Lecture 6.

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Finish Euler's Formula.

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Planar Five Color theorem.

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Types of graphs.

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Finish Euler's Formula.

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Types of graphs.

- Complete Graphs.
- Trees.
- Hypercubes.

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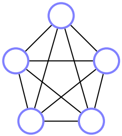
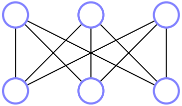
Planarity and Euler





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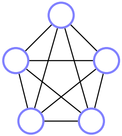

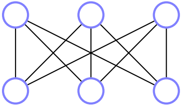

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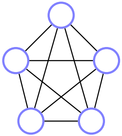

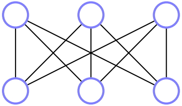

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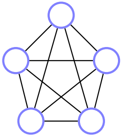

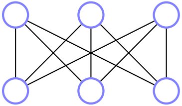

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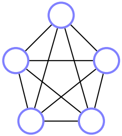
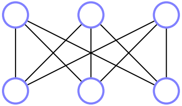
Planarity and Euler





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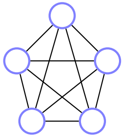

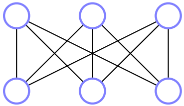

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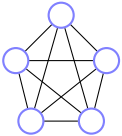

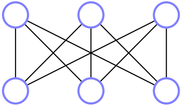

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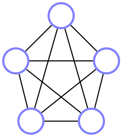

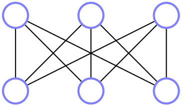

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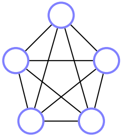

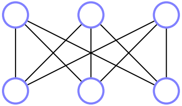

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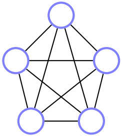

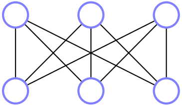

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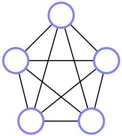

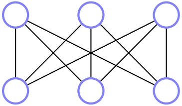

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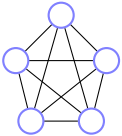

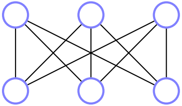

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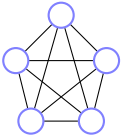

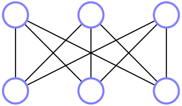

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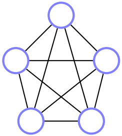
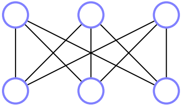
Planarity and Euler





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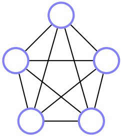

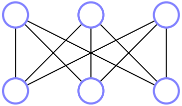

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So......so ...Cool!

Tree.

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To tree or not to tree!

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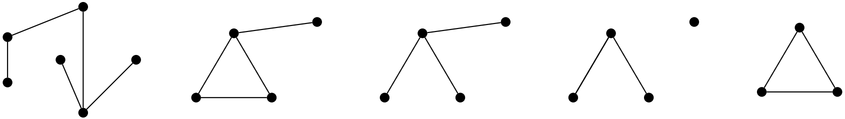

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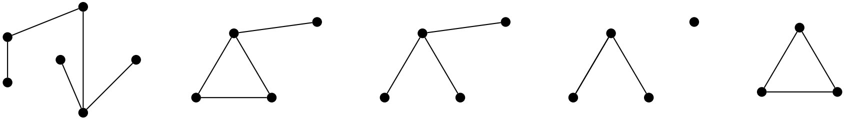

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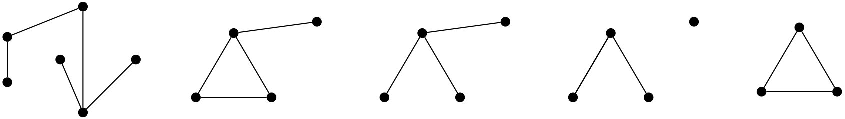

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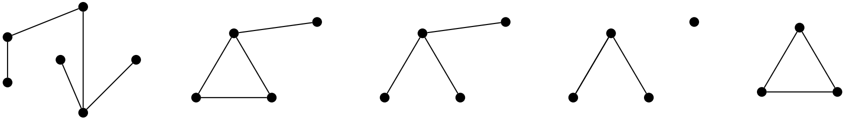

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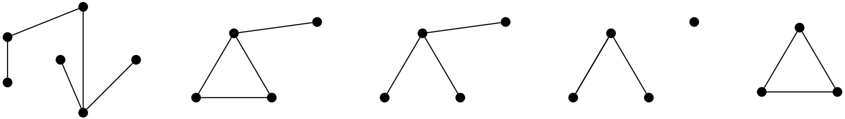

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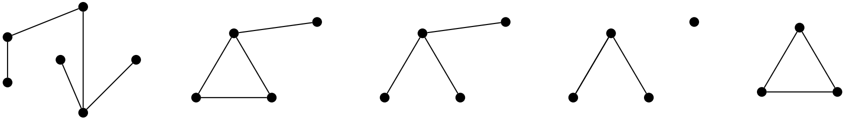

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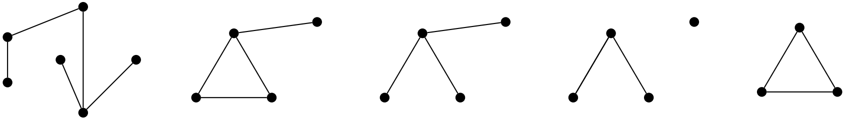

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Faces?

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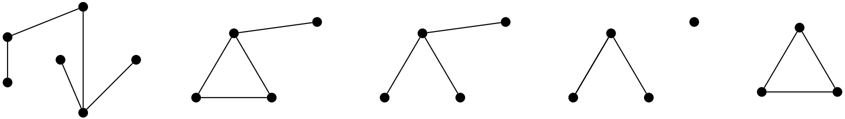

Yes. No. Yes. No. No.

Faces? 1.

