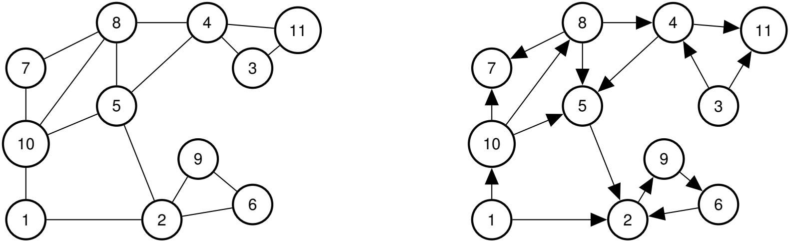
Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

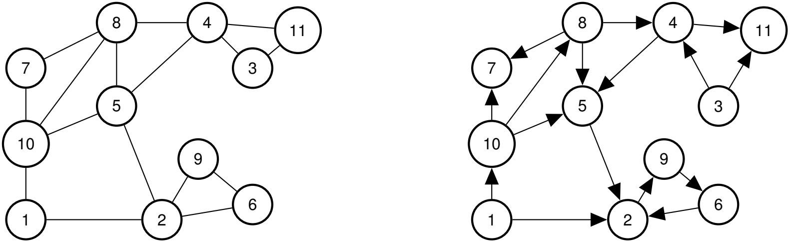


Neighbors of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

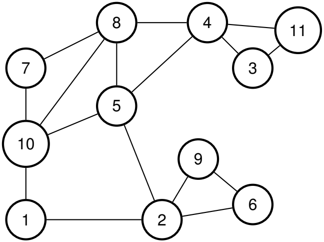


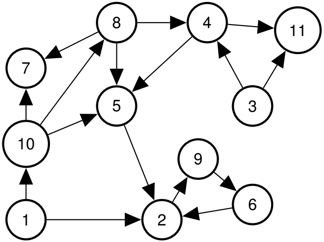
Neighbors of 10? 1,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



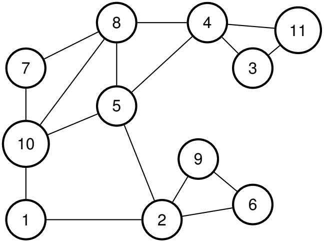


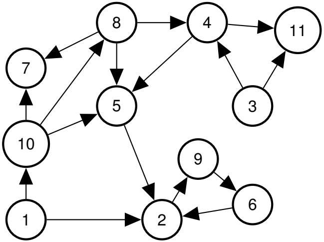
Neighbors of 10? 1,5,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



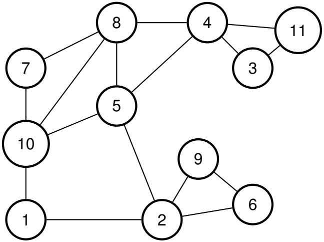


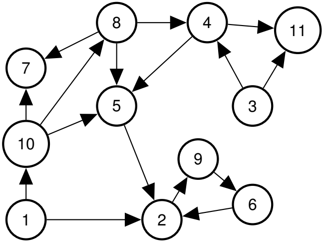
Neighbors of 10? 1,5,7,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





Neighbors of 10? 1,5,7, 8.

Graph Concepts and Definitions.

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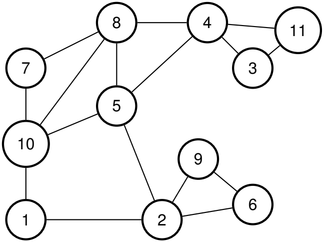

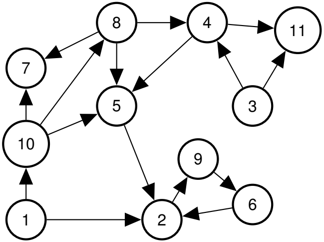

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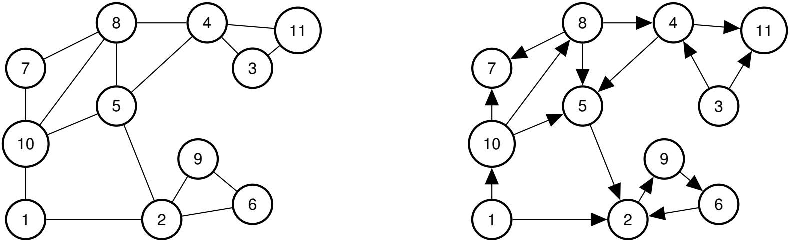

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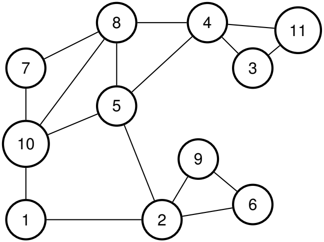

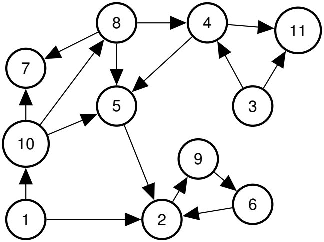

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Degree of vertex 1?

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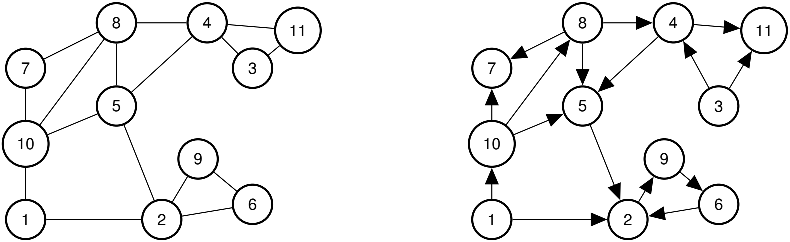

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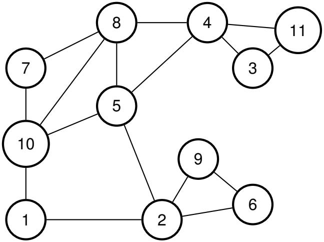

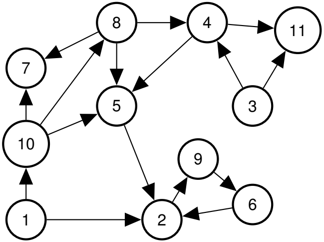

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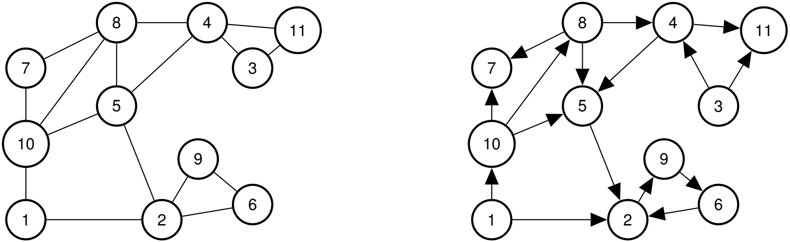

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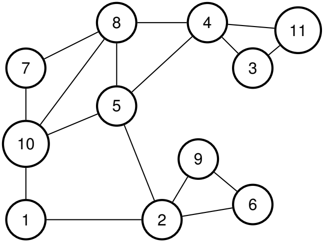

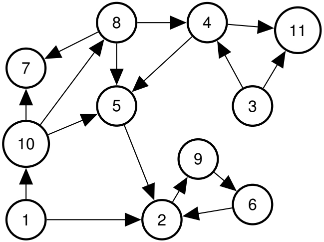

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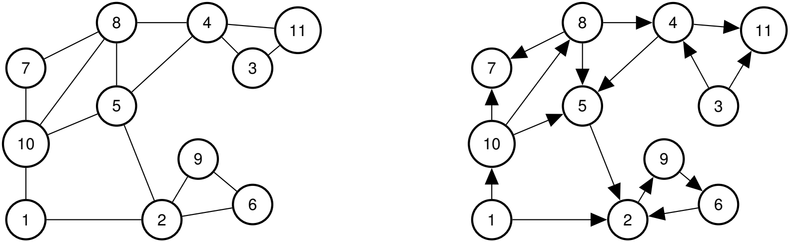

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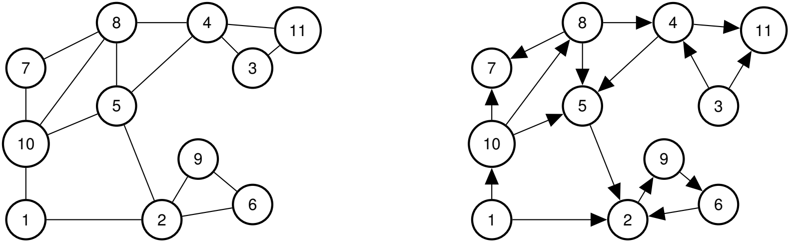

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In-degree of 10?