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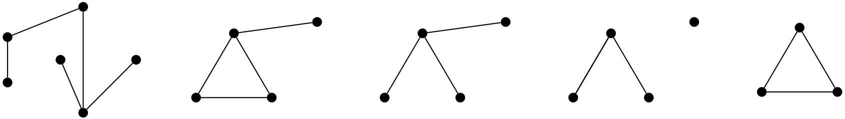

Yes. No. Yes. No. No.

Faces? 1. 2.

Tree.

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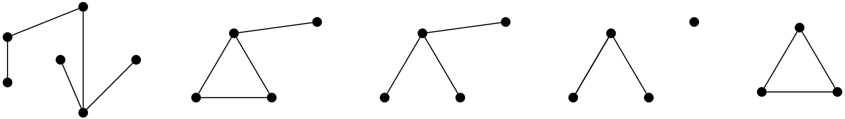

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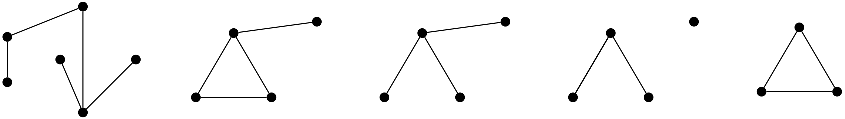

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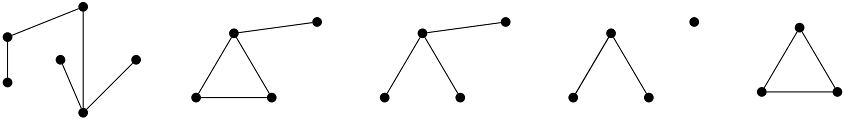

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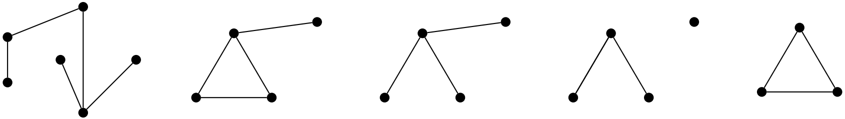

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Vertices/Edges.

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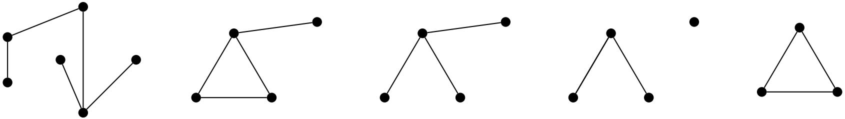

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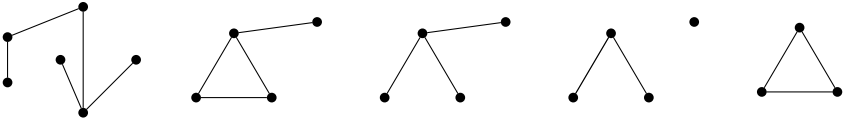

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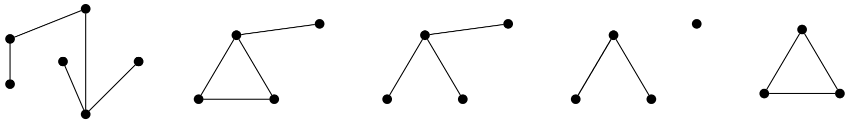

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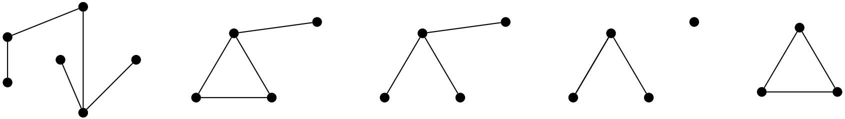

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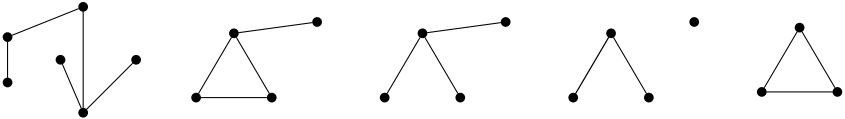

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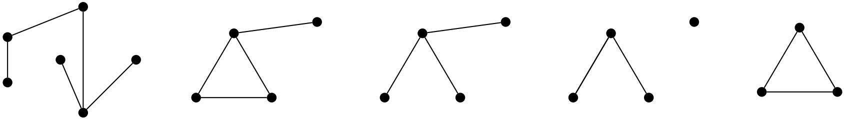

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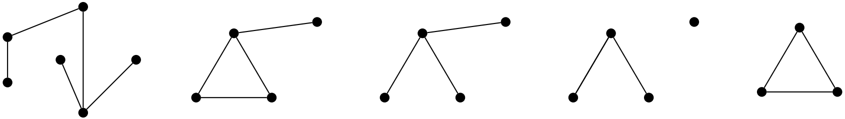
To tree or not to tree!



Yes. No. Yes. No. No.

Faces? 1. 2. 1. 1. 2.
Vertices/Edges. Notice: $e = v - 1$ for tree.

One face for trees!

Euler's Formula: $v + f = e + 2$ for any connected planar drawing.

Euler works for trees: $v + f = e + 2$.
$v + 1 = v - 1 + 2$

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

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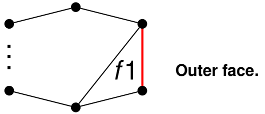

Joins two faces.

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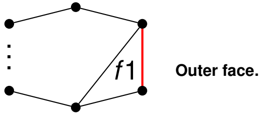

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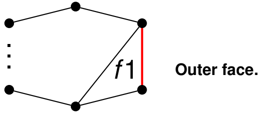

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New graph: v-vertices. $e - 1$ edges.

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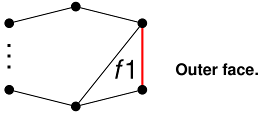

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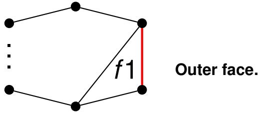

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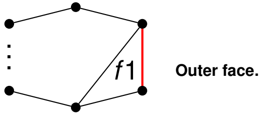

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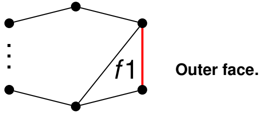

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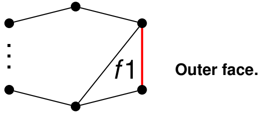

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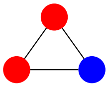

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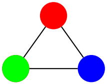

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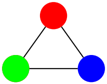

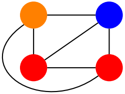

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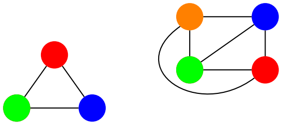

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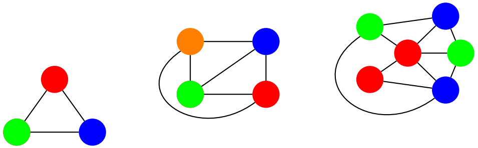

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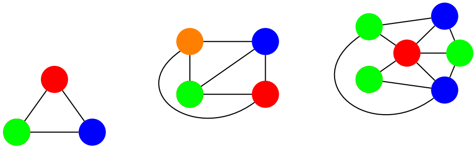

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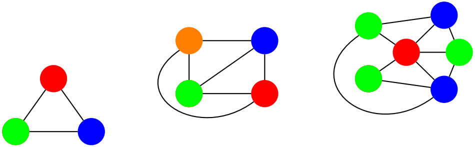

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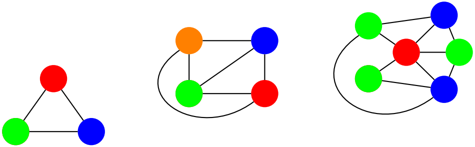

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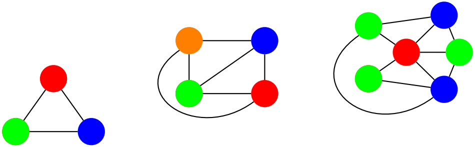

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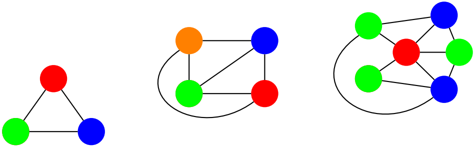

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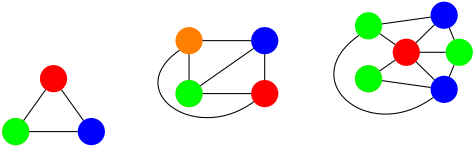

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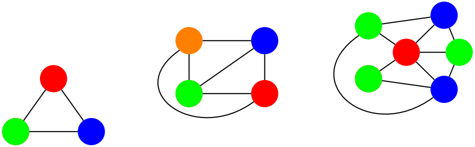

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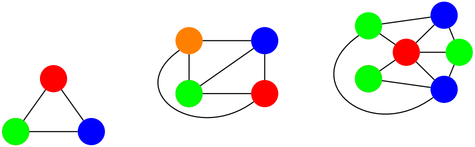

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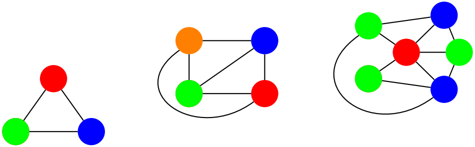

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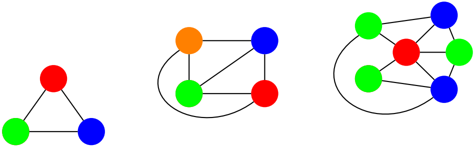

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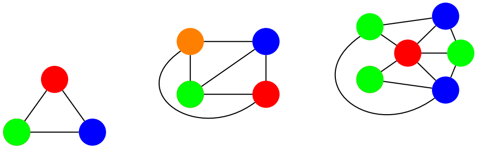

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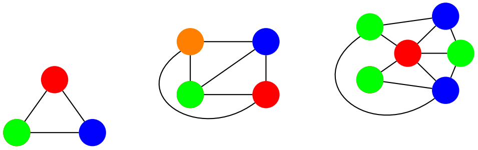

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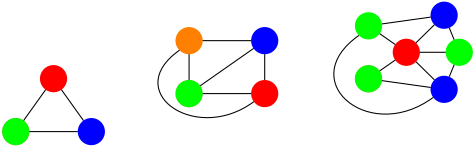

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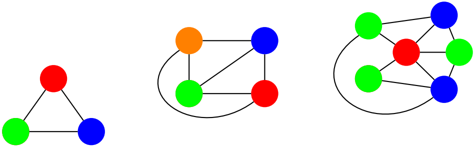

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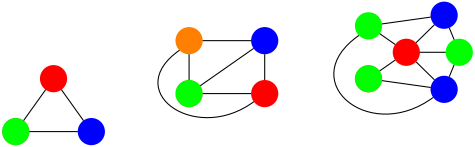

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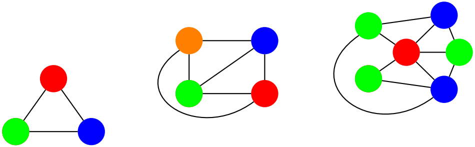

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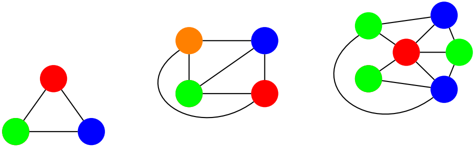

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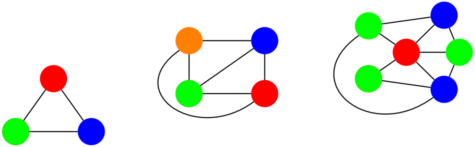

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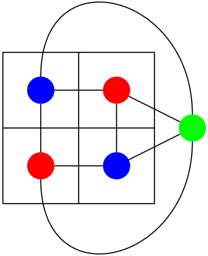
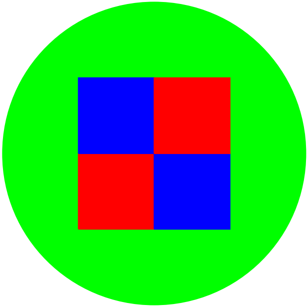
Planar graphs and maps.