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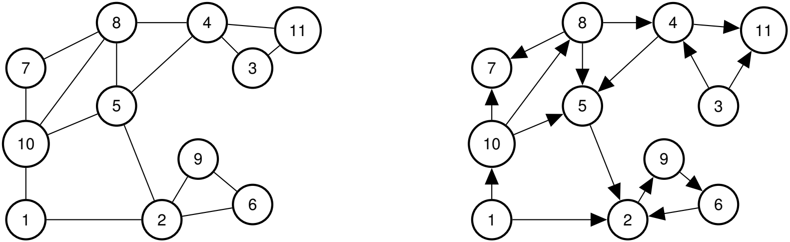

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In-degree of 10? 1

Graph Concepts and Definitions.

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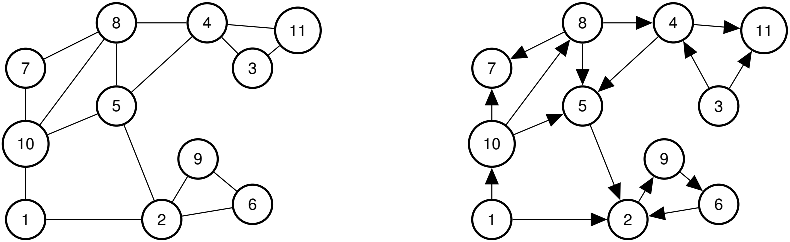

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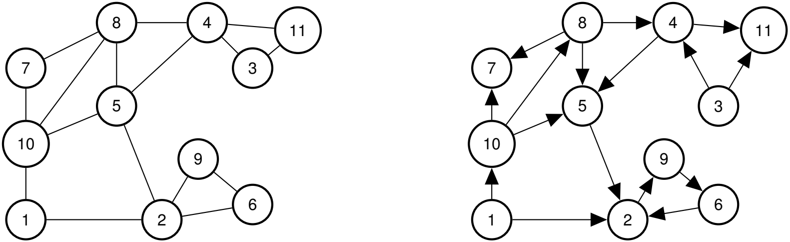

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In-degree of 10? 1   Out-degree of 10? 3

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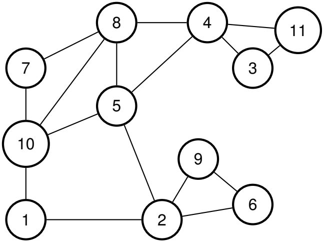

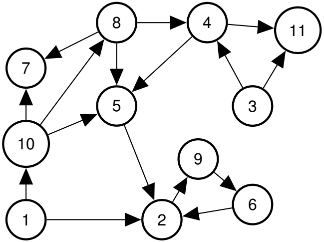

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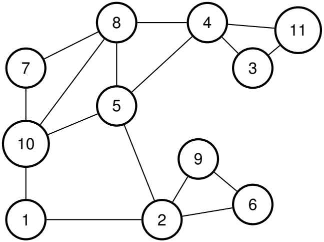

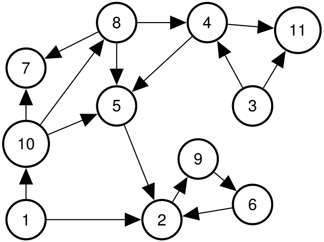

Edge $(8,5)$ **is incident to:**

(A) Vertex 8.
(B) Vertex 5.
(C) Edge $(8,5)$.
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(E) Vertex 10.

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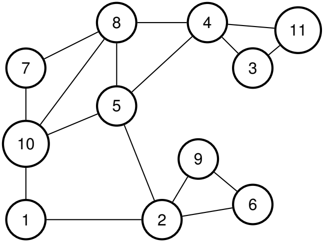

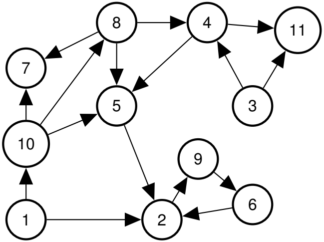

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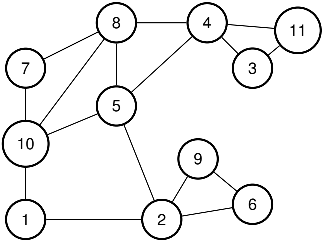

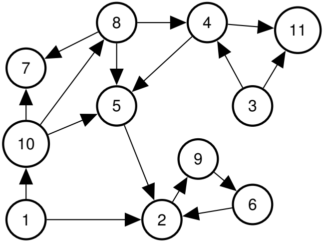

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The degree of a vertex is:

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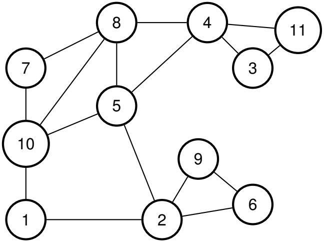

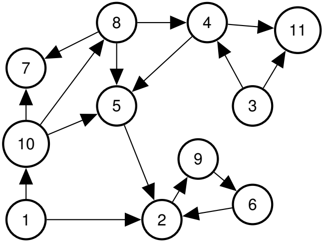

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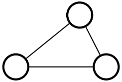
Not (A)!



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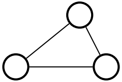

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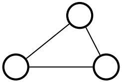

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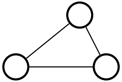

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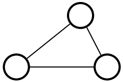

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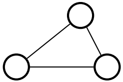

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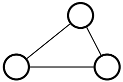

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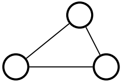

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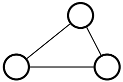

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Could sum always be...

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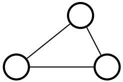

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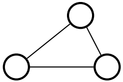

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(A) is true.

Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

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Recall:

Quick Proof of an Equality.

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Let's count incidences in two ways.

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How many incidences does each edge contribute?

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The sum of the vertex degrees is equal to ??

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edge, (u, v), is incident to endpoints, u and v.

degree of u number of edges incident to u

Let's count incidences in two ways.

How many incidences does each edge contribute? 2.

Quick Proof of an Equality.

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Total Incidences? $|E|$ edges, 2 each. $\rightarrow 2|E|$

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Total Incidences? The sum over vertices of degrees!

Thm: Sum of vertex degress is $2|E|$.

Poll: Proof of “handshake” lemma.

What’s true?

(A) The number of edge-vertex incidences for an edge e is 2.
(B) The total number of edge-vertex incidences is $|V|$.
(C) The total number of edge-vertex incidences is $2|E|$.
(D) The number of edge-vertex incidences for a vertex v is its degree.
(E) The sum of degrees is $2|E|$.
(F) The total number of edge-vertex incidences is the sum of the degrees.

Poll: Proof of "handshake" lemma.

What's true?

(A) The number of edge-vertex incidences for an edge e is 2.
(B) The total number of edge-vertex incidences is $|V|$.
(C) The total number of edge-vertex incidences is $2|E|$.
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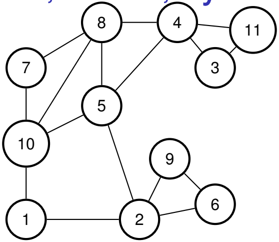
(E) The sum of degrees is $2|E|$.
(F) The total number of edge-vertex incidences is the sum of the degrees.

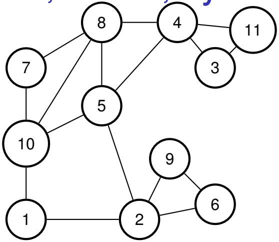
(A),(C), (D), (E), and (F).