Planar graph coloring $\equiv$ map coloring.





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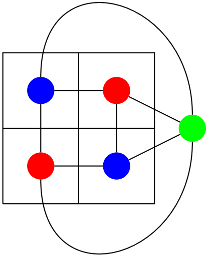

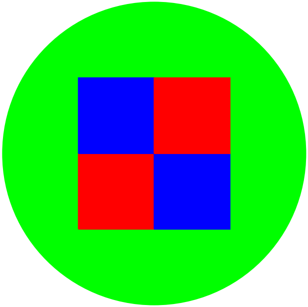

Four color theorem is about planar graphs!

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Average degree: $= \frac{2e}{v} \leq \frac{2(3v-6)}{v}$

Six color theorem.

Theorem: Every planar graph can be colored with six colors.

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Recall: $e \leq 3v - 6$ for any planar graph where $v > 2$.
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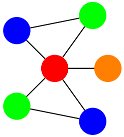

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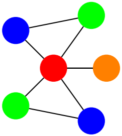

Look at only green and blue.

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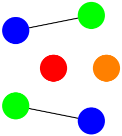

Look at only green and blue.
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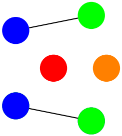

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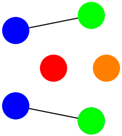

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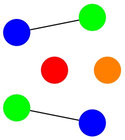

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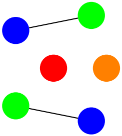

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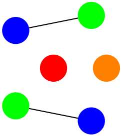

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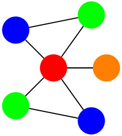

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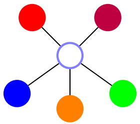

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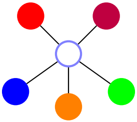

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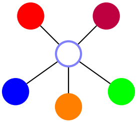

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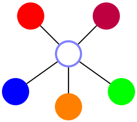

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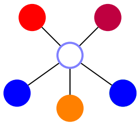

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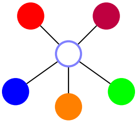

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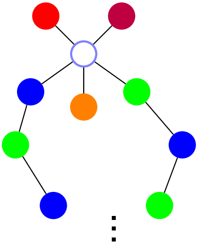

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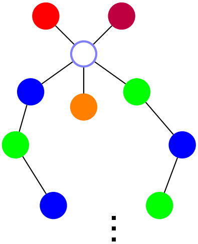

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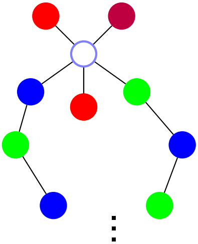

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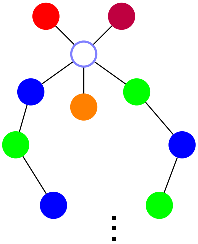

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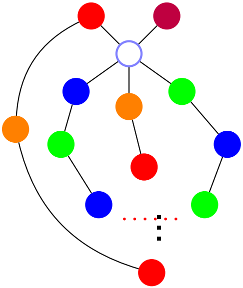

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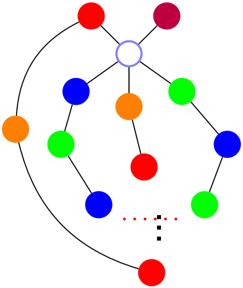

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Planar.

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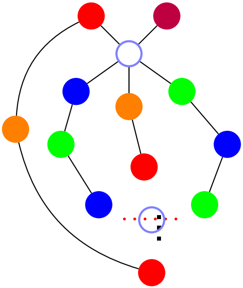

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Planar. $\implies$ paths intersect at a vertex!

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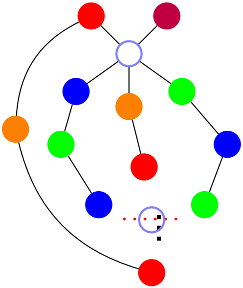

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What color is it?

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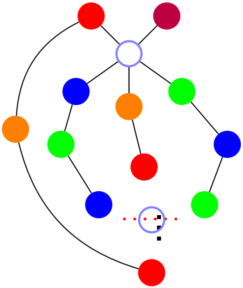

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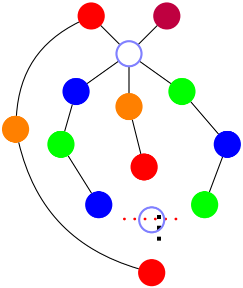

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What color is it?
Must be blue or green to be on that path.

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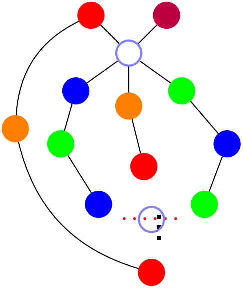

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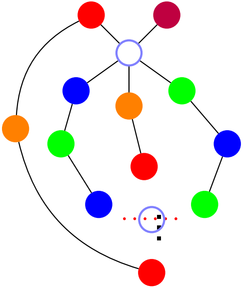

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What color is it?
Must be blue or green to be on that path.
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Contradiction.

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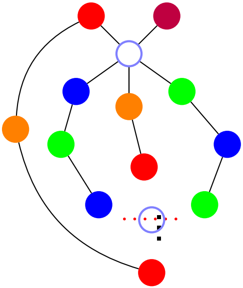

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What color is it?
Must be blue or green to be on that path.
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Contradiction. Can recolor one of the neighbors.

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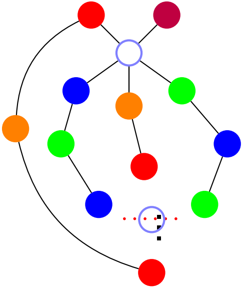

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Must be blue or green to be on that path.
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Gives an available color for center vertex!

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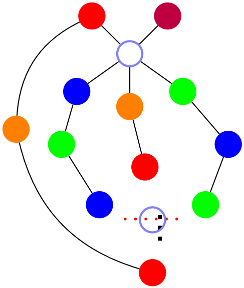

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Four Color Theorem

Four Color Theorem

Theorem: Any planar graph can be colored with four colors.

Four Color Theorem

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Proof:

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Proof: Not Today!

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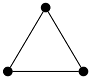
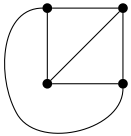
Proof: Not Today!

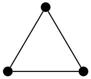
Complete Graph.

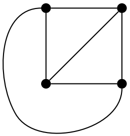




K_n complete graph on n vertices.

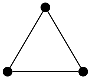
Complete Graph.

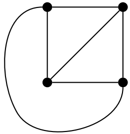




K_n complete graph on n vertices.
All edges are present.

Complete Graph.



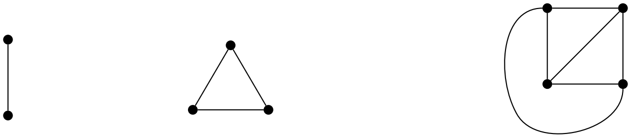


K_n complete graph on n vertices.

All edges are present.

Everyone is my neighbor.

Complete Graph.



K_n complete graph on n vertices.

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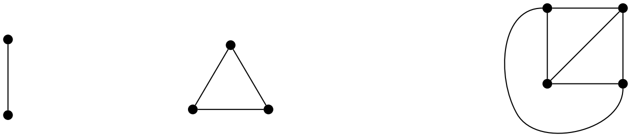
Each vertex is adjacent to every other vertex.

Complete Graph.



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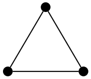
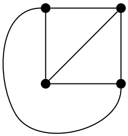

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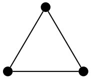
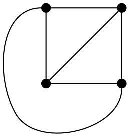
How many edges?

Complete Graph.





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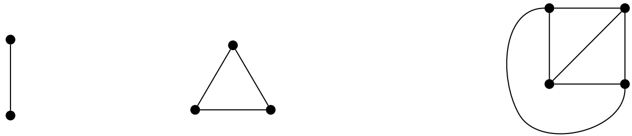
How many edges?

Each vertex is incident to $n-1$ edges.

Complete Graph.



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All edges are present.
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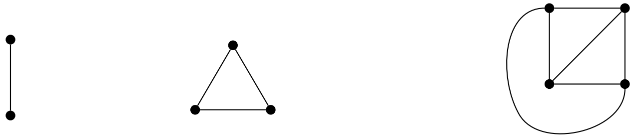
How many edges?

Each vertex is incident to $n-1$ edges.
Sum of degrees is $n(n-1)$

Complete Graph.



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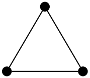
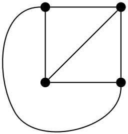
Each vertex is adjacent to every other vertex.

How many edges?

Each vertex is incident to $n-1$ edges.

Sum of degrees is $n(n-1) = 2|E|$

Complete Graph.

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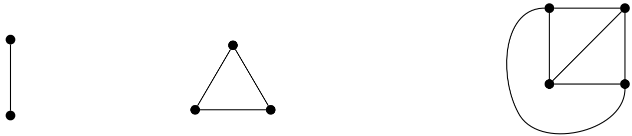
Each vertex is incident to $n-1$ edges.

Sum of degrees is $n(n-1) = 2|E|$

$\implies$ Number of edges is $n(n-1)/2$.

Complete Graph.



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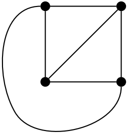
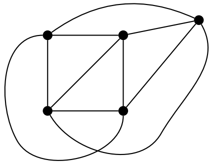
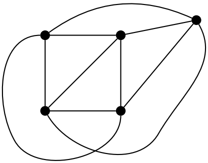
K_4 and K_5



K_4 and K_5

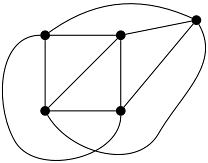


K_4 and K_5



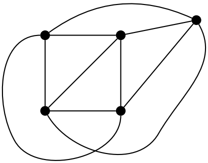
K_5 is not planar.

K_4 and K_5



K_5 is not planar.
Cannot be drawn in the plane without an edge crossing!

K_4 and K_5



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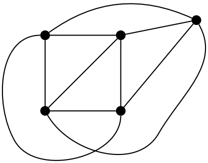
Prove it!

K_4 and K_5



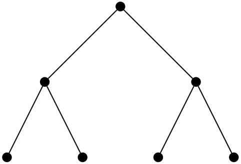
K_5 is not planar.

Cannot be drawn in the plane without an edge crossing!

Prove it! We did!

A Tree, a tree.

Graph $G = (V, E)$.
Binary Tree!



More generally.

Trees.

Definitions:

Trees.

Definitions:

A connected graph without a cycle.

Trees.

Definitions:

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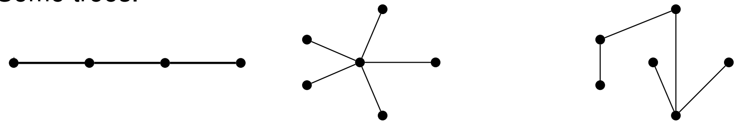
Some trees.



no cycle and connected?

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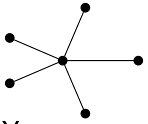
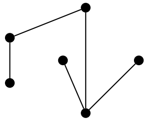

no cycle and connected? Yes.