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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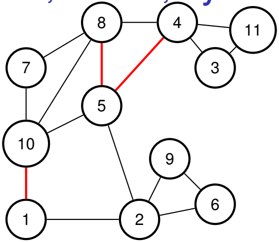

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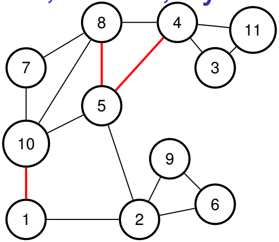

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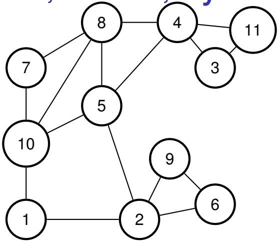

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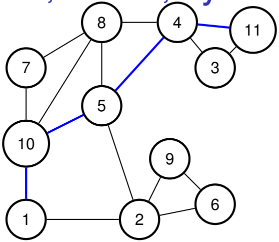

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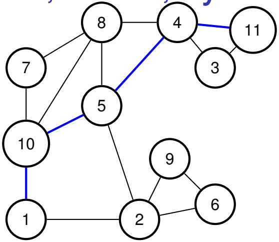

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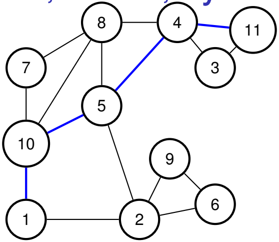

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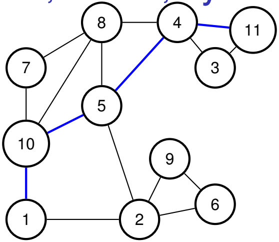

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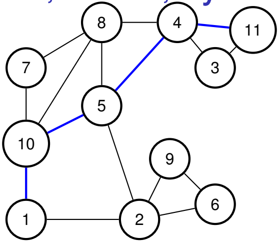
Quick Check!

Paths, walks, cycles, tour.



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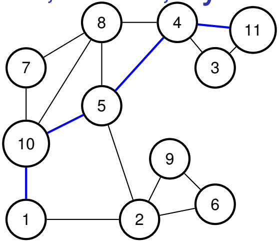
Quick Check! Length of path?

Paths, walks, cycles, tour.



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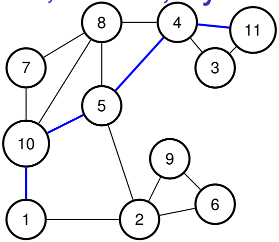
Quick Check! Length of path? k vertices

Paths, walks, cycles, tour.



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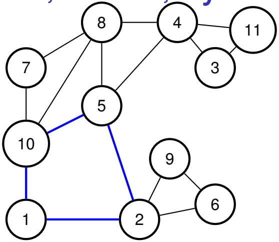
Quick Check! Length of path? k vertices or $k-1$ edges.

Paths, walks, cycles, tour.



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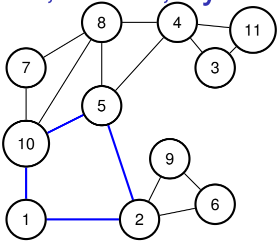
Paths, walks, cycles, tour.



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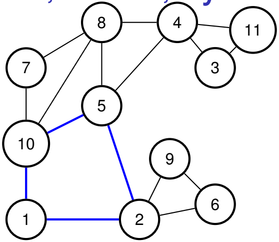
Paths, walks, cycles, tour.



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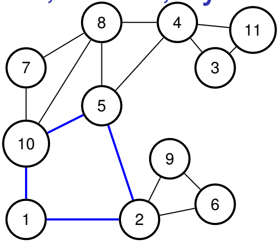
Paths, walks, cycles, tour.



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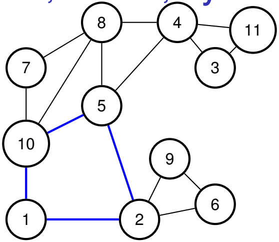
Paths, walks, cycles, tour.



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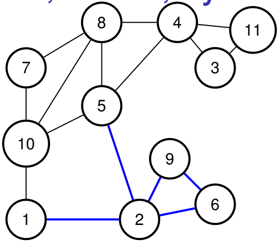
Paths, walks, cycles, tour.



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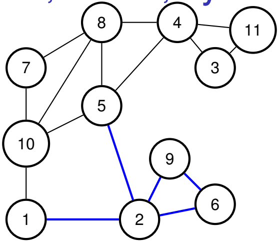
Walk is sequence of edges with possible repeated vertex or edge.

Paths, walks, cycles, tour.



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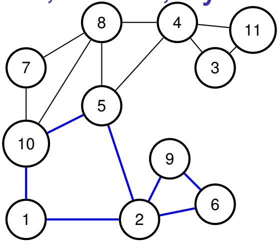

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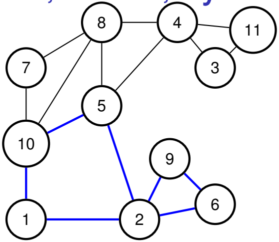
Paths, walks, cycles, tour.



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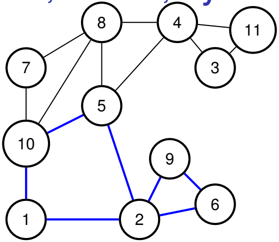

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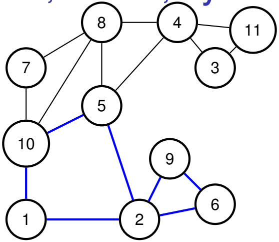

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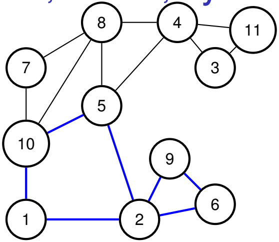
Quick Check!

Path is to Walk as Cycle is to ??

Paths, walks, cycles, tour.



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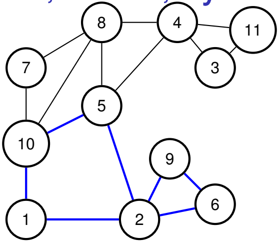
Paths, walks, cycles, tour.



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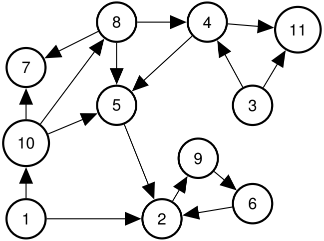

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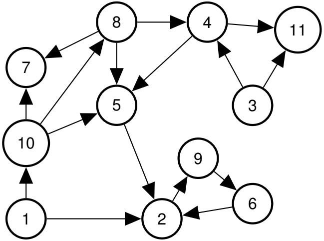

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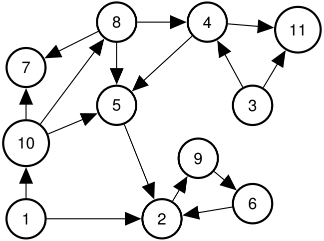

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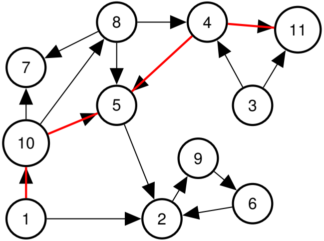

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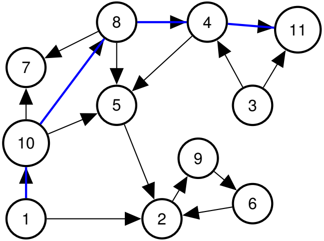

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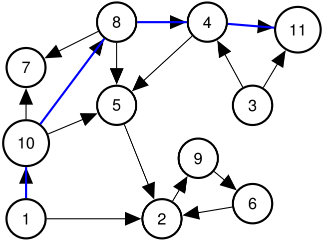
Directed Paths.



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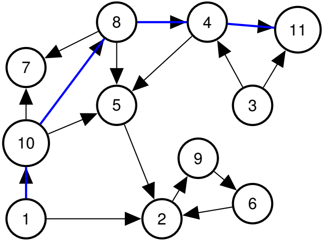

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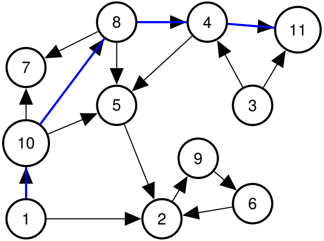

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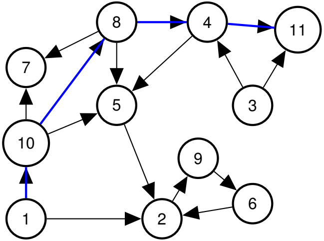

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Paths, walks, cycles, tours

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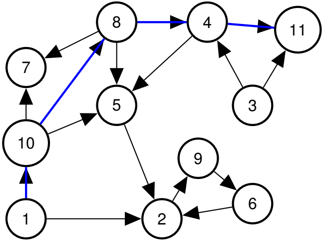

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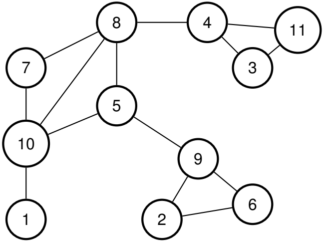
Paths, walks, cycles, tours ... are analagous to undirected now.