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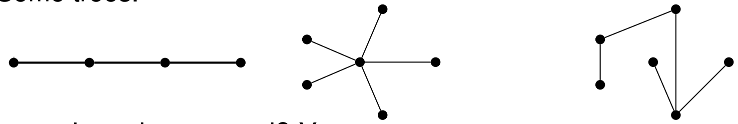

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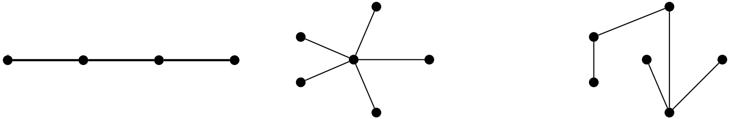

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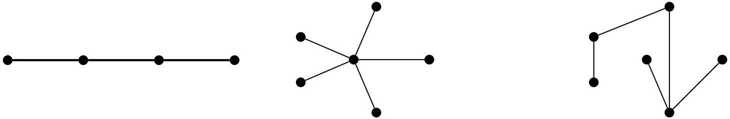

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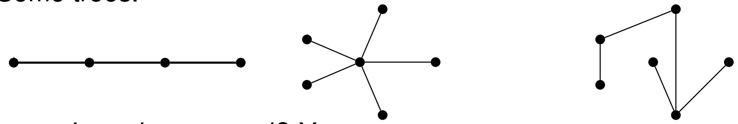

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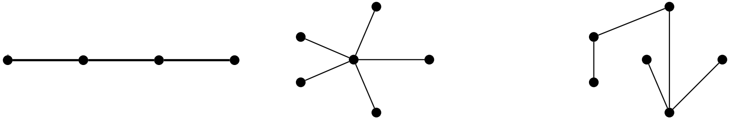

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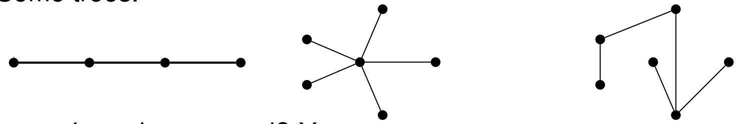

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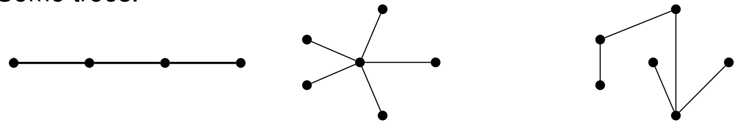

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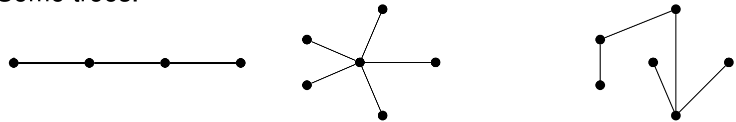

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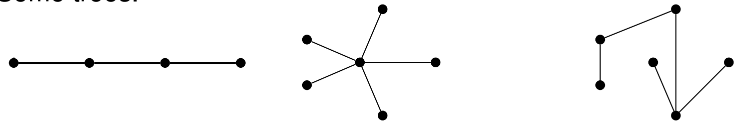

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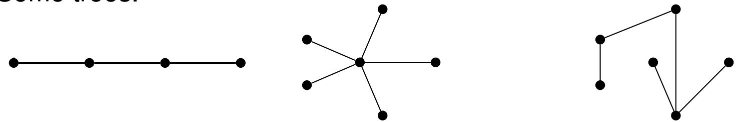

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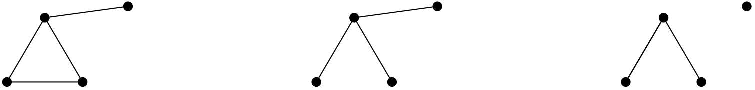
To tree or not to tree!



Equivalence of Definitions.

Theorem:
"G connected and has $|V| - 1$ edges" $\equiv$
"G is connected and has no cycles."

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Lemma: If v has degree 1 in connected graph G, $G-v$ is connected.
Proof:
For $x \neq v, y \neq v \in V$,

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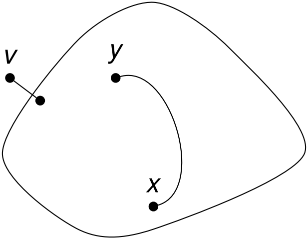

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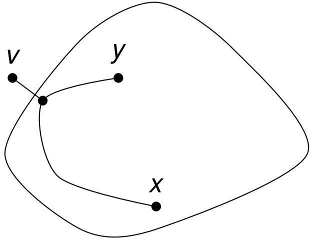
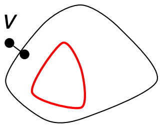


Proof of only if.



Thm:
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Proof of $\implies$:

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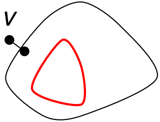
Proof of $\implies$: By induction on $|V|$.



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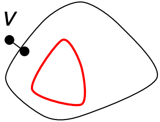

Proof of $\implies$: By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

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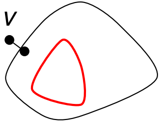

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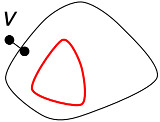

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Induction Step:

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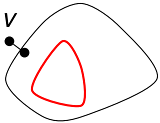

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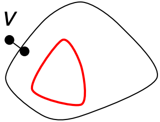
Claim: There is a degree 1 node.

Proof of only if.



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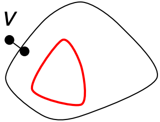
Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.

Proof of only if.

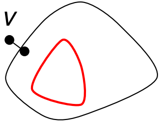


Thm:
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

Proof of $\implies$: By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
Sum of degrees is $2|V|-2$

Proof of only if.



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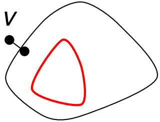
Proof of $\implies$: By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
Sum of degrees is $2|V|-2$
Average degree $2-2/|V|$

Proof of only if.

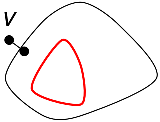


Thm:
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

Proof of $\implies$: By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
Sum of degrees is $2|V| - 2$
Average degree $2 - 2/|V|$
Not everyone is bigger than average!

Proof of only if.



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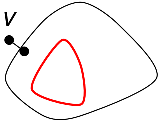
Proof of $\implies$: By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V|-1$ edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
Sum of degrees is $2|V|-2$
Average degree $2-2/|V|$
Not everyone is bigger than average! $\square$

Proof of only if.



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Proof of $\implies$: By induction on $|V|$.
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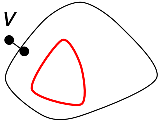
Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
Sum of degrees is $2|V|-2$
Average degree $2-2/|V|$
Not everyone is bigger than average! $\square$
By degree 1 removal lemma, $G-v$ is connected.

Proof of only if.



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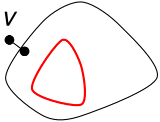
Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
Sum of degrees is $2|V|-2$
Average degree $2-2/|V|$
Not everyone is bigger than average! $\square$
By degree 1 removal lemma, $G-v$ is connected.
$G-v$ has $|V|-1$ vertices and $|V|-2$ edges so by induction

Proof of only if.



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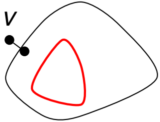
Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
Sum of degrees is $2|V|-2$
Average degree $2-2/|V|$
Not everyone is bigger than average! □

By degree 1 removal lemma, $G-v$ is connected.
$G-v$ has $|V|-1$ vertices and $|V|-2$ edges so by induction
$\implies$ no cycle in $G-v$.

Proof of only if.



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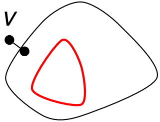
Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
Sum of degrees is $2|V| - 2$
Average degree $2 - 2/|V|$
Not everyone is bigger than average! □
By degree 1 removal lemma, $G - v$ is connected.
$G - v$ has $|V| - 1$ vertices and $|V| - 2$ edges so by induction
$\implies$ no cycle in $G - v$.
And no cycle in G since degree 1 cannot participate in cycle.

Proof of only if.



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And no cycle in G since degree 1 cannot participate in cycle. □

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
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Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
Proof:
Walk from a vertex using untraversed edges.

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
Proof:
Walk from a vertex using untraversed edges.
Until get stuck.

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.
Entered.

Proof of if

Thm:
"G is connected and has no cycles"
$\Longrightarrow$ "G connected and has $|V| - 1$ edges"
Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.
Entered. Didn't leave.

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge.

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
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Walk from a vertex using untraversed edges.
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"G is connected and has no cycles"
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Proof:
Walk from a vertex using untraversed edges.
Until get stuck.

Claim: Degree 1 vertex.

Proof of Claim:
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. $\square$

Removing node doesn't create cycle.

Proof of if

Thm:
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Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. □
Removing node doesn't create cycle.
New graph is connected.

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. $\square$
Removing node doesn't create cycle.
New graph is connected.
Removing degree 1 node doesn't disconnect from Degree 1 lemma.

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. □
Removing node doesn't create cycle.
New graph is connected.
Removing degree 1 node doesn't disconnect from Degree 1 lemma.
By induction $G - v$ has $|V| - 2$ edges.

Proof of if

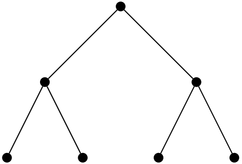
Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V|-1$ edges"
Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. □
Removing node doesn't create cycle.
New graph is connected.
Removing degree 1 node doesn't disconnect from Degree 1 lemma.
By induction $G-v$ has $|V|-2$ edges.
G has one more or $|V|-1$ edges.

Proof of if

Thm:
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. □
Removing node doesn't create cycle.
New graph is connected.
Removing degree 1 node doesn't disconnect from Degree 1 lemma.
By induction $G - v$ has $|V| - 2$ edges.
G has one more or $|V| - 1$ edges. □

Tree's fall apart.

Thm: There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



Idea of proof.

Tree's fall apart.

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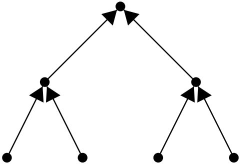
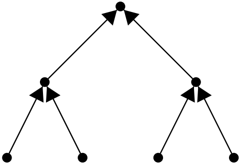


Idea of proof.
Point edge toward bigger side.

Tree's fall apart.

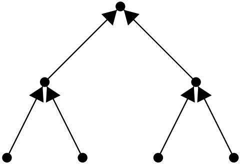
Thm: There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



Idea of proof.
Point edge toward bigger side.
    Remove center node.

Tree's fall apart.

Thm: There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



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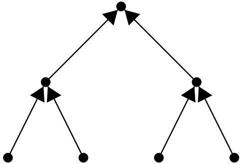

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Point edge toward bigger side.
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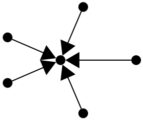

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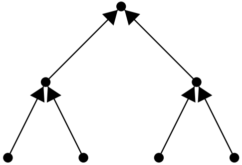

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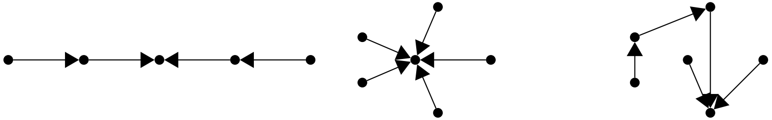

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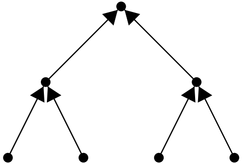

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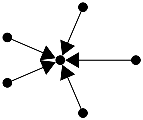

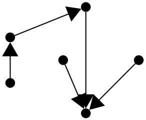

Hypercubes.

Complete graphs, really connected!

Hypercubes.

Complete graphs, really connected! But lots of edges.