Connectivity: undirected graph.



u and v are connected if there is a path between u and v.

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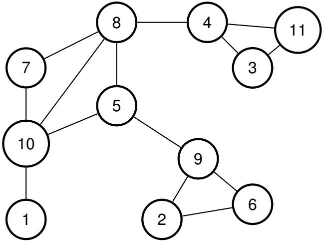

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A connected graph is a graph where all pairs of vertices are connected.

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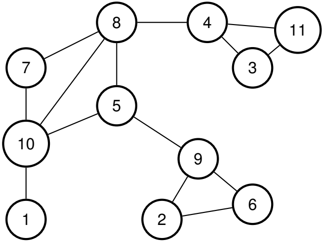

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If one vertex *x* is connected to every other vertex.

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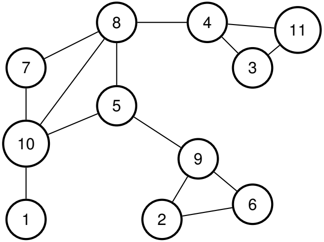

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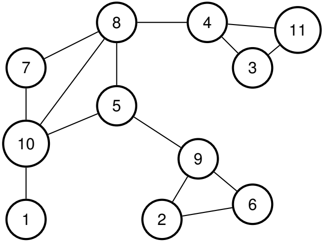

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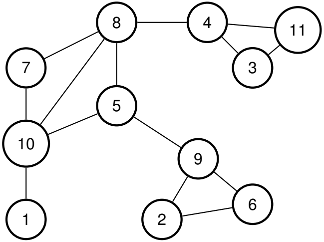

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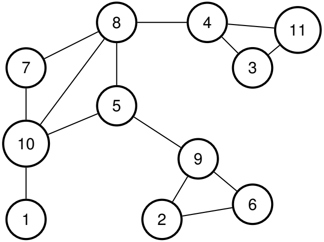

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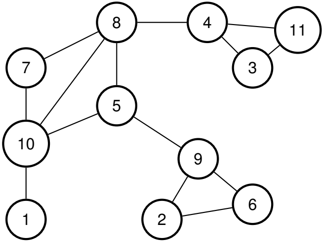

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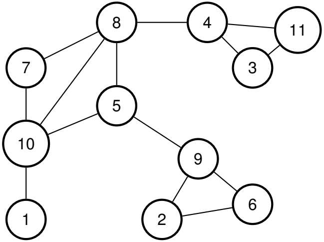

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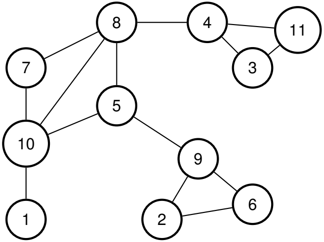

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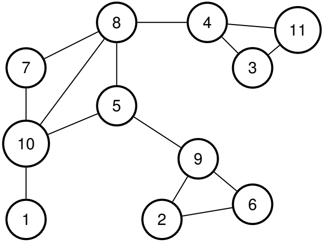

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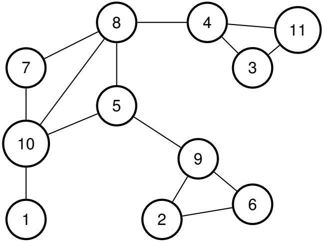

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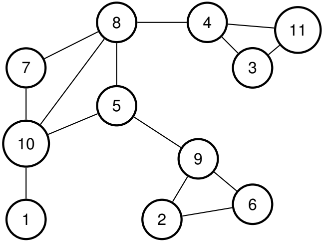

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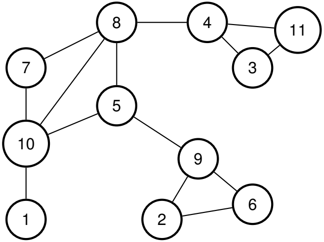

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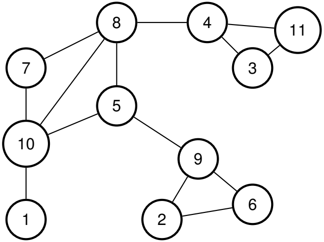
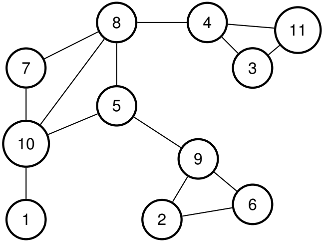
Connected Components: Quiz.



Is graph above connected?

Connected Components: Quiz.



Is graph above connected? Yes!

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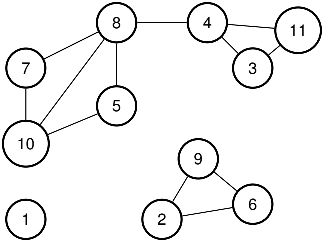

Is graph above connected? Yes!

How about now?

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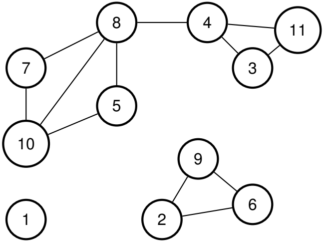

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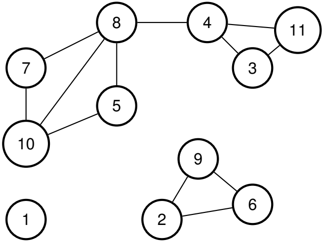

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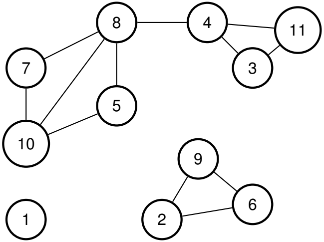
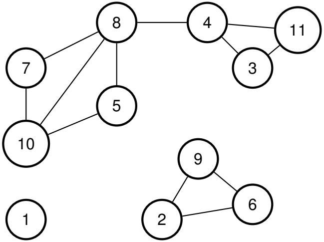


Is graph above connected? Yes!

How about now? No!

Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.

Connected Components: Quiz.



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Connected component - maximal set of connected vertices.

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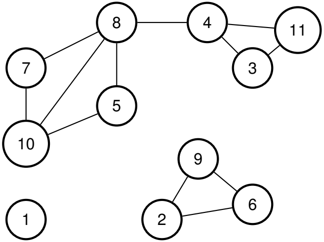

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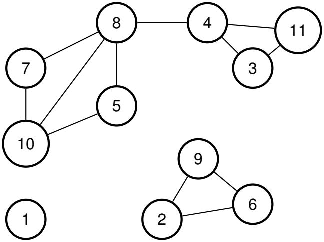
Quick Check: Is $\{10, 7, 5\}$ a connected component?

Connected Components: Quiz.



Is graph above connected? Yes!

How about now? No!

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Connected component - maximal set of connected vertices.

Quick Check: Is $\{10, 7, 5\}$ a connected component? No.

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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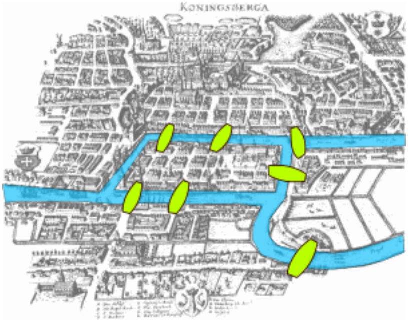

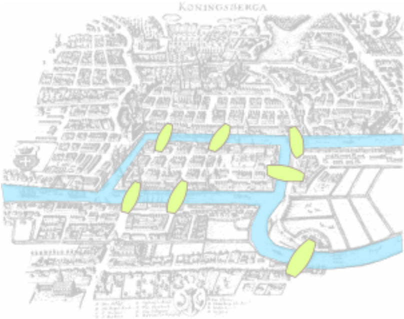

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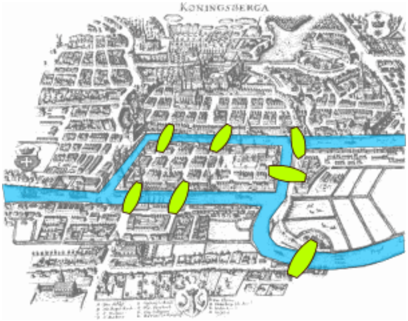

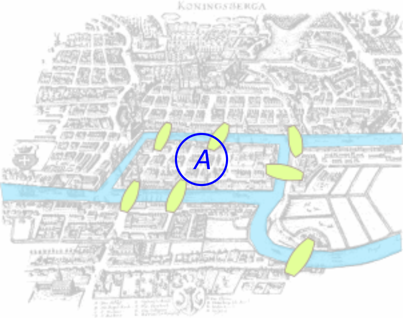

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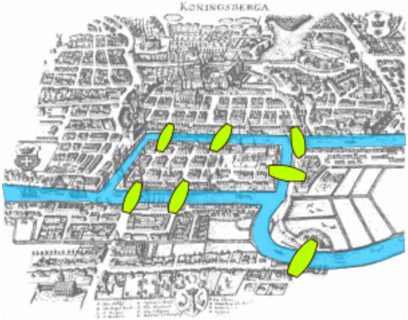

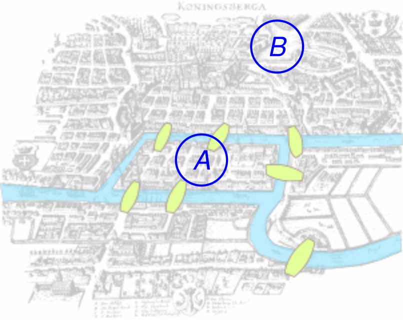

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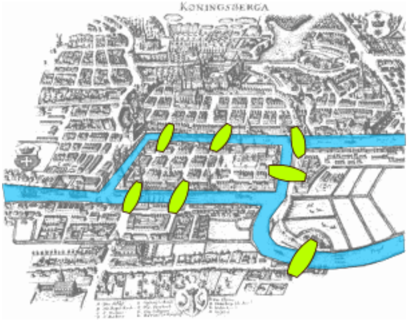

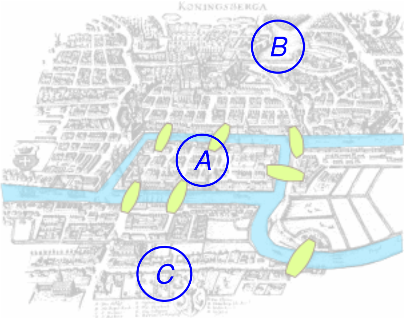

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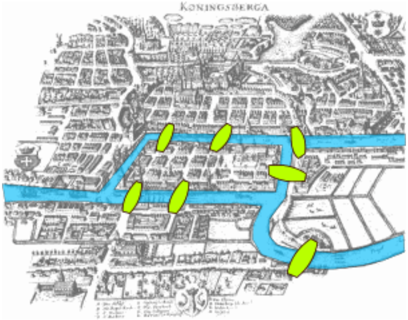

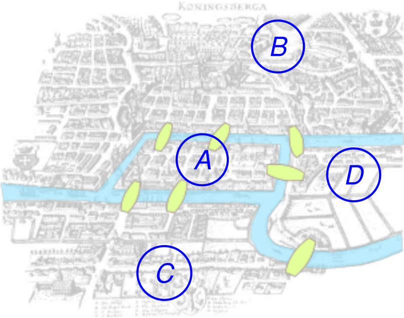

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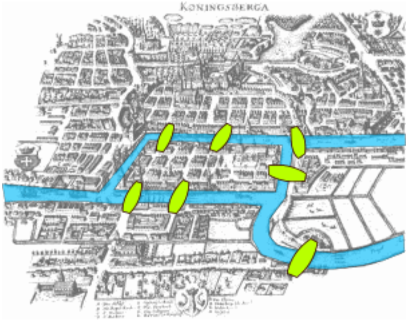

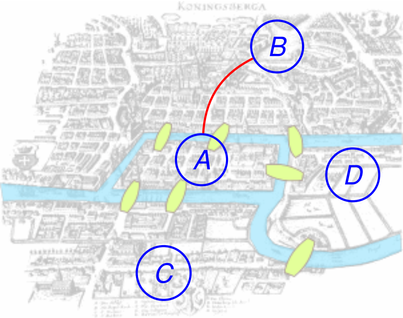

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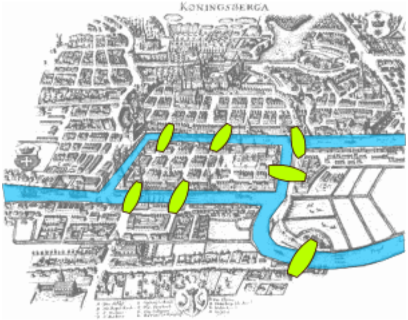

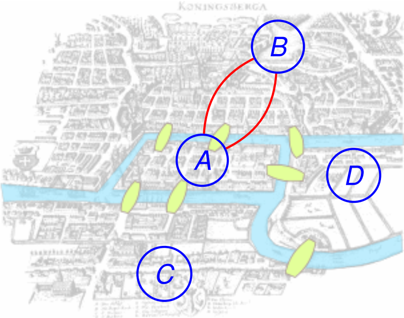

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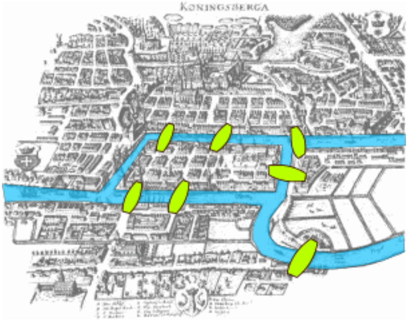

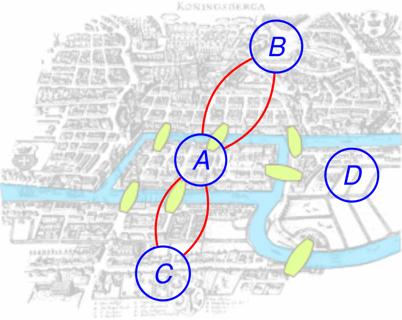

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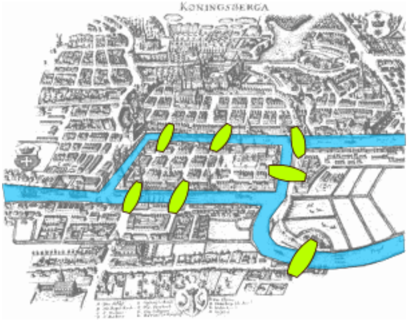

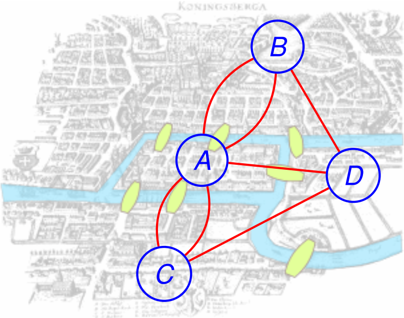

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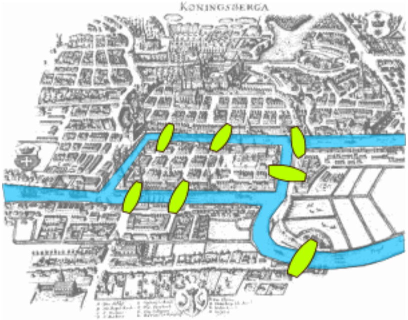

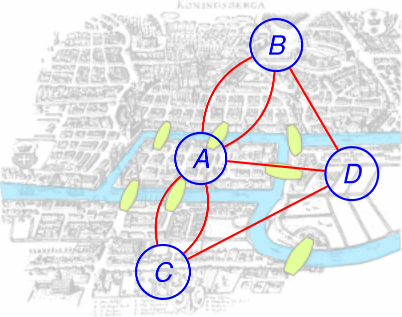

Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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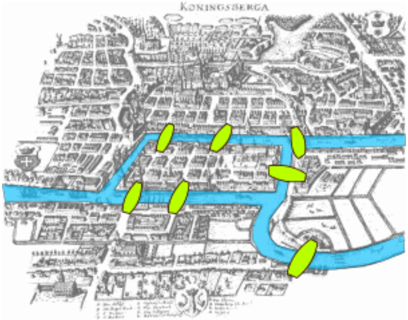

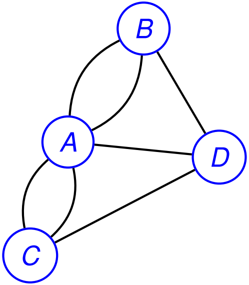

Can you draw a tour in the graph where you visit each edge once?
Yes?