$|V|(|V|-1)/2$

Hypercubes.

Complete graphs, really connected! But lots of edges.

$|V|(|V|-1)/2$

Trees,

Hypercubes.

Complete graphs, really connected! But lots of edges.

$|V|(|V|-1)/2$

Trees, few edges. $(|V|-1)$

Hypercubes.

Complete graphs, really connected! But lots of edges.

$|V|(|V|-1)/2$

Trees, few edges. $(|V|-1)$

but just falls apart!

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Trees, few edges. $(|V|-1)$

but just falls apart!

Hypercubes. Really connected. $|V|\log|V|$ edges!

Hypercubes.

Complete graphs, really connected! But lots of edges.

$|V|(|V|-1)/2$

Trees, few edges. ($|V|-1$)

but just falls apart!

Hypercubes. Really connected. $|V|\log|V|$ edges!

Also represents bit-strings nicely.

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$G=(V,E)$

Hypercubes.

Complete graphs, really connected! But lots of edges.

$|V|(|V|-1)/2$

Trees, few edges. $(|V|-1)$

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Hypercubes. Really connected. $|V|\log|V|$ edges!
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$G = (V, E)$
$|V| = \{0,1\}^d$,

Hypercubes.

Complete graphs, really connected! But lots of edges.

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Trees, few edges. $(|V|-1)$

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Hypercubes. Really connected. $|V|\log|V|$ edges!
Also represents bit-strings nicely.

$G = (V,E)$
$|V| = \{0,1\}^d$,
$|E| = \{(x,y)|x$ and y differ in one bit position.$\}$

Hypercubes.

Complete graphs, really connected! But lots of edges.

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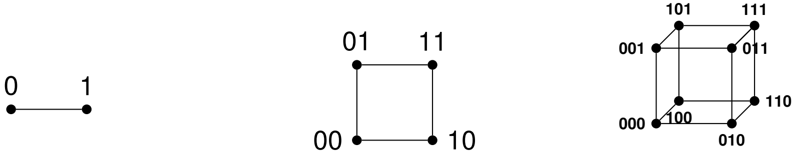

Dimension - d.

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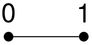

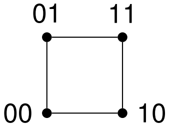

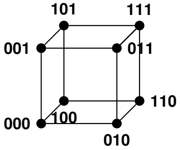

Dimension - d.

2^d vertices.

Hypercubes.

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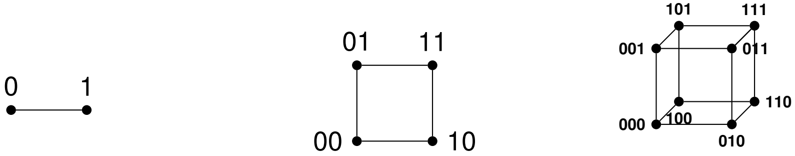
$|V|(|V|-1)/2$

Trees, few edges. $(|V|-1)$

but just falls apart!

Hypercubes. Really connected. $|V|\log|V|$ edges!
Also represents bit-strings nicely.

$G=(V,E)$
$|V|=\{0,1\}^d$,
$|E|=\{(x,y)|x$ and y differ in one bit position.$\}$



Dimension - d.

2^d vertices. number of d-bit strings!

Hypercubes.

Complete graphs, really connected! But lots of edges.

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Trees, few edges. $(|V|-1)$

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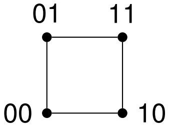

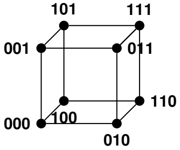

Dimension - d.

2^d vertices. number of d-bit strings!

$d2^{d-1}$ edges.

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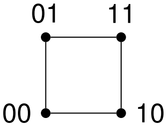

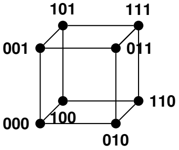

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2^d vertices. number of d-bit strings!

$d2^{d-1}$ edges.

2^d vertices each of degree d

Hypercubes.

Complete graphs, really connected! But lots of edges.

$|V|(|V|-1)/2$

Trees, few edges. $(|V|-1)$

but just falls apart!

Hypercubes. Really connected. $|V|\log|V|$ edges!

Also represents bit-strings nicely.

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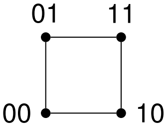
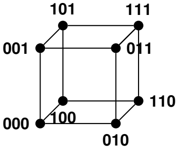
$|V| = \{0,1\}^d$,

$|E| = \{(x,y) | x$ and y differ in one bit position.$\}$





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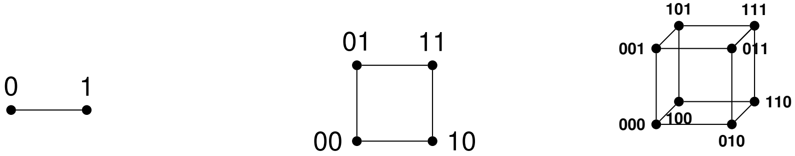

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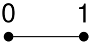

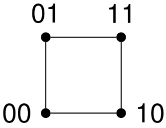

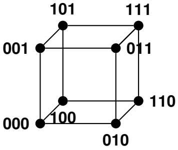

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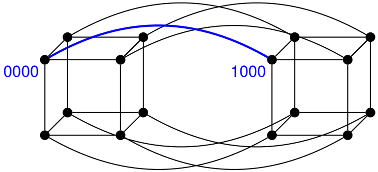

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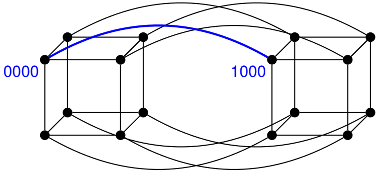

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Restatement: for any cut in the hypercube, the number of cut edges is at least the size of the small side.

Proof.

Next week.

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Have a nice weekend!