Konigsberg bridges problem.

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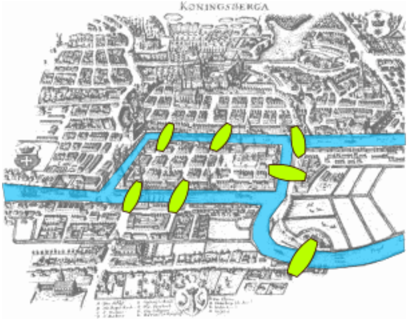

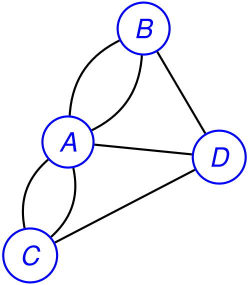

Can you draw a tour in the graph where you visit each edge once?
Yes? No?

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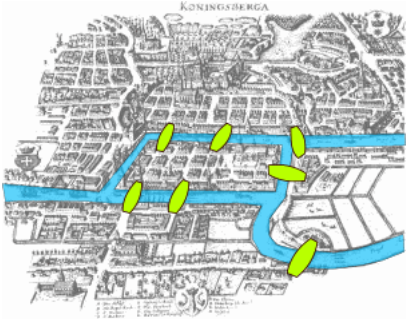

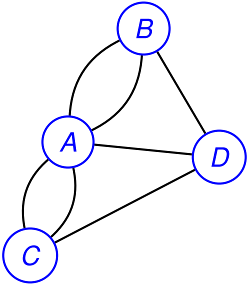

Can you draw a tour in the graph where you visit each edge once?
Yes? No?
We will see!

Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

Eulerian Tour

An Eulerian Tour is a tour that visits each edge exactly once.

Theorem: Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

Eulerian Tour

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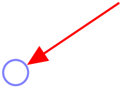

When you enter,

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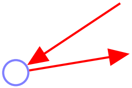

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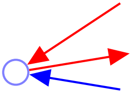

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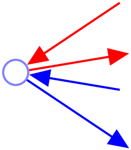

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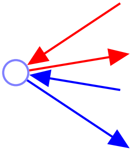

When you enter, you can leave.
For starting node,

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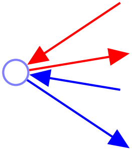

When you enter, you can leave.
For starting node, tour leaves first

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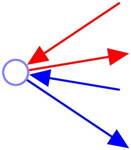

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.

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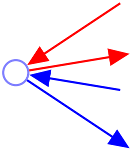

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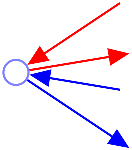

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.
Not The Hotel California.

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.

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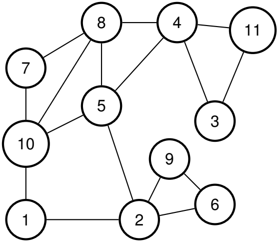

1. Take a walk starting from v (1) on "unused" edges

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
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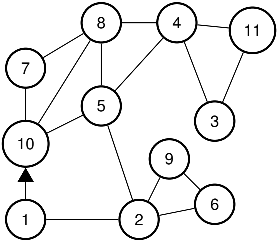

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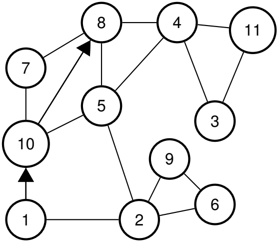

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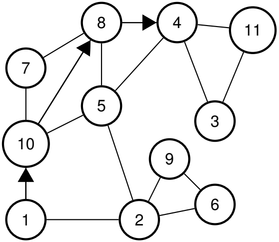

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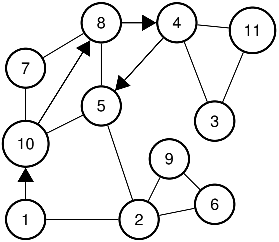

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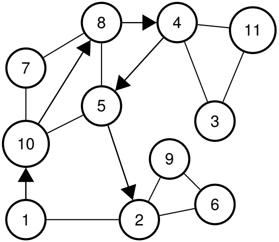

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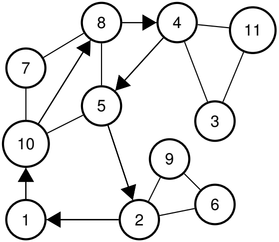

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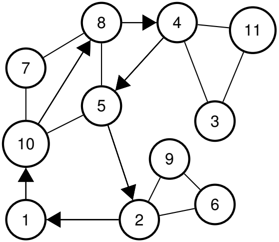

1. Take a walk starting from v (1) on “unused” edges
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2. Remove tour, C.

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
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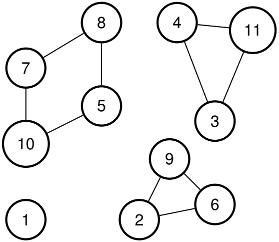

1. Take a walk starting from v (1) on "unused" edges
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2. Remove tour, C.
3. Let $G_1, \ldots, G_k$ be connected components.

Finding a tour!

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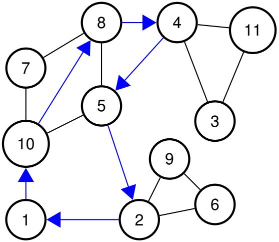

1. Take a walk starting from v (1) on "unused" edges
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2. Remove tour, C.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by C.

Finding a tour!

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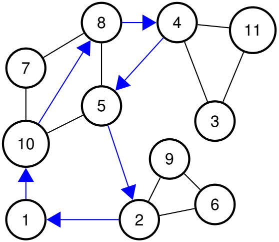

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      Why?

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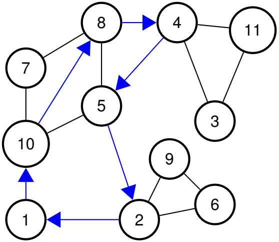

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   Why? G was connected.

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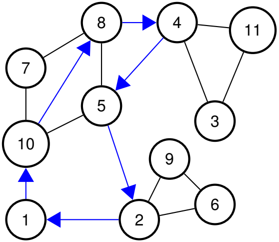

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   Let v_i be (first) node in G_i touched by C.

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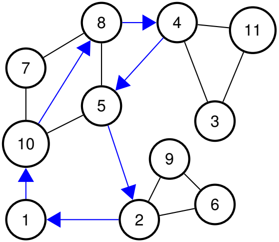

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    Example: $v_1 = 1$,

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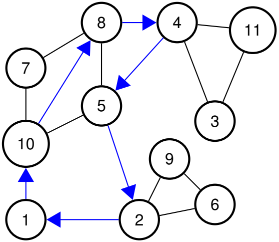

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      Why? G was connected.
   Let v_i be (first) node in G_i touched by C.
    Example: $v_1 = 1$, $v_2 = 10$,

Finding a tour!

Proof of if: Even + connected $\Longrightarrow$ Eulerian Tour.
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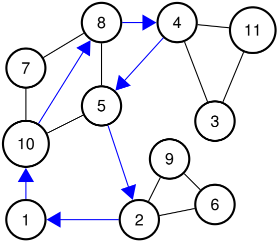

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    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$,

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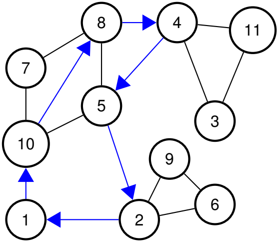

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    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.

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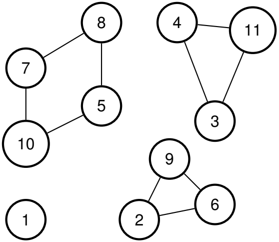

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   Let v_i be (first) node in G_i touched by C.
    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from v_i

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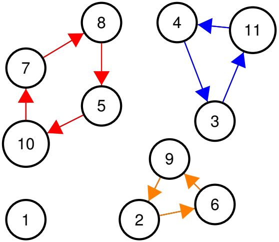

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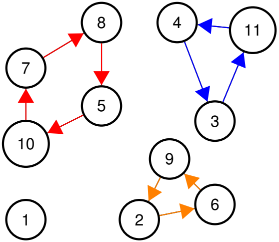

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    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from v_i
5. Splice together.

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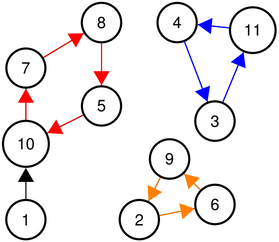

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   1,10

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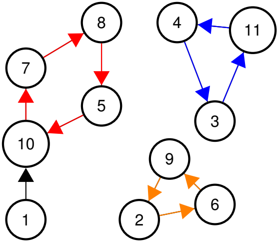

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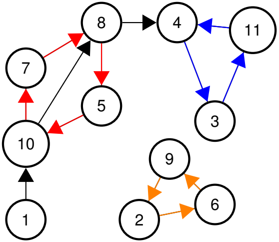

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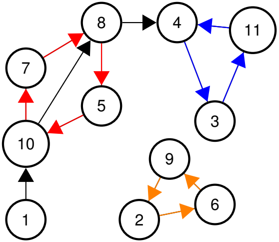
    Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$, $v_4 = 2$.
4. Recurse on $G_1, \ldots, G_k$ starting from v_i
5. Splice together.
   1,10,7,8,5,10 ,8,4

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm. First by picture.



1. Take a walk starting from v (1) on "unused" edges
... till you get back to v.
2. Remove tour, C.
3. Let $G_1, \ldots, G_k$ be connected components.
   Each is touched by C.
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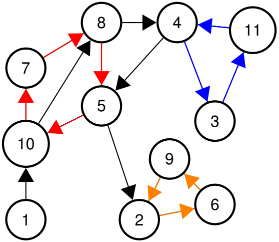

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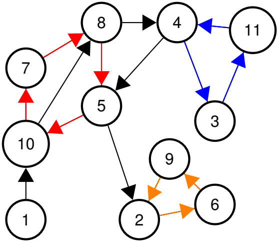

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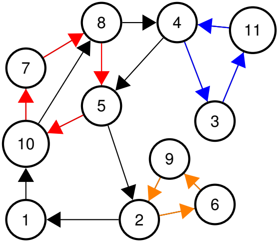

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Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

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Proof of Claim: Even degree.

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Why is there a v_i in C?

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Visits every edge once:
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Poll: Euler concepts.

Mark correct statements for a connected graph where all vertices have even degree. (Below, we use tour to mean uses an edge exactly once, but may involve a vertex several times.

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(A) Removing a tour leaves a graph of even degree.
(B) A tour connecting a set of connected components, each with a Eulerian tour is really cool! Eulerian even.
(C) There is no hotel california in this graph.
(D) After removing a set of edges E' in a connected graph, every connected component is incident to an edge in E'
(E) If one walks on new edges, starting at v, one must eventually get back to v.
(F) Removing a tour leaves a connected graph.

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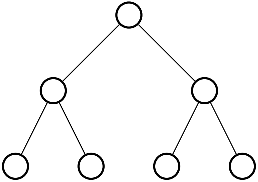
Only (F) is false.

A Tree, a tree.

Graph $G = (V, E)$.
Binary Tree!



More generally.

Trees.

Definitions:

Trees.

Definitions:

A connected graph without a cycle.

Trees.

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A connected graph with $|V| - 1$ edges.

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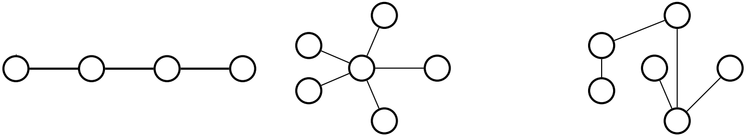

no cycle and connected?

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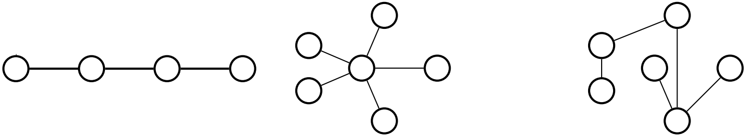

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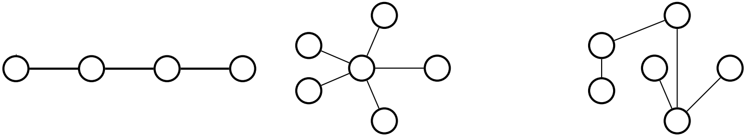

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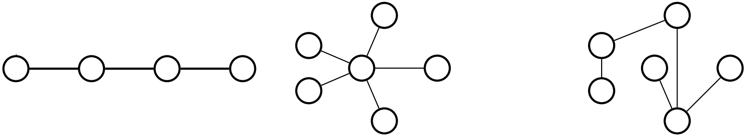

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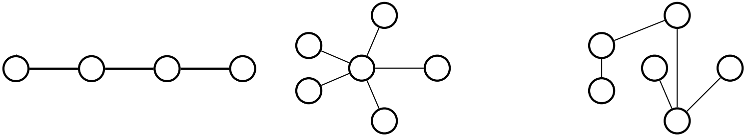

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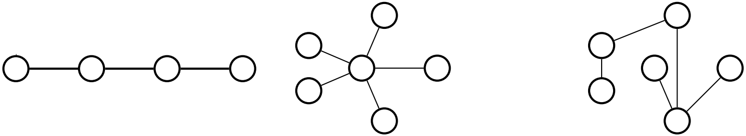

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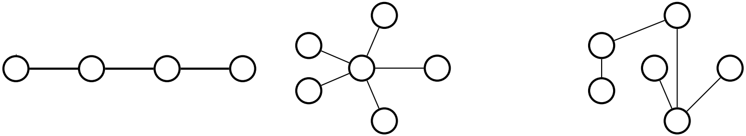

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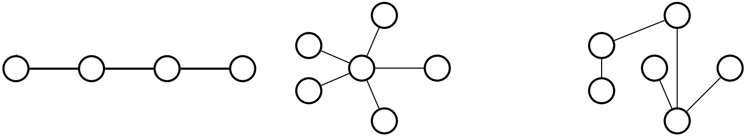

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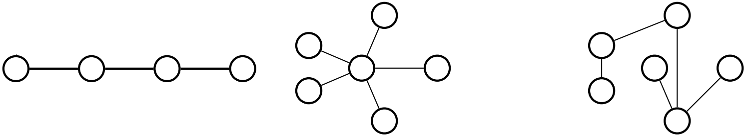

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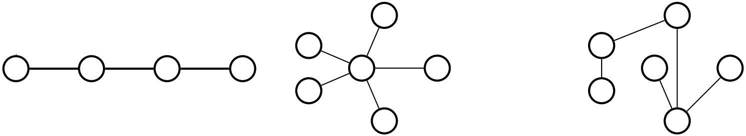

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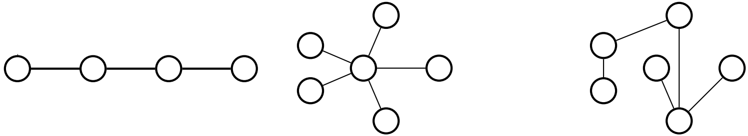
Some trees.



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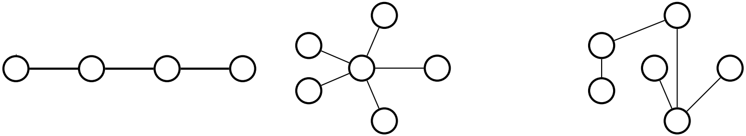

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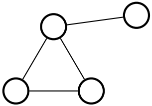

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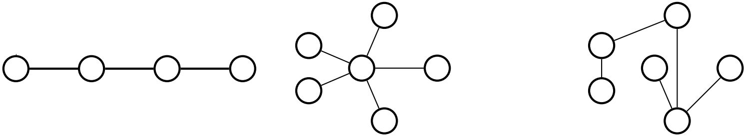

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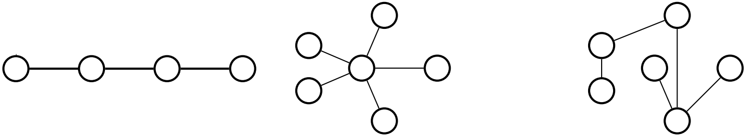

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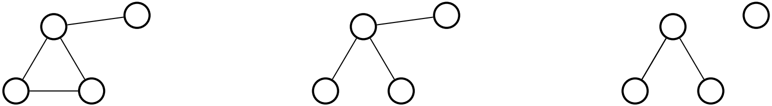

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Theorem:
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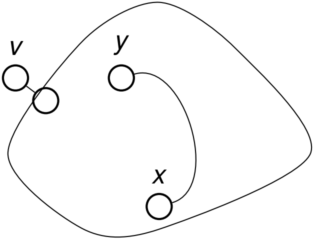

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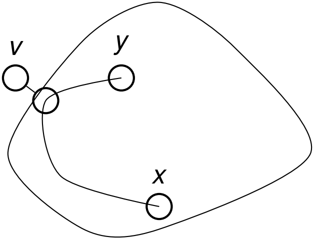

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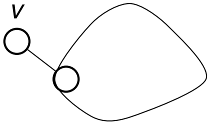

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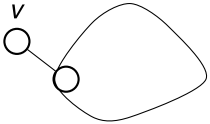

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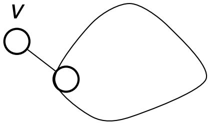

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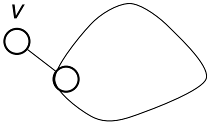

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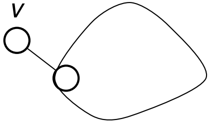

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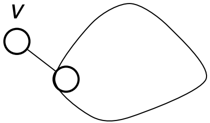

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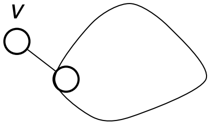

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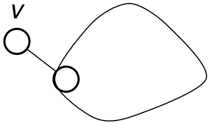

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Sum of degrees is $2|E| = 2(|V| - 1) = 2|V| - 2$

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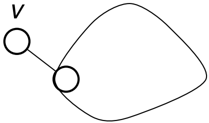

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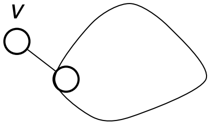

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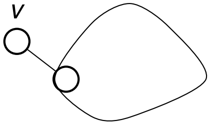

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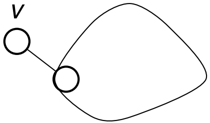

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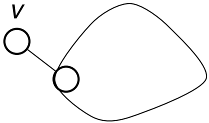

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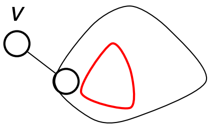

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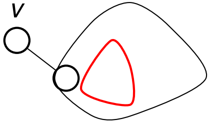

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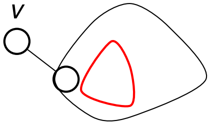

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Can't visit more than once since no cycle.

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Poll: Oh tree, beautiful tree.

Let G be a connected graph with $|V| - 1$ **edges.**

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Let G be a connected graph with $|V|-1$ edges.

(A) Removing a degree 1 vertex can disconnect the graph.
(B) One can use induction on smaller objects.
(C) The average degree is $2-2/|V|$.
(D) There is a hotel california: a degree 1 vertex.
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(B), (C), (D) are true

Planar graphs.

A graph that can be drawn in the plane without edge crossings.

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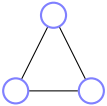

Planar?

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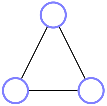

Planar? Yes for Triangle.

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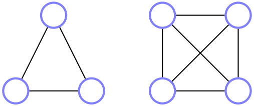

Planar? Yes for Triangle.
Four node complete?

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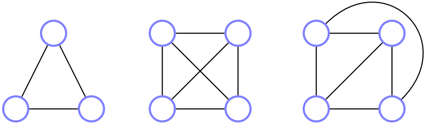

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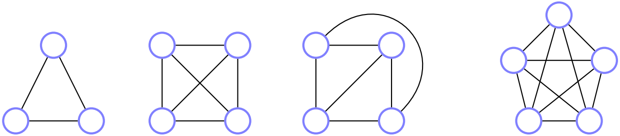
Planar graphs.

A graph that can be drawn in the plane without edge crossings.



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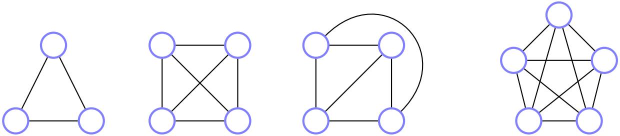

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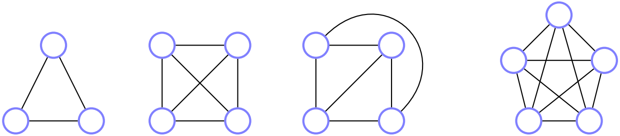

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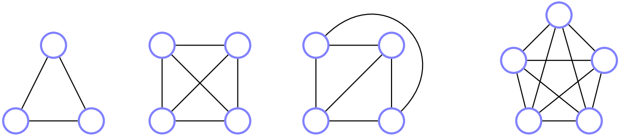

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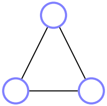 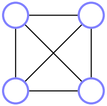 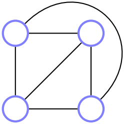 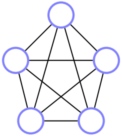

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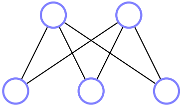

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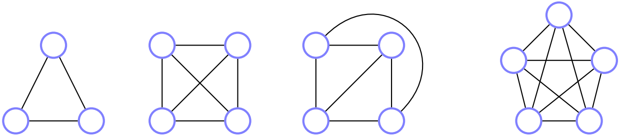

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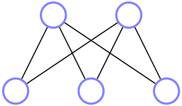

Two to three nodes, bipartite?

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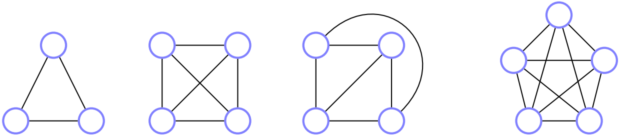

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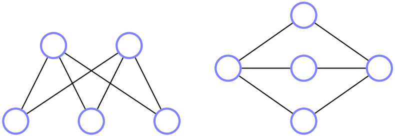

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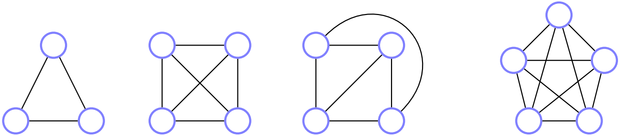

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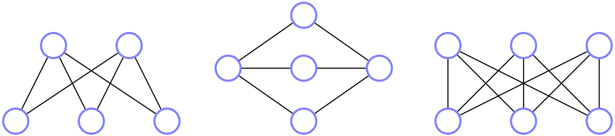

Two to three nodes, bipartite? Yes.
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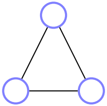
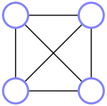
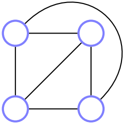
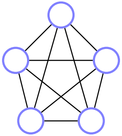

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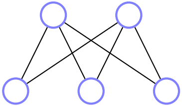
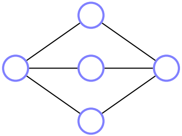
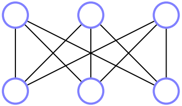

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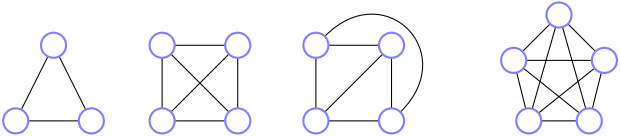

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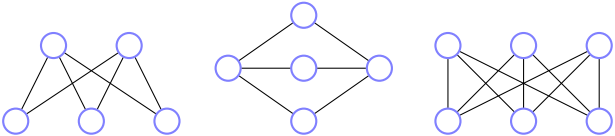

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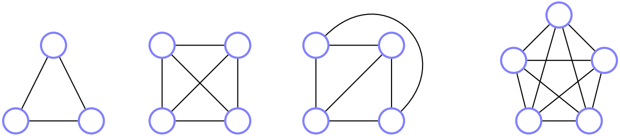

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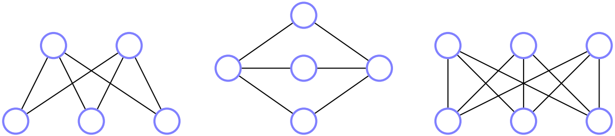

